CONVERSATION THEORY

APPLICATIONS IN EDUCATION AND EPISTEMOLOGY

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Elsevier

AMSTERDAM–OXFORD–NEW YORK 1976

ELSEVIER SCIENTIFIC PUBLISHING COMPANY
335 Jan van Galenstraat
P.O. Box 211, Amsterdam, The Netherlands

Distribution for the United States and Canada:

ELSEVIER/NORTH-HOLLAND INC.
52, Vanderbilt Avenue
New York, N.Y. 10017

ISBN: 0-444-41424-X

Printed in The Netherlands

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Preface

The argument in this book aims to apply a body of cohesive and interpretable ideas, developed over the last dozen years or so, to issues of significance in educational psychology and epistemology. The history and development of these ideas, which emerged from experiments on perceptual motor learning, group interaction and sequential choice (as well as more obviously relevant studies of learning, subject matter structuring and cognition), are described in two previous books (Pask 1961, 1975a). But the main themes are crystallised in a monograph (henceforward called "the previous monograph"), Pask 1975b, *Conversation, Cognition and Learning*, which is part of the present series. In fact, the previous monograph marks a point of departure, for the notions cling together well enough to count as an empirically supportable theory: Conversation Theory.

Ideally, perhaps, *Conversation, Cognition and Learning* should be read first. But there are some 600 odd pages of it, including some lengthy appendices, and provided the reader will take various statements on trust, it is quite possible to start with this book. *Conversation, Cognition and Learning* can be regarded, with equal legitimacy, as an essay in man/man and man/machine symbiosis or as an essay upon education, learning and the like. In contrast, the present book is an application study and is unambiguously oriented towards the areas of education, its psychology and epistemology. The Introduction provides the essential groundwork, and for those who have read *Conversation, Cognition and Learning*, it bridges the gap between the two volumes.

Technical jargon has been minimised and examples have been stressed in order to increase readability. But it is also true that a good deal of fresh ground is broken. There has been progress both in the theoretical and empirical areas since 1973, and the picture which can now be drawn is more readily comprehended and rather more comprehensive. The theoretical and experimental work is focussed upon learning strategy and style, upon innovation and "learning to learn," and upon the representation of knowledge by teachers, students or subject matter experts. The enquiries in both areas lead to some novel perspectives and discoveries.

Right at the outset I would like to qualify this pretentious word "discovery". One of the lessons continually relearned by our research group is that most of the "discoveries" amount to a restatement (with suitable backup) of the intuitions and covert opinions entertained by well-informed educators; so their surprise value is less than it might be. For example, styles and strategies of learning and problem solving are known to exist: Understanding is often conceived as *some* kind of reproductive process. All we do, in this respect, is to assert that there are particular kinds of strategy and that an understanding is a particular type of reproduction; that is, to render the common belief explicit. The position is a little different in the epistemological arena; some of the comments upon the nature of knowledge are surprising and uncover an interesting cognitive pattern. Moreover, the *methods* used both for subject matter structuring and the detailed study of individual or group learning are (I think) genuinely novel and merit attention as candidates for general employment.

Another lesson we continually relearn is that originality is something of a snare if not a positive delusion. Other people have thought the same thoughts and sometimes done the same things while using different idioms and methods, which frequently obfuscate the unmistakable similarities. Some debts and dependencies were picked up in the previous monograph; in this book there is a determined and fairly systematic attempt to establish the proper linkages and set the work in the context of the entire field.

Sometimes this is a difficult task. Commonly enough one is unaware of an intellectual debt except in retrospect and this is especially true when the donor speaks from a different platform. For example, all system theoretic and information process oriented psychologists owe an immense amount to Craik, working chiefly with Bartlett; but the magnitude of this particular heritage only became evident when Craik's notes, essays and memoranda were edited for publication by Sherwood (Craik, 1966). By the same token most people concerned with knowledge and its representation have (often quite unconsciously) garnered ideas from Meredith (1966). It is clear, at any rate, that many of my own "original ideas" recapitulate the argument in his *Epistemics* and were probably born in discussion with the author some 15 years ago. Much the same comments apply to Grey Walter. Nearly everything worth saying that is said in this book about concurrency and local synchronicity (of a priori asynchronous systems) is contained in a prescient article (Walter 1956; the paper was presented in 1953) where the mechanisms in question underlie a phenomenon happily named "Abcission". Moreover, transplanted from cognitive studies into neurophysiology, the experimental methods he devised for displaying and quantifying this phenomenon are virtually identical with our own methods.

Other acknowledgements are quite easily tracked down. The likeness of conversation theory to the theoretical underpinning of the Vygotsky-Luria school and the Piaget school was evident from the outset but only became obtrusive after lengthy and illuminating discussions with Michael Huberman. Chapter 1 is mostly concerned with bringing the pertinent methods and techniques into register with the standard experimental conditions appropriate to conversation theory.

Substantial portions of the book were rewritten after a series of seminars and discussions with Gergely (a collaborator of Ivanhanko) and Nemeti occasioned by their recent visit to Great Britain; clearly, they and their colleagues are saying the same things (more elegantly from a mathematical point of view) insofar as they have consistently applied their concepts to social systems and the development of science and have pursued their research over more than a decade. In view of this fact it would plainly be impertinent to construct an ad hoc "string and sealing wax" calculus to replace well-developed notions. In using these notions, in grossly simplified form, as a cornerstone of the argument I hope I have neither misrepresented their position nor distorted a very beautiful theory. Their own books on the subject are in preparation.

I am particularly indebted to John Daniel both for helpful criticism and inspiring ideas; for example, that entire educational systems can be characterised on a par with individual or group learning (his remarks on the divergence between educational styles, Daniel 1974, is well worth consulting). Marvin Minsky's theory of "Frames" turns out, on discussion, to be more or less the same as our theory of concepts. Nicholas Negroponte, in many ways, is responsible not only for the basic ideation but for extending it to the wider horizon of design and architecture — quite apart from his role as tutor in how to implement man/machine interaction.

In the previous monograph I stressed the conjoint origins of much of this research and noted that it stemmed from the intellectual mandates of McCulloch and Ashby. That is still true. Most of the research lines have also been pursued simultaneously but more or less independently by Von Foerster and his collaborators (Loefgren, Gunther, Weston, for example). Over and above these dependencies which were mentioned in the previous monograph, this book owes a great deal to the fresh efforts of other colleagues. As often in the past, Prof. Brian Lewis and others at the Open University have commented upon and criticised the manuscript; Lewis has read it in detail and his revisions have been freely incorporated. I owe a debt to my associates at System Research (to the extent that this is really a compound document): in particular Robin Bailey, David Ensor, Dionysious Kallikourdis, Robert Newton, Elizabeth Pask, Valery Robinson, Bernard Scott, and Tony Watts. Most of the ideas have been refined and several of them instigated by faculty members (especially Laurie Thomas) and graduate students at Brunel University and at the University of Illinois at Chicago Circle (where a couple of chapters were written). There I am specially indebted to Professors Conant, De Fanti, Laxpati, Bruce McCormick, Manacher; to Ted Nelson and others in the Department of Information Engineering; to participants in the Applied Epistemology Seminars; for example, Sally Drogue, Professor McNeil, Professor Miller, Dr. Joe Lipson, the Tiemans, and to Larry Leske, Dave Douglas, and Randy Walser. Laxpati, Leske, Douglas and Walser made detailed and valuable criticisms of the manuscript; in addition they have realised an implementation of several operating systems within a slightly modified form of PLATO.

I would like to thank Isaac Haissman of System Research for scientific editing and the preparation of an index.

By a conventional impropriety the most important people come

last. Our research group is a Social Science Research Council Research Programme: "Learning Styles, Educational Strategies and Representations of Knowledge: Methods and Applications," and the research is carried out at System Research Ltd. For the most part this book is an account of this programme, its ambitions and its achievements (occasionally it goes beyond our brief though not, perhaps, our endeavours). These patrons are not only sponsors but valued advisors.

Linda Barsby has typed manuscript drafts repeatedly, corrected them and often the author. Bernard Scott and Robin Bailey have read it and Scott is responsible for the detailed referencing.

Gordon Pask

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Introduction

The previous monograph (*Conversation, Cognition and Learning*, Elsevier, 1975) deals with the history and implementation of techniques designed to exteriorise cognitive operations, especially those of learning and of teaching, so that they can be observed as segments of dialogue and behaviour. One method of exteriorising cognition is to engage in a verbal conversation, with a learner for example, and to *discuss* the way he learns *as* he learns.

This method has several obvious defects. The dialogue interferes with progress. The experimenter loses his status as an external observer, since he participates in and biasses the learning process. Natural language expressions are hard to interpret and may be inherently ambiguous. Even so, the amount of information about mental events which can be obtained by this means greatly exceeds the amount obtainable by the classical type of stimulus/response or input/output experiment. In fact we proposed that as the classical type of experiment is improved to approximate the ideal (the respondent is isolated in controlled and replicable conditions), the information available to an external observer regarding conscious operations will decrease very rapidly to the vanishing point. Conversely, the information about conscious operations is maximised by establishing an appropriate kind of dialogue which is overlooked by an external observer.

Some of the difficulties mentioned in the last paragraph can be surmounted. For example, it is possible to distinguish the roles of external observer and participant experimenter; the observer gives instructions to a participating agent (how to act and what to discuss), after which he looks on dispassionately. The agent in question may, for many purposes, be either a human interviewer or a mechanised system. Much of the argument in the previous monograph was couched in terms of a mechanised system (CASTE: "Course Assembly System and Tutorial Environment"): partly to make a clear distinction between what can and cannot be mechanised and partly as a practical expedient (human beings are unable to sustain the role of participant experimenter if the conversation ranges over sizable subject matter areas, if the "instructions to the agent" are precisely obeyed and if transactions are to be recorded).

To meet another objection, the conversational language need not (for many purposes) be natural language. The conversational language (henceforth designated L to distinguish it from the metalanguage L*, employed by an external observer to *talk about* the conversation) may be a graphic or non-verbal symbolic language. Certainly, L must be quite a rich language. For instance it must be a programming language as well as a descriptive or assertoric language; there are genuine L questions and L commands (not just formal surrogates for questions and commands); L statements must refer to *persons* "I" and "you" as well as objects; L must have an unusually liberal interpretation or *semantic*. Even so, it is often possible to realise the L transactions as sequences of concrete operations and in that case to replace verbal utterances by behaviours which can readily be computer monitored and recorded.

Some caution is needed when using the word "behaviour" in this context. The necessary caveats were stressed in the previous monograph to produce an almost obsessive notation in which behavioural terms like "stimulus" and "response" were generally eschewed. Having made the point, it is legitimate to relax the nomenclature provided that the behaviours attending L transactions are recognised as *many sorted*. (In contrast, the most extreme forms of behaviourism view behaviours as *one sorted*; a precondition for synthesising complex entities out of simple ones, or conversely, for an atomistic analysis of complex behavioural events.) To illustrate the *many sortedness* of behaviour we should distinguish between simple behaviours (causally, albeit probabilistic-causally determined) which are the *one sort* of classical behaviours; model-building or rule delineating behaviours (a sort of behaviour that delineates an explanation or a demonstration); and *learning strategy* behaviours which represent, by a concrete tracing, how an explanation is derived from other explanations.

Depending upon the form of L, there are many types of dialogue which will exteriorise mental events and they are graded or typed in a series extending from free natural language dialogue, via restricted natural language dialogue, to situations in which L transactions are mechanised. All of these types are called *conversations*; the necessary experimental methods are called "conversational techniques".

The objection which cannot be eliminated, whichever technique is used, is that any conversation takes place within a contractual or normative framework. The respondent *agrees* to engage in the conversation, for example, in order to learn about a subject matter, and this agreement or contract is *negotiated* in natural language L*, though it may also be *expressed* in L. Further, the participating agent, either man or machine, biasses the conversation: literally an external observer looks at a *conversation* not at unfettered response (whatever that may be). In aggregate, these objections are not very serious. The price paid for observation is no greater than the price paid in a classical experiment though the biasses and constraints are manifested differently. Moreover, at least in systems like CASTE, the amount and type of bias can be estimated *after the event*, though it cannot be accurately predicted beforehand.

We now come to the underpinning contention of the previous monograph. Psychological phenomena, especially those involved in learning and education, stem from or are related to states of *consciousness.* Using the argument which relates the information available about conscious processes to the type of experimental situation, we maintain that the basic unit of psychological/educational observation *is* a *conversation.* In order to test hypotheses and explicate the conversational transactions, it is necessary to invoke various tools and explanatory constructs. These are coherent enough to count when interlocked as a theory, and this theory was dubbed *conversation theory.*

1. PREREQUISITES

Certain prerequisites are demanded of any worthwhile theory.

1.1. Observation

To begin with, there must be at least one *standard condition* for measurement and observation; other conditions, usually less restrictive, being systematically derived from it. The standard condition of *conversation theory* is called a *strict conversation* and it is possible to instruct or program participating agents so that if *any conversation takes place,* then it is certainly a *strict conversation.* The main features of a strict conversation are as follows:

(a) The participants, as part of a contract, agree to obey the rules of the conversational language L and the participating agent makes sure that the L syntax is respected.

(b) The conversation is focussed, or anchored, upon a conversational domain: typically, a representation of the topics in a subject matter.

(c) The conversational domain involves a particular and canonical type of representation, both of what may be known and what may be done or discussed: hence, conversation theory has an epistemological commitment, and about half of this book is devoted to an exploration of what this commitment is.

(d) Each topic, said to be learned or assimilated in a strict conversation, is understood.

(e) In this connection *understanding* is given a theory specific and technical connotation though the imputed meaning tallies with and probably amplifies the usual meaning. We say that a topic T is understood by a participant if and only if T is explained and if T can also be derived from other topics in the conversational domain, i.e., a derivation is an explanation or systematic justification, *of* an explanation. It is crucial that understandings can be detected.

The explanation need not be verbal. If not, then it is called a model-building operation and is a satisfactory explanation insofar as the model can be executed in an external facility to bring about the formal relation underlying topic T. Nor need the derivation be verbal. If not, it is a *learning strategy* (a concrete depiction of one or more topic derivations).

(f) A strict conversation is punctuated by *understandings* and the intervals occupied in reaching an understanding are called occasions.

1.2. Framework admitting inference

Another prerequisite for admissibility is that a theory shall have predictive power and that its predictions can be empirically falsified when tested under the standard condition. The predictive capabilities of conversation theory chiefly emerge from psychological or systemic postulates introduced in order to furnish a mechanism of understanding.

The critical mechanism-postulates developed in the previous monograph are as follows: Concepts and memories are regarded as dynamic constituents of the mind. Specifically a concept is regarded as a procedure that realises or satisfies a topic and the topic itself is an interpreted (formal) relation. For generality, we say that a concept is a procedure that reconstructs or reproduces a topic (T). By virtue of this definition it is natural and in line with ordinary language usage to assert that a memory is a procedure that reconstructs or reproduces a concept. We contend that stable concepts, for all practical purposes the concepts existing in a mental repertoire, are those which can be reconstructed or reproduced by at least one (usually many) memory-procedures in the same repertoire. It follows that learning is an evolutionary type of process in which concepts and memories are constructed, *ab initio*, and an understanding signifies the generation and existence of a stable concept, i.e., a concept associated with a memory which either exists or is created in the process.

These definitions fit in quite neatly with the events observed in the conduct of a strict conversation (which is not surprising since the postulates were advanced as plausible and worthy of serious consideration just because certain kinds of dialogue can be observed). Notably, if we looked at the execution of a concept inside some processor (programmable computing system) such as a brain, then the reproduction of a concept would appear as a cycle or series of execution steps and the instructions making up the procedure as a "listing," i.e., a series of linked statements which specify the intention or rule (in the same way that a computer program specifies a rule which, on execution, performs a computation). Similarly, if we looked inside the processor, the reconstruction of a concept (by a memory) will be manifest as a cycle or series of execution steps and the memory itself, as a series-like "listing". All this is a straightforward consequence of regarding concepts and memories as replicative and reconstructive operations which can be described, in the abstract, in terms of several theories of self-reproducing automata. (Much of the previous monograph was concerned with the hedges and conditions needed to fit existing versions of self-reproducing automaton theory to mental reality; for example, mental operations are not generally serial or completely synchronous.)

Suppose that the cycles which might be examined by probing inside a brain, qua processor (call it α) are literally pulled out so that some of the cycle is executed in α as before and some of it in a distinct brain or processor called β. Under these circumstances, scrutiny of the interaction between α and β will expose the cycles to view. In particular, there will be one cycle corresponding to the execution of a concept, one to the listing of the concept and others corresponding to the listing and the execution of a memory. These are identified with stretches of dialogue or behaviour, as follows: the execution of a concept (to realise or satisfy a topic) is an exemplification (dialogue term) or a simple behaviour; the listing of a concept is an explanation (dialogue term) or a model-building operation (behavioural term): the memory cycle is a derivation (dialogue term) or a *learning strategy* (behavioural term).

A strict conversation gives rise to a series of transactions that are characterised as occasions, insofar as each topic learned is associated with an *understanding* (in its technical sense; a linguistic event involving the explanation and derivation of a topic or the construction of a satisfactory model for a topic within the framework of a learning strategy). α and β figure as the brain of a participating respondent and either the brain of a participant experimenter or a suitably programmed mechanical agent. The observable event of understanding is held to signify or evidence the construction of a stable concept due to a very specific kind of cooperative interaction between the conversing participants.

The circumstances under which cycles of explanation and derivation may be "pulled out" or (equisignificantly) "exteriorised for external observation" are precisely those set up by the contract and conduct of a strict conversation. In particular the learning participant must have a need to cooperate (implicitly identified with "procedure sharing" or "program sharing") in order to learn the topics in a conversational domain, which he has agreed to do in the initial contract. The other participant must be in a position to provide this cooperation and foster understanding. Finally, insofar as "procedure sharing" or "program sharing" depends upon local synchronisation of the brains or processors involved, the occasions of a strict conversation are intervals of partial synchronisation between the participants during which they both attend to the same topic. Notably, such occasions are rare in nature. Brains, unlike computing machines, are not a priori synchronised by a master clock and it takes an act of attention (one type of "provocative transaction" noted in the previous monograph) to secure synchronicity.

A satisfying relation is thus established between the dialogue or behaviour of a strict conversation and a fundamental notion of information-transfer, due to Petri (1964). According to Petri's argument, information-transfer takes place if and only if two or more dynamic systems which do not have a common master clock (i.e., a priori asynchronous systems) come into local synchronicity (local with respect to topics in the conversational domain). Under this interpretation, the *occasions* of a strict conversation are, as they intuitively should be, indicators of information transfer.

1.3. Observable Units

We pointed out, in the previous monograph, that a theoretical framework of this kind permits several alternative definitions of a participant. Which definition is adopted is, to some extent, a matter of elegance and convenience.

Two perfectly valid alternatives are as follows:

(a) A participant is identified with a brain able to act as a processor for L-Procedures (henceforward, an L-Processor). The brains (L-Processors; α and β) are spatio-temporally demarcated on biological or mechanical grounds, the usual criteria for isolating an integral object in the environment. If participants are identified in this way, they are mechanically individuated (for brevity M-Individuated) by the external observer and count as Mechanical Individuals or M-Individuals. * By the same token other parts of the

* For most purposes "biological Individuals" would be just as acceptable. However, the class of L-Processors is larger than the class of brains, and conversely, brains have functions other than L-Processing.

environment (usually having less computational versatility than an L-Processor) can be M-Individuated, for example, various components of CASTE or any other design of experimental situation.

(b) A participant can be identified with the set of stable concepts that are, or may be, part of his mental repertoire. To obtain this characterisation of a participant it is only necessary to extend the sequence of formulations "concept, memory, . . ." until it is possible to answer the question, "what reproduces the memories that stabilise the concepts, thus yielding a unitary and recognisable repertoire." Since the answer to this question consists in a series of interlocking and compatible L-Procedures that are executed to realise a system of coherent beliefs or hypotheses, we say that the external observer has psychologically individuated (for brevity P-Individuated) the participant. If the constituent procedures are actually executed in some L-Processor, the participant is characterised as a Psychological Individual,or as a P-Individual. Although there must be some L-Processor to realise a P-Individual, we need not dogmatise about which processor it is (α, say, or β) and it often turns out to be impossible to do so. In this sense, P-Individuation is "processor independent". For example, the strict conversation is a P-Individual and is the direct object of observation. The participants, call them A and B, form the factor P-Individuals (A, B) of the conversational P-individual. Clearly, the execution of the conversational P-Individual is distributed (by "procedure sharing") over the M-Individuated processors α, β and its factors may be. This identification scheme also accommodates such obvious and important internal (and *not* directly observable) conversations as "thinking to oneself" or "learning on one's own account" (the coexistent execution of A and B in one brain, α) and group learning (where A, for example, is distributed over several brains α, β). As hinted already α, β need not necessarily even be brains (there are some inanimate L-Processors).

Although either formulation is legitimate the P-Individual is usually a more convenient unit for conversation theory; for example, a strict conversation is a protypical P-Individual and the use of this formulation avoids a number of puzzling pseudo questions like "where did the concept come from?" or "which brain does it belong to?"

1.4. Changed Emphasis in the notation

In this book we do not make much explicit use of M-Individuation though such an act is implicit whenever brains are considered as distinct and recognisable entities. Moreover, the discussion often rests upon entities that are identified by M-Individuation: notably L-Processors (human brains and certain inanimate systems) and modelling facilities. The latter are vehicles in which models are manufactured as non-verbal explanations and (facilities of a distinct kind) in which derivations are reified as learning strategies. All modelling facilities have a dynamic component, they are computers and execute the models built in them to realise or satisfy relations: however, they are much more restricted computers than a human brain.

P-Individuation is, however, used quite extensively and the P-Individual, as a unit, is ubiquitous. As in the previous monograph, a P-Individual is realised by execution in an L-Processor and, generally, one or more L-Processors are assumed to be available. If that is not the case, we distinguish (notably in Chapter 11) between the representation of P-Individuals A, B written Π_A, Π_B and the P-Individuals undergoing execution (just A, B, simpliciter).

Since a great deal of the argument is concerned with the creation and learning of analogy relations of a much more general and useful kind then those discussed in the previous monograph, we often need to pay special attention to the interpretation of a topic (its realisation in some universe, in contrast to the formal systemic or syntactic topic relation). This trend penetrates to all levels of the argument and motivates a change in notation, though not in principle, from the standards established in the previous monograph. In order to treat interpretations and analogies intelligibly, it is desirable to discriminate between programs as syntactic entities, the compilation of programs (the configuration set up in an L-Processor or any other computer, which is open to execution), and the execution of the compiled program.

Whereas before, programs written in a modelling facility (as non-verbal explanations) were identified piecemeal with models as "compiled programs, compiled in the modelling facility," it is now more expeditious to distinguish the program (i.e., the listing as a syntactic entity, to call "the program compiled in a modelling facility" a *model*, and to consider the execution of the model.

This usage accords with recent theories of semantic interpretation that are usually called "model theory". *

Under these circumstances, the notation Exec (used in the previous volume to designate the execution of a procedure in order to produce a program *listing* which was compiled in an external modelling facility to produce a model) is positively misleading and is herewith discarded. Its main virtue, in any case, was to complement the explanatory response Expl. Throughout this book (unlike the previous monograph) we speak explicitly of non-verbal explanations as the production of program listings which represent the class of programs making up a concept. We refer to the compilation of such representative programs in a modelling facility as *models* and, when necessary, refer to the execution of models in a modelling facility (under the control of the facility and not under cognitive control). This brings the argument back into kilter with the previous monograph. But the revision admits a relatively uncomplicated account of analogy relations and their models (the non-verbal explanation of analogies), a topic which often dominates the present discussion.

Corresponding notational adjustments are required in respect of concepts, memories and P-Individuals. Whereas, in the previous monograph, these were regarded piecemeal as procedures under execution in an L-processor, it is now expedient to discriminate a syntactic component of each entity (its formal specification) which is called a program, a Procedure being a compiled program. Hence a concept is respecified as the stable compilation of a program in a brain or other L-Processor; a memory as the stable compilation of a different kind of program, and with one caveat, the P-Individual as the stable compilation of properly adjoined programs. Hence, concepts may be selectively executed provided they are stabilised (as compilations) by memories. Similarly, memories may be selectively executed to stabilise concepts. For the P-Individual there is an additional requirement; namely, that some of its programs (compiled as concepts and memories, in general, as procedures) are invariably undergoing execution.

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* Until recently model theory was mostly concerned with static models. In contrast the current argument is almost exclusively focussed upon dynamic and executable models (i.e., compilations of programs of one kind or another either in brains or mechanical artifacts).

We refer, throughout, to L-Processors (in which concepts, memories and P-Individuals are executed as programs and procedures). This usage may *seem* to be (in fact it may *be*) eccentric since an L-Processor is nearly always a brain. But on balance it has value insofar as it does bring home the following facts: (a) An L-Processor may be a brain or a collection of brains or a man/machine system, without prejudice; (b) Not all of the brain can act as an L-Processor and a brain has other functions to perform.

1.5. Testability of Postulates

Conversation Theory should have predictive power and be open to falsification under its standard condition, the strict conversation. It indubitably does have predictive power and its predictions are open to falsification. For example, we predict that the concepts of understood topics shall be indelible within one conversation if it is anchored upon one conversational domain, and that they should be relatively resilient to the interference effects encountered if they are recalled or executed in a different and perhaps incompatible conversational domain. We also predict the existence of classes of learning strategies which become mutually exclusive in a strict conversation, for example, the previous monograph emphasised holist learning strategies and serialist learning strategies. A fair body of empirical evidence, supporting these and more subtle hypotheses, is collected in the present book, and the tenure of hypotheses in conditions that deviate from the standard conditions is examined at some length because many educational situations *do* deviate quite markedly.

On the whole the salient hypotheses are supported by the experimental data and many of the tenets of conversation theory continue to hold (sometimes with modification) under circumstances that are very realistic (and often very deviant). It is wise, however, to stress the status of conversation theory and to consider what it can and cannot be expected to do.

1.6. The Scope of Systemic Theories

As outlined so far conversation theory is a systemic microtheory or molecular theory. It refers to concepts, memories and the like manifest in detailed transactions: either stretches of dialogue, or stretches of many faceted behaviour. Although the theory is detailed and mechanism-oriented, the mechanics are systemic (i.e., patterns of organisation) and the theory is not intended to discriminate particular biological processes (for example, any or none of the very different memory mechanisms proposed by Bogoch, John and Ungar and discussed by Libassi [1974] may be responsible for the compilation of our "memories" or our "concepts"); the theory is neutral on this score.

To some extent, this degree of neutrality is maintained with respect of functional distinctions as well. To illustrate the point, consider the learning theory proposed by Atkinson and Shiffrin (1965, 1967). This theory provides a bridge between statistical learning theories and the "artificial Intelligence" approach to mental activity (for example, Feigenbaum 1959, Feigenbaum and Simon 1962, or Simon and Feigenbaum 1964). It posits a structural demarcation of storage media; a sensory buffer, a short-term store and a long-term store, with appropriate connections. Although these storage locations have specific properties and capacities they are functional loci; not (except by indirect inference using other evidence) sites in a brain. Within these locations and the constraints they impose, control processes, which are identically "compiled programs or procedures, undergoing execution" set up and manipulate data structures — for example, a rehearsal buffer is maintained in short-term store and other control processes, which generally compete with rehearsal buffer operation, select symbols for acceptance into the rehearsal buffer.

Of course, this is also a *systemic* theory. However, its validity (there is strong evidence that it provides an excellent picture of short-term storage, at any rate) neither confirms nor denies our theory, or vice versa. Notably there *might* be competition; it simply happens that some mechanism of this type is mooted as part of our own theory and almost any mechanism would do. Moreover, the detailed transfer patterns (between structurally demarcated compartments) are represented statistically as a result of which the content of the theory neither confirms nor denies the kind of cyclic reconstruction we posit (though the rehearsal buffer process could surely be regarded as *one example* of a cyclic reconstruction process).

1.7. Comparison between conversation theory and other systemic theories

In the previous monograph, Chapter 11, we noted that a macrotheory or molar level conversation theory is possible and some effort was made to relate subjective uncertainties (sampled by confidence estimates and the like) to the activity of mental systems. The macrotheoretic variables are, or are derived from, degrees of doubt and certainty. We distinguished in particular, certain *kinds* of doubt: d_a or doubt about what topic is being attended to; d_1 or retrospective doubt, given that a topic is understood, of which one of several methods (all belonging to the topic's concept) *is used* to solve problems posed under the topic on a particular occasion; and d_2, or prospective doubt, given that a topic is in the field of attention but is not understood, about alternative outcomes or solutions to be obtained by applying the existing, and perhaps partially formed, concept. Moreover, we specified a "look ahead" uncertainty; namely, a doubt, given that *one* learning strategy *must* be selected from a set of possibilities, about which one will be selected.

All of these quantities are measurable, and from time to time, we take advantage of this fact. Also, at the macrolevel, conversation theoretic predictions do mingle with the predictions of other information processing and systemic theories whenever the experiments are comparable. So far as we can see (and there is, as yet, rather little experimental overlap) our own results are in accord with those of other researchers. This is especially true in the context of recent results on the perceptual and cognitive psychology of recall and recognition, a body of data far richer than our own limited scope experiments. Though I have not attempted to do so in this book, it appears that our findings can be transformed, by change of idiom and context, into substantial agreement with these results (e.g., the *Attention and Performance* publications).

Results from experiments in conversation theory, as the theory stands at the moment, are directly comparable with results from information processing theories and the psychology of "Decision Formulation" (that is "Decision Making" insofar as it refers to heuristics or mental operations, rather than the art of weighing up alternatives).

Some representative information processing theories are those of Broadbent's later work (in and after Broadbent 1971), or of

Cohen (1964, 1972) on subjective probability and choice tactics; theories of cognitive mechanisms (for example, Conrad 1974); the work of Daniel (1974) or Dirkzwager (1974) and his group (both the latter include replications).

It is quite possible that conversation theory can be developed to yield predictions/data compatible with the psychology of more elementary information processes; underlying the kind of cognition/behaviour in Welford's (1968) summary of the field, earlier with Broadbent and the Cambridge Applied Psychology Unit, or even the "signal in noise" treatment of perception and recognition pioneered by Tanner and Swets (1954). In order to bridge the gap between complex phenomena such as understanding and elementary mass phenomena (signal detection in a noisy background), it is necessary to provide a statistical treatment of memories, concepts, etc. This turns out to be a statistical mechanics (with some peculiarly psychological features) in which the dynamic systems making up the canonical ensemble are P-Individuals. On interpretation, the members of the ensemble may either represent students in a class (when the condition of the ensemble represents a state of general knowledge) or factor P-Individuals in one student (when the condition of the ensemble represents a state of knowing). Work in this direction has just started and parallels very closely the approach to the regulation of cellular metabolism adopted by Goodwin (1963). In Goodwin's case the dynamic systems are basic units involved in enzyme production (DNA, RNA, ribosomes and feedback from products produced by the action of the synthesised enzymes). Hence the equations of the dynamic systems are quite different and their often oscillatory interaction has a different form. But, in other respects we encounter very similar difficulties and insights. At least the approach is a workable and potentially useful way of viewing mental activity, and it is at this level that direct comparison between conversation theory and the statistically interpreted structural theories is logically sensible.

To illustrate "Decision Formulation," where complex mental operations, heuristics, and the like are in the foreground, we cite the work of Tversky and Kahneman (1971, 1973), of Philips (1973), or Edwards (1968). The only difficulty in comparing hypotheses or results is that "Decision Formulation" theorists generally concentrate upon the use and nature of heuristics, concepts, or whatever, whereas conversation theory is generally focussed upon their development.

Entwistle (1975) points out that quantitative information theoretic and decision theoretic methods could with advantage be employed in educational psychology, and it is clear from his paper that he means methods which are founded upon structural principles or mechanisms and consequently have a commitment to information processing. These methods are based on systemic theories (on a par with the examples just cited) and, although information measures are used as a common currency (as they are in any application), the methods are inherently more powerful than "information theory" used only as a metrical device. Entwistle's reasoning would (in our view, it *should*) find general acceptance. The trouble is that few relevant studies of this kind have yet been published though many of them are in progress.

Conversational domains (and, with them, the epistemological aspect of conversation theory) are also represented in systemic terms. Comparison with other work is relatively easy; in fact, an almost embarrassing number of comparisons are possible (many noted in the previous monograph and some to be introduced). For example, both data base design (at one extremity) and the semantic networks and data structures of cognition science (at the other) have features in common with our own formulation.

Probably the chief differences between conversation theory and other systemic theories are as follows: Conversation theory is explicitly relativistic; this is evident on inspecting the standard condition. Measurements are made relative to a conversation, or of one participant, relative to another, in the context of a conversational domain. Most of the other theories do not make the point explicitly, though some of them probably involve relativistic estimation. For the other outstanding point of difference, conversation theory is, with the possible exception of some events in a strict conversation, overtly reflective. It permits personalised statements "I" or "you" not just impersonal statements about objects and makes an attempt to explicate their nature. This, of course, is part and parcel of our general concern with consciousness as the distinctively psychological phenomenon. *

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* The justification for relativistic and reflective theories is discussed at much greater length in Pask (1961) and in Pask (1975a), in particular, the development of pertinent experimental methods from studies of perceptual motor learning.

For psychology in general the merits of an orientation to conscious phenomena, to relativism and reflectivity are frequently debated. But whatever the outcome, it seems that a theory of this kind is required in order to deal with practical problems in educational psychology and the wider educational issues of course design, the structure of institutions and media, and the origin of creativity and innovation.

1.8. Unification

An incidental but valuable claim for conversation theory is that it unifies a number of psychological theories which otherwise appear entirely different. In the previous monograph, we examined several representative schools of thought in this light and tried to show the points of systemic identification between Personal Construct, Information Processing, Cognitive, Transactionalist, Behaviourist, and other psychologies by mapping them onto a conversation theoretic image. The present book goes a good deal further. On the one hand, the argument extends the domain of application from educational psychology to epistemology. On the other hand, the argument unifies various essentially conversational techniques (thus acknowledging the roots of conversation theory) and various theories of thinking, innovation, social learning and development.

2. A PLAN OF THE BOOK

Chapter 1 provides a survey of other conversational methods (Piaget, Vygotsky, Luria, for example). Although the present theory was developed independently (deliberate isolation in an attempt to integrate ramifying thoughts), it owes whatever value it has to precedents established in the culture and we try to trace the real origins, in retrospect. We also take the opportunity to describe the operating systems used in the experimental work: INTUITION (a transportable modification of CASTE, used in schools) and several others.

Chapter 2 very briefly reviews the structure of conversational domains as set out in the previous monograph, but most of the material is novel; we report work that has been done since 1973 to yield an enriched and more generally useful product. In particular, the notion of an analogy relation is broadened (whilst the analogy is still represented systematically in a conversational domain). The significance of this manoeuvre is partly epistemological and partly practical. We posit that the rate of learning is materially influenced by the number (or density) of analogies a learner can appreciate, the quality of learning by the number of valid analogies that the learner comes to understand.

Chapter 3 reports a number of recent studies to do with learning strategies and styles; in the light of these results the holist/serialist distinction of the previous monograph is seen as an important but special case of more fundamental and pervasive mental processes.

Chapter 4 is concerned with theoretical developments bearing upon agreement and understanding and also upon the character and origin of analogies as "petrified agreements". The discussion in this chapter hinges upon independent work in two main fields; non-classical model theory and the coherence theory of truth. Both fields appear to be of the utmost importance to any rational theory of education—conversation theory or any other theory.

Chapter 5 furnishes a series of condensed notations or schemes for the description of learning. By adopting these notations, it is possible to avoid a great deal of symbolism (such as the symbols for complex transactions used in the previous monograph) whilst remaining in a position to describe the types of learning discussed in Chapter 3 and the acts of invention discussed later in the book.

Chapter 6 introduces the topic of conversations in which there are two or more simultaneous foci of attention, either on the part of several coupled participants (a group) or just one participant (a transient phenomenon, believed to underpin innovation).

Chapter 7 contains a description of a course assembly system, THOUGHTSTICKER, much more versatile than EXTEND (of the previous monograph) in which one or more subject matter experts maintain distinct foci of attention, from time to time, whilst building up a conversational domain.

Chapter 8 is also concerned with THOUGHTSTICKER but especially with transactions that lead to innovation.

Chapter 9 is devoted to an argument relating the art of course assembly as it is practised by experts (delineating knowables, constructing a conversational domain), to the art of "learning to learn," as practised by students. We maintain that "learning to learn" is a crucial accomplishment and that a student who can do so effectively is (amongst other things) able to impose a personal structure upon otherwise unstructured information or upon an often perversely structured environment. Experimental data are cited to support this view.

Chapter 10 makes explicit a theory of creativity and innovation developed at various points in the preceding discussion and shows its relation to several other theories of innovation. It appears to tally with them all but is, in a systemic sense, more general (i.e., in this sense, it encompasses them as special cases suited to particular kinds of innovation).

Chapter 11 is speculative. It deals with work in progress and sometimes far from completion. But the issues addressed, such as characterisation or dramatisation, the nature of the media, the scope of developmental studies, strike me as fascinating and I hope the reader will find some of the novel perspectives both interesting and useful.

In conclusion, there is one general caveat. By disposition, I like to think as a philosopher (or a philosophical psychologist). To justify this mode of thought and to implement the conclusions experimentally, it is often helpful to build physical systems (INTUITION and THOUGHTSTICKER, for example). Under some conditions these are essential experimental tools, under other conditions they are valuable tutorial devices. Often, however, it is possible to realise the principles derived from experience with these systems in human terms, with human teachers in a classroom, subject matter experts working in a group, and in various other ways involving no machinery at all. On balance, we believe that most if not all of the findings and principles discussed in this book can be employed without invoking machinery (even though the discussion itself is machinery laden). Such non-mechanical implementations are usually of greater practical significance and may even be inherently more effective.

Chapter 1

A Comparative Survey of Conversational Methods

1. INTRODUCTION

The basic theme underlying a conversation theory has been voiced repeatedly. There are precedents for many of the tactics adopted in the work of Piaget and (independently though contemporaneously) Vygotsky (see, for example, Piaget's 1962 *Comments* upon reading the English translation of *Thought and Language*). These pioneers, their colleagues, and students, including Inhelder, Papert, Luria and Minskaya, developed conversational methods for probing, observing and exteriorising cognitive events which normally remain concealed. All the techniques rely upon a participant experimenter in the role of a tutor, an interviewer or an interrogator; in each case, of someone who shares in the mental activity of the respondent. Two special methods are representative of their studies: the "paired experiment" and the "questioning interview", and two aspects of these methods are of special interest: the elicitation of explanatory responses and the representation of thoughts and discoveries.

1.1. "Paired Experiments" and Concrete Operations

The "paired experiment" is a paradigm chiefly exploited by the "Russian school". A respondent faces a problem situation in concert with the participant (who is there to aid, abet, provoke and encourage the respondent, as well as to record what goes on). The problem situation is embodied in a physical artifact such as a puzzle or a mechanical gadget. Whatever the artifact may be, it is jointly perceived by the participants (respondent and experimenter) and is open to external observation. The experimenter poses problems, some of them designed to place insuperable obstacles in the respondent's path, concerned with the function of the artifact or extensions of the artifact. The respondent replies either verbally or by manipulating the artifact. In a typical session the questions are "How" and "Why" questions and the answers, if forthcoming, are explanations or constructive responses that refer to the artifact or a conceivable modification of the artifact. Insofar as some enquiries are designed to pose unsolvable problems, there are occasions upon which the respondent appeals for help and the experimenter then performs a demonstration or points out a principle or suggests some way in which the artifact could be modified to serve a different purpose. The immediately relevant point is that all statements, whether verbally uttered or not, can be interpreted either with respect to the problematic artifact as it stands or some other construction which could (at least ideally) be constructed from a similar apparatus. By this means, the participants are able to reach an agreement and the basis for their agreement is exteriorised for impartial scrutiny.

In the mid 1920s Piaget employed similar techniques; children (the participating respondents) focussed their dialogue upon physical situations. Though experimentally convenient, such an arrangement may also hamper flights of fancy and imagination which are just as important constituents of thinking as sober minded essays. Hence it was noted (and similar comments recur, from time to time, in the literature) that the physical realisation is optional. Experience seems to have shown, however, that an anchor of some kind is nearly mandatory if the dialogue is to make sense; for example, Piaget himself stresses the importance of a concrete situation with metric rods, water jars, or whatever to reify abstractions like the conservation of quantity, area and volume. One line of argument lays emphasis upon the respondents' age. *Children* need to concretise their operations; the requirement for a manipulable artifact is bound up with a well-established developmental phase (concrete operational/formal reasoning). No doubt there is a great deal of truth in the suggestion that children must explain manipulatively because they are unable to give coherent verbal explanations (we return to this matter in Chapter 11). But the truth is almost certainly qualified. Age or developmental phase exacerbates a difficulty latent in any participant experiment, even using respondents old enough to reason formally and probably embarrassed by the requirement to map (for them) natural abstractions onto the manipulation of an artifact. Unless an intermediary exists and responses are referred to it, certain sorts of agreement are unachievable and certain (participant) agreements, even if achieved, are inherently ambiguous to an external observer.

It is worth investigating what this intermediary (so far, represented as an artifact) must be. Need it, for example, be a physical contrivance (puzzle, water jars, playing board)? Could it be something far less restrictive? An affirmative reply is furnished by a recently translated body of work by Landa (1971) which made systematic use of paired experiments (though the phrase "paired experiment" is not employed in the description).

Landa is concerned with the way that older children and adolescents learn the logic of sentance manipulation, subject to grammatical and semantic constraints. In particular, he is anxious to show that expertise depends upon knowing and using valid inference and exclusion principles represented as algorithms. At one stage in the discussion, Landa ponders over the question of whether he is teaching "grammar" or "logic" (he notes, for example, that grammarians might think it odd or even wasteful to incorporate logic in the syllabus). His conclusion is extremely telling. You cannot teach logic. You can only teach an interpretation of logic and one such interpretation is in the universe of grammatical transformations (other universes of interpretation include engineering systems or mathematical structures). Logic can surely be learned in any interpretation, conceivably a specialised variety of logic. It cannot be learned *in vacuo*. The converse argument also applies: Unless logic *is* learned there is *no* learning.

Clearly, "learning" is used in a special sense in this statement (and the statement is a terse accentuation of Landa's point of view). However, this sense is quite defensible and is an implicitly accepted tenet of the argument presented in this book (in Chapter 6). "Logic" is used in rather a specialised way also, and this usage uncovers the depth of Landa's commentary. For, although he appears to be talking about a logic of classes and propositions (and sometimes is doing so) the logical schemes interpreted in the universe of grammatical transformations are themselves algorithmic.

The interpretations are processes. The logic is a logic of questions and actions and the universe is a dynamic entity capable of accommodating events (the logic is amongst the non-classical logics of Chapter 4, the interpretation is like the universe of compilation and interpretation introduced at this juncture).

The critical feature of the intermediary problem situation is thus seen to be a semantic interpretation of the language employed for the dialogue. In order that a conversation shall take place, the rules of the language must be understood, in the notation of the previous monograph the rules and syntax of L (this condition being part of the experimental contract). In addition, there must be a semantic interpretation, whether concrete or symbolic, and this interpretation is generally more than the "interpretation" of classical model theory and mathematics (i.e., a set or sets of objects). It is an interpretative medium in which programs (algorithms) may be compiled and executed. By far the most versatile and well developed concrete medium is a computer equipped with Papert's (1970) LOGO peripherals and able to intrepet LOGO programs. As noted in the previous monograph, execution of the program is either a visually displayed, or mechanical, activity (depending upon the peripheral devices that are used).

1.2. Representing Knowables

In the Piagetian interview and to some extent the paired experiment, the participant experimenter probes the respondent in order to draw out his concepts of the problem situation; for example, by asking why an event takes place or what would happen if some feature of the situation changed. Such exploratory questioning must be backed up by knowledge of a subject matter field if corrective assistance is to be furnished. It may or may not be the case that this knowledge is functional and in this respect the experimenter's brief is quite liberal. For example, if we want to discover what the respondent knows about physics, then (since this is an empirically-based subject) the experimenter must be abreast of things as they are. But it is just as legitimate to follow an imaginative trail and discuss how the respondent thinks. Here, and in general, it is only necessary that the experimenter has a greater cognitive facility than the respondent, supported, if possible, by a broader knowledge of history, mythology or the possibilities of invention.

Although this specification is pleasingly flexible, it suffers from the defect that the data structure in the experimenter's head is inaccessible to an external observer, except that some of it is externalised in dialogue. Moreover, this data structure is inaccessible to the respondent, except for the information he gains by questioning the experimenter.

Ideally, both participants should be able to point out items in the data structure in a mutually comprehensible manner so that lines of explanation can be started and questioning initiated by either party. Various schemes have been adopted and do not in practice unduly restrict the interchange of ideas, since in any actual experiment the possible topics are limited (if only as a result of having an interpretive medium as the intermediary problem situation). In particular, a subject matter specification, especially if redundant, is completely unobjectionable for studies of learning, where the respondent is a student, and the specification stipulates what may be learned. So, for example, it is possible to stack up index cards or pictures bearing on a redundantly specified subject matter, these cards or pictures being accessed by either participant.

The obvious and valid objection is that the indexing which, in effect, describes the data items is arbitrarily imposed upon the conversation. It is due to an outsider, rather than the participants themselves. This objection, which bears just as strongly upon tutorial/learning experiments as any others, can to some extent be met. At least it is possible to play various tricks which effect a compromise between allowing for a participant-based description and an acceptable standard of observability.

1.3. Descriptions of Data Base

Most studies which employ explicit representations of knowables take it for granted that a description is given and understood by the participants. Commonly this description is just sensibly chosen (Bruner, Goodnow and Austin's 1956 study of concept acquisition); sometimes, it is based upon a factor analytic resolution of semantic scales evaluated by a population of respondents (for example, using Osgood et al. 1957) "semantic differential" techniques. Amongst the exceptions to this rule is work by Thomas and his associates in which exploratory conversations, often concerned with learning, are based upon mutually generated descriptions. Such descriptions are obtained from one respondent by applying the repertory grid sampling procedure (see previous monograph or Chapter 3, 6 and 7) to elicit personal constructs (Kelly 1955). * If the situation warrants serious attention to the description schemes of both participants, it is possible to use a more sophisticated routine (exchange grids) in order to compare their personal constructs and to circumscribe a region of mutual agreement. By iterating the routine, it is also possible to generate a shifting description scheme in which the area of mutual agreement moves around as the conversation proceeds.

One study will exemplify the method and indicate its main features. The term projects of art school students were the topic of conversation (between an experimenter and the students in a class). Each project produces a crop of artifacts, usually bits of sculpture. These artifacts, made by the participating students to crystallise their work, form the objects over which the personal constructs are elicited. If the conversation ranges over a wider compass, the set of objects is augmented, commonly by other pieces of artwork, from museums, galleries and representative practitioners in the field.

Each respondent determines his own personal constructs over the entire set of objects. During the conversation the constructs of the participants are compared, as a rule with the aid of exchange grids formed by requiring one participant (A) to rate or evaluate construct names used by another participant (B), and vice versa requiring B to rate A's construct names. Various means are employed to limit the proliferation of constructs and to condense those parts of the description that are agreed as mutual (i.e., to arrive at a core of possibly novel constructs which A and B rate in a similar manner).

Without going into the technical details, it is clear that this procedure gives rise to a participant-generated description scheme which, by rating the core of constructs over any desired objects, can be extended to cover any dialogue bearing even remotely upon the term project; hence, a description of the sort looked for in the last section. However, there is more to it than that.

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* It will be recalled that a personal construct is elicited by presenting triads of objects, requiring a predicate (the personal construct) which separates one member of the triad from others, rating the values of this predicate over all objects, and iterating triad selection.

The conversation refers to a term project and its intellectual ramifications; in general to artifacts that might be produced under comparable conditions rather than the gaggle of artifacts that were produced and are used as objects. The conversation refers, in other words, to an interpretative medium: the concrete or symbolic production system in which art school students are able to model their notions of reality. This universe of interpretation is not given, as it would be in a classical experiment. It is specified by the participants who choose properties (the personal constructs or the mutually acceptable core) and later instantiate their values. Let us say the classical experiments determine a description scheme "from down to up"; that is, a set of objects or events are chosen as a universe of interpretation, together with predicates that name properties or relations between these elementary entities. Conversely, an experiment such as Thomas's determines a universe "from up to down". Certain knowables, signified by the (repertory grid) objects, are ostended by the participants; personal constructs are elicited as predicate names which are rated or given values. Instances of these values (or, by repeating the procedure, the values of an arbitrarily fine grained mesh of constructs) are instated as elements of one or more universes. The universe of interpretation is thus generated by the participants, rather than being given. Usually the several participants have distinct universes. Some areas remain private but others are placed in a common domain by dint of mutual agreement about a core of constructs. This core is the conversational universe of discourse and it may change, both in extent and refinement, as the conversation proceeds.

1.4. Interpersonal Interaction Techniques and IPM

In Piagetian interviews, the conversation sometimes refers to the problem situation, the knowables, or the interpretative medium; sometimes, to the participants. So, for example, some stretches of dialogue express hypotheses due to the respondent or the experimenter about solving a problem; other stretches of dialogue express hypotheses due to the respondent about the experimenter (or *his* view of the problem); and vice versa, hypotheses due to the experimenter about the respondent. Since the discourse takes place in a (possibly restricted) natural language, it is difficult to disentangle hypotheses about facts or depersonalised ideas (objective or *it* referenced hypotheses) and interpersonal ideas (*I* or you referenced hypotheses).

If the IPM ("interpersonal communication test") method and its associated comparisons (previous monograph and Chapters 6 and 7) are used as communication aids in a conversational experiment, this method provides a filter that isolates interpersonal hypotheses for special scrutiny; for example, A's hypothesis about solving a problem (P) and B's hypothesis about solving P figure as personal-objective hypotheses A(P) and B(P) that are duly matched for factual agreement. In contrast, A's hypothesis about B's hypothesis about P, written A(B(P)) is an interpersonal hypothesis and so is B(A(P)). They, and higher level interpersonal hypotheses A(B(A(P))) and B(A(B(P))) are matched to determine mutual comprehension and appreciation. When the idea of a conversation is analysed, the segregation of the interpersonal component in dialogue is very important; just why will be discussed in Chapters 4, 6, and 8. It is doubtful whether an interchange devoid of an interpersonal component should be deemed a conversation at all. Yet one of the outstanding hazards attached to refining the conduct of a conversation is as follows: the well-intentioned refinements produce an arid situation stripped of interpersonal exchange.

This danger is present even when imposing the modest codification required to elicit and make sense of personal constructs. Hence, it is noteworthy that the experiments mentioned in the last section avoid this danger by incorporating a tacit IPM interchange. The exchange grid procedure *is* such a thing. To see this, replace P by a repertory grid (G). Let A(G) be the grid elicited from A: a matrix with columns labelled by objects, rows labelled by A's construct names and entries that are the values given by A to each of his constructs on each of the objects. Let B(G) be the grid elicited from B, with columns identical to A(G), but with rows labelled by B's construct names and entries comprising the values given by B to these constructs. Mutual agreement over a description (of what may be known or discussed) is obtained by requiring A to rate (give values to) B's constructs — which results in an extended matrix A(B(G)) — and requiring B to rate A's constructs — yielding, as a result, an extended matrix B(A(G)). Now, instead of independently eliciting a further level of mutual hypotheses (the trick employed in the IPM test), the participants compare and consider the matrices A(B(G)) and B(A(G)) in order to select or generate constructs that belong to the mutually agreed core; in the light of the mutual information, fresh constructs are invented and the ratings of the existing constructs are modified, as the exchange grid procedure is repeated.

1.5. Conversation Theory and Conversational Methods

The conversation theory described in the previous monograph unifies these well-established conversational methods. To some extent, it adds to the repertoire of techniques and increases the precision with which postulates about conversations are stated.

Surely, these claims require qualification. Of the experimental or tutorial arrangements described in the previous monograph, one ("teachback") is a specialisation of the natural language interviews used by Piaget, Vygotsky, and their followers. The special condition secured (namely, an understanding of each topic addressed by the participant) is believed to be fundamental, but that belief could be faulted. The other arrangement, a computer monitored Course Assembly System and Tutorial Environment (CASTE) does unequivocally secure understanding. But the mechanisation which is a practical prerequisite for this much rigour and objectivity may be unwelcome. Although the system does exteriorise hidden cognitive events, it imposes certain restrictions upon the participants. Though these restrictions are vastly less hampering than the constraints imposed by an other-than-conversational method of enquiry, it can be argued that CASTE conversations are oddly stilted ones. We are sensitive to the potential criticism and feel it is sometimes justifiable. Hence, much of this book (notably Chapter 6 onwards) is devoted to a systematic relaxation of the constraints upon the dialogue. This endeavour pays an unexpected dividend: the emancipated system allows for transactions that are, in practice at any rate, prohibited during fettered conversation. For example, even in the Piagetian interview, there is a tacit presupposition that the participants have one and only one focus of attention at once, corresponding in CASTE to one and only one aim at once. Our relaxations permit many aim operations and, in practice, several sorts of many aim transactions are realised.

Concerning unification, the other claim for conversation theory, the experimental arrangements ordained by the theory embody and integrate the components highlighted in previous sections of this chapter. It is true that a complete embodiment only occurs in relatively sophisticated arrangements of the kind discussed later in this book. But all the experimental systems (CASTE, for instance) are derived as specialisations of the general and sophisticated case; conversely, the general case is presaged by "teach back" and the course assembly system (EXTEND) described in the previous monograph.

For instance, the modelling facility featured in all the systems and spawned by conversation theory is the interpretative medium of Section 1.1 (for housing problem situations). The entailment structure is a representation of what may be known (Section 1.2), and insofar as it is used in an evolutionary fashion (EXTEND or a system to be introduced called THOUGHTSTICKER), its description is both personalised and "from up downwards" (rather than "from down upwards") as proposed in Section 1.3. Many of the experimental or tutorial systems incorporate an IPM like component (Section 1.4). In fact, this component so underpins the operation of the complex systems that the "interpersonal interaction" paradigm gains a novel significance.

Finally, conversation theory maintains that the basic unit for psychological experiments is a conversation, and carries this dogma to a rational conclusion in the hypotheses about cognitive organisation and P-Individuals. The other theorists do not seem to make this point as definitively or to pursue its consequences to the same extent. Our thesis is, perhaps pedantically, explicit. Hopefully, it reflects the views of our coworkers in the field, amplifying rather than distorting their meaning.

2. OPERATING SYSTEMS

The various experimental "arrangements" such as "teach-back" and CASTE are henceforward clustered under the title "operating systems". This section describes the operating systems currently in use for "one aim at once" conversations; roughly for conversations in which the conversational domain is fixed and the student has only "one focus of attention at once". Evolutionary systems in which the conversational domain, represented as an entailment structure, may be enlarged or modified are discussed in Chapter 3, where the discussion is again confined to "one aim at once" operation. Many-aim systems are considered in Chapter 6, after some theoretical prefatory material has been presented (Chapter 4 and Chapter 5 mostly).

Section 2.1 is a brief recapitulation of the work reported in the previous monograph. In Section 2.2 to 2.5 we describe the mechanically regulated one aim operating system employed in recent studies. Both this system and CASTE have been augmented as a result of experience by incorporating several features; notably, a much richer semantic interpretation obtainable by explore transactions, a procedure called aim validation, and a series of special transactions for dealing with analogical topics.

2.1. Recapitulation of Basic Features

A strict conversation takes place between participants using a conversational language L. For convenience, L is stratified into levels $L = L^1, L^0$. On theoretical grounds, the unit of a strict conversation is held to be an event called an *understanding* of a topic. An understanding is evidenced by an explanation of the topic and the derivation of the topic; the former in terms of L^0 transactions and the latter in terms of L^1 transactions. An explanation specifically evidences the existence of a concept and *is* the listing of a program which represents this concept. A derivation specifically evidences either a memory (defined as a concept that reconstructs a concept) or else the construction of the concept as it is acquired in learning. The period occupied in reaching an understanding is called an occasion, and if occasions are to be ordered so that topics (though accessed in any order) are understood in sequence, then it is necessary to introduce the caveat "one and only one focus of attention at once".

In "teachback" the explanations and derivations required to substantiate an understanding are elicited humanly, using a slightly stilted form of English in place of L. The subject matter which contains the topics is represented in a description scheme (a map like display where each topic has a location and the locations are classified by descriptive properties). Under these circumstances some essential aspects of the subject matter data base are out of sight in the participant experimenter's head.

CASTE is a largely mechanised system. Explanations are elicited non-verbally as model-building operations in one or more model-ing facilities, which are dynamic processors in which programs or models are executed.

It is particularly important to notice that an explanatory model built by a student is a program listing; so is a demonstration offered by a teacher or obtained from the regulatory heuristic on request. The behaviour of these programs when they are executed is quite distinct; it is their behaviour which does or should (if correct) satisfy the topic being explained (modelled) insofar as it computes or "brings about" or "satisfies" the underlying topic relation.

The subject matter representation for CASTE is a *conversational domain.* This consists in a formal (or syntactic) network imaging a thesis expounded by a subject matter expert; the topic relations appear in this network as nodes standing for derivations. Since a thesis is any orderly collection of derivation paths, a topic relation is linked by derivation chains to others. As a matter of convenience, the student sees, on a display called the *entailment structure*, only a simplified form of this network (the details of derivations are smudged under a common entailment connection). This simplified mesh is permissive. It represents what may be known if certain other topics are understood. Explicitly, the mesh asserts what may be known with the guarantee (obtained by processing a thesis before it is deemed legal and represented) *that the known topic(s) is (are) learnable and memorable.*

To each node in the mesh is attached through a data link (not an entailment connection), a structure which says *what may be done* to bring about the topic relation represented at the corresponding node. In the previous monograph these structures are referred to as Task Structures *TS*; since they act as a source of demonstrations. They may also be used for comparative purposes (a student's explanatory model is matched against the *TS* to determine its rectitude). The task structure is literally a collection of programs or sequences of commands for setting up *models = compiled programs* in one or more modelling facilities (either as demonstrations given to the student or as explanatory models he submits) and mere correctness, unqualified, is secured if the model can be executed and if, on execution, it satisfied the relevant topic relation.

In the previous monograph we distinguished the prescriptive and descriptive role of such a structure attached to the node of *topic* i by the notations *TS*(i) (as above) and $D^0(R_i)$. This notation becomes cumbersome when considering analogical topic relations which have been shown to play a crucial part in learning. Since much of this book is devoted to a discussion of analogy relations, I propose to change the notation and to call the graph representing what *may be done to model topic i in a particular independent modelling facility* a behaviour graph *BG*(i), which is simply a more familiar name for a program graph. That is, *BG*(i) determines or advises or recommends *model-building behaviours* (not the behaviours that take place if the model is executed). *TS*(i) will be reserved for the imperative or prescriptive use of *BG*(i), the set of commands or instructions which may be issued when the student receives a demonstration. Similarly, since most conversational domains are necessarily associated with many independent modelling facilities, I shall use the phrase *Lumped Modelling Facility = Set of Independent Modelling Facilities* in place of the terminology employed in the previous monograph namely, "modelling facility", for "Lumped Modelling Facility" and "component of a modelling facility", for "Independent Modelling Facility". In the long run, these changes of notation (not of meaning) are well worth the trouble taken in "translation".

Finally, the conversational domain is described (previous monograph $D^1(R)$) by means of descriptive predicates or *descriptors* which assume particular values on different nodes. The description performs two tasks: (a) It provides an indexing scheme, with meaningful indices, for gaining access to topics in the course of transactions initiated either by the student or the teacher/regulating heuristic. (b) It distinguishes and describes the several universes of interpretation proper to the independent modelling facilities in the Lumped Modelling Facility, i.e., it gives a semantic interpretation both to what may be known and to what may be done (by way of explanatory modelling).

The entailment structure which is displayed to a student thus consists in a mesh (a simplified image of the underlying thesis), its description, and the data links connecting each node for *topic* i to the associated structure *BG*(i). Finally, each node standing for a topic in the entailment structure is provided with electronic storage devices and indicators which display its state. The state depends upon the transactions which have taken place in a strict conversation and the possible states are shown in Table 1.1 (recalled from the previous monograph).

TABLE 1.1
States of Nodes

State	Resulting Transformations
Explore	Gives examples of descriptor values
Aim (validated)	Marks aim topic. Provides display of entailment set and permissible topics
Goal (legal)	Marks topic to be learned about. Permits demonstration and requires non-verbal explanation
Subgoal	Particular goal transactions are monitored
Understood	Determined by operating system and displays student progress

Just as an explanation in the CASTE operating system is a model built in a modelling facility, so a derivation (an explanation of how the explanation was obtained) is modelled as a sequence of state distributions upon the entailment structure. These taken together delineate the learning strategy adopted by the student. As a practical point, it is crucial that the state markers are displayed continually to the student as well as to the regulating heuristic/teacher and an external observer.

The CASTE transactions are shown in Table 1.2 (again recapitulating the previous monograph) together with their status as L statements. One of the transactions in Table 1.2 (aim validation) is novel; the reply to an explore transaction is also augmented by further descriptive data.

The rules for transactions in this operating system are designed to secure an understanding (i.e., the evidence of an explanation and a derivation) for each topic learned. This is the least biassed mode of operation, referred to in the previous monograph as a cognitive reflector. Tutorial arrangements are obtained by embellishing the cognitive reflector; namely, adding constraints to ensure that the student's learning strategy is dominated by an imposed teaching strategy.

TABLE 1.2

Transaction Types (as in the previous monograph)

		Commands	Questions	Executions	Explanations
L^1		$\underline{\text{Comm}}^1_A$ i	$\underline{\text{EQuest}}^1_A$ i	$\underline{\text{Exec}}^1_A$ i (Learning strategy)	$\underline{\text{Expl}}^1_A$ i
	Base	Aim Specification: Student (A) stipulates a desired aim by citing descriptor names and descriptor values sufficient to identify topic node.		Aim Validation: If BOSS testing validates aim, then aim specification becomes Aim, as below. Failing that, student must explore for further information.	
	Base	Aim i	Aim i	$\underline{\text{Exec}}^1_B$ i (EntSet display)	Cooperative Transactions
	Qualified	Explore i	$\underline{\text{EQuest}}^1_A$ ji	$\underline{\text{Exec}}^1_A$ ji (Learning Strategy)	$\underline{\text{Expl}}^1_A$ ji
	Qualified	Accept aim i	Tagaim i	$\underline{\text{Exec}}^1_B$ i	Cooperative Transactions
L^0	Base	$\underline{\text{Comm}}^0_A$ i	$\underline{\text{EQuest}}^0_A$ i	$\underline{\text{Exec}}^0_A$ i	$\underline{\text{Expl}}^0_A$ i
	Base	Goal $ji_k jm_{ki}$	Goal $ji_k jm_{ki}$	$\underline{\text{Exec}}^0_B$ i (Demonstration)	Cooperative Transactions
	Qualified	$\underline{\text{Comm}}^0_A$ ji	$\underline{\text{EQuest}}^0_A$ ji	$\underline{\text{Exec}}^0_A$ ji	$\underline{\text{Expl}}^0_A$ ji
	Qualified	Subgoal ji	Subgoal ji	$\underline{\text{Exec}}^0_B$ ji (Demonstration)	Cooperative Transactions

2.2. Use of CASTE and Its Field Station Relative INTUITION

In the studies to be described, CASTE has been employed to maintain a minimally biassed conversation; namely, as a cognitive reflector.

In this capacity the equipment exteriorises a strict conversation between a student and a regulating heuristic which could be humanly executed but is normally computer implemented. Whereas, previously, the heuristic was executed by a time shared system based on a PDP10 and interfaced from the modem into a special purpose computer (rather than the terminal normally connected for time sharing), it has proved more convenient to run the heuristic in a computer located in the laboratory. This is a small and inexpensive minimachine with fast operation, an LSI 2 with 16k of store and digital tape cassette backup. The system has two functions:

(a) To secure the understanding condition for each topic said to be learned (and to regulate learning over the entire conversational domain).

(b) To provide cooperative assistance, by way of demonstrations and other help-giving transactions, so that learning is possible. This operation is programmed (in pursuit of minimal bias) to provide as little cooperation as the student needs and, in any case, to record details of the cooperation furnished.

Apart from this, the equipment keeps a record of all transactions and the entire sequence of state marker distributions on the entailment structure.

2.3. Requirement for an Inexpensive Version

Our research has moved towards schools and colleges; most of the current programme of experiments is based upon remotely located field stations. On the one hand, it would be physically difficult to install bulky equipment (CASTE) in a field site. On the other hand, it would be quite undesirable to do so.

The main object of the field research, to investigate conversational methods and principles applied in the context of real educational institutions, depends upon securing cooperation from the teaching staff, and, so far as possible, their active involvement in the ongoing experiments with a view to developing courses and further applications. To work with pieces of equipment that are manifestly too costly to fit an academic budget would defeat the purpose. If the equipment is to be seriously considered, it must be perceived as potentially available as a scholastic tool.

2.4. The INTUITION Operating System

For these reasons, a good deal of effort was devoted to developing an operating system with the main characteristics of CASTE, which has components that are moderately inexpensive and can be readily conceived as items on a budgeting par with laboratory demonstrations. The result of the development is a system called INTUITION, an acronym for "Individual Tuition System".

Any subject matter can be encoded; the most refined course being an appreciable extension of "Probability Theory" (the main example of the previous monograph). However, the encoding, in the interests of inexpensive realisation, leads to prewired modular units called miniature entailment structures, each containing the nodes of 30 or 35 topics. The miniature entailment structures have state markers like CASTE but most of the transactions involve inserting plugs (to determine explore or aim or goal as the case may be) and these operations activate the computing equipment prewired into each modular unit.

The modelling facility, STATLAB II, is more elaborate than STATLAB I (previous monograph). Amongst other things it accommodates many stage experiments, several independent universes (both in the real and abstract worlds), and incorporates the distinct notions of causation, probabilistic causation and correlation, as well as complex conditional probabilities. This component is quite expensive, but it can be seen as a "Statistics laboratory" in toto, and it is not difficult to imagine separate bits of equipment concerned with the different demonstrations and explanatory models that are fabricated in the whole laboratory.

As in CASTE, demonstrations are given with the aid of overlay cards placed on the modelling facility and bearing instructions that tell the student how to build a model. The demonstrations used by a student are registered electrically and listed so as to check for and prohibit mere copying. Explanations are elicited as models and are marked for rectitude and progress by a check and instruction list recorder which also recycles the student according to the outcome.

Descriptive materials are provided, as before, in the form of slides arranged in a random access projector which is centrally controlled and sensitive to explore or aim transactions. Confidence estimates are obtained by a miniature form of BOSS (the Belief

Fig. 1.1. The INTUITION System. Typical working station. A = Entailment structure display (probability theory). B = Node with state marker lamps, sockets, contacts, labels; used in transactions described in the text. C = Screen for display of descriptor examples and counterexamples through random access projector. D = Random access projector. E = Check list device. F = Modelling facility for subject matter or probabilistic theory, STATLAB. G = "Boxes" used for conditional probability experiments. H = Tape recorder for inputting random or quasi random "natural results" together with spoken commentary on the external demonstration. I = Files containing "layover cards" for STATLAB, other demonstration material and aim validation cards for insertion in mini BOSS (not shown in this photograph). J = Control and recording mechanism which may be interfaced with minicomputer for class (not shown in this photograph). K = Student position.

and Opinion Sampling System of the previous monograph). A typical field working station is shown in Fig. 1.1; BOSS in Fig. 1.2.

Together with the recording and control facilities required for experiments, it is quite an elaborate installation. But a great deal of the complexity can be abandoned for teacher monitored tutorial applications (where direct involvement is encouraged) and the

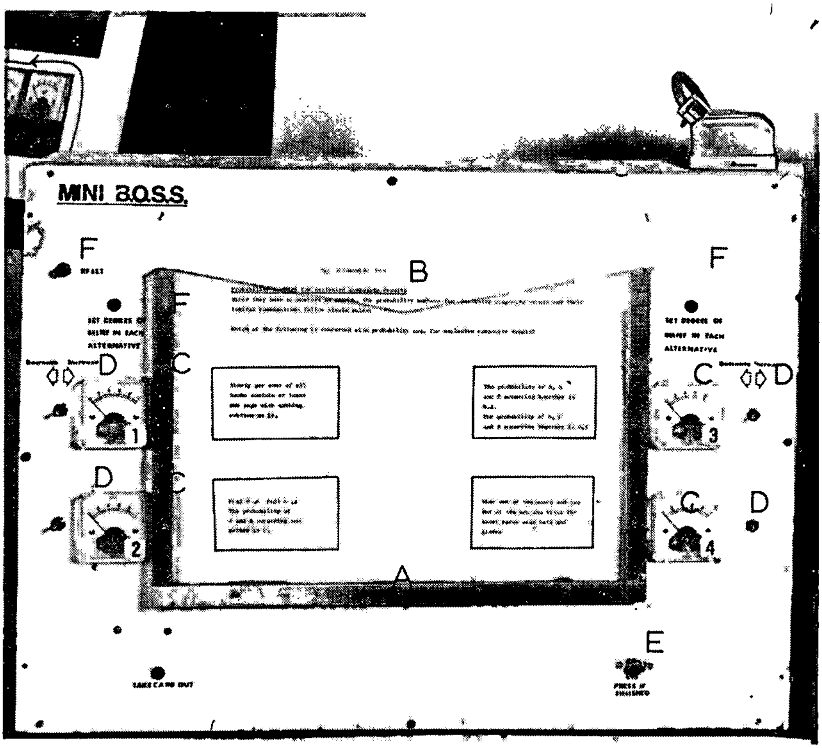


Fig. 1.2. Mini BOSS confidence estimation equipment. A = Card holder (reads punched hole code on the question card inserted and displayed for response elicitation). B = Card inserted. C = Meters showing result of automatic normalisation of response to guarantee that it is a valid confidence estimate. D = Buttons manipulated by student and used to increase and decrease his estimate of belief about "correct" alternative answer to the question. E = Submission button pressed by student if displayed confidence estimate is in agreement with his "correct belief". F = Signal lamps for control of response process.

other components appear as embellishments, necessary only for experimentation.

In order to operate INTUITION, the student must subscribe to a number of "game rules" (about how to put in plugs, what indicator lamps mean, how to instrument the transactions). These rules are stated in Appendix A and seem burdensome. But, learned by experience, they are not hard to comprehend or obey and the system is presented as a "learning game" rather than a tutorial device.

2.5. *Discussion of System*

With full implementation, including the elaborate recording equipment which serves in addition as a controller, the INTUITION system is able to accommodate nearly all CASTE transactions. Aim and goal selections are differently implemented (by plug insertion), and the Tagaim routine, which searches for understood subordinates, cannot be executed because the logical circuitry of the entailment structure is prewired. However, it is possible for a student to use "explain of explain". That is, suppose he wishes to assert his understanding of a topic other than the primitives (as a side comment, this possibility is quite essential), he can do so by announcing his intention, giving a non-verbal explanation, stating a derivation path, and finally, explaining the topics that are prerequisites for this derivation.

Mechanical checking of the "explain of explain" transaction calls for the computer; otherwise, a manual check must be instituted. In practice, the computer is a very useful adjunct in any experimental run, and it is virtually mandatory for monitoring and supervising the group learning discussed in Chapter 6 (several students with the same entailment structure and a learning strategy consensually selected, or several stduents and duplicated entailment structures so that several learning strategies proper to individuals or subgroups coexist in the system). Apart from sorting out who did what, the computer acts as a device for distributing explanations amongst the members of the group. The algorithm takes advantage of the redundancy which exists in any conjunctive substructure of an entailment structure and its associated *BG*s. That is, if *topic* k is superordinate (in a substructure) to *topic* i and *topic* j then the explanation of *topic* k will involve repeating the explanations of *topic* i and *topic* j and, of course, giving some novel explanation; *BG*k embodies *BG*i and *BG*j together with some fresh exercises.

This redundancy is quite advantageous for the individual learner (though we have a procedure that condenses explanations to reduce their redundancy if it becomes excessive). In a group situation, however, repetition holds up progress and soon becomes intolerable. The algorithm thus distributes parts of the explanation of a topic among the members of a group working together so that: (a) Each member has finally explained the head topic of a substructure, perhaps in part by explaining subordinate topics, before selecting a further head topic. (b) The burden of repetitious explanation is distributed equally amongst the members of the group who are working together.

The criticism (on cost grounds) suggested in Section 2.3 is not too troublesome. The computer is often regarded as part of the recording equipment and it is seen as unnecessary (as, for individual operation, it *is*). If people wish to enquire more deeply into the cost benefit of the system, it can be honestly pointed out that just as the computer programs can supervise a group of students, so also, the same machinery can be used to regulate conversational activity in a class of up to 10 or 11 students, only the inexpensive parts of the hardware being dedicated to students individually.

2.6. Recording of Data

All Explore, Aim, Goal, Subgoal, Understood transactions are recorded on digital magnetic tape; so are the check and instruction list transactions and the demonstrations received. The BOSS equipment used in Aim Validation is electrically traced, acceptable correct certainty is determined, and the confidence estimates recorded. Several spare recording inputs are available; these are used in group operation for monitoring the FRIM transactions (which realise IPM like interactions between students) noted in Chapter 6. Key features and states of STATLAB II are recorded to detect crass misuse of the check list facility, and (in a group) recorded segments are prefaced by student identifiers.

2.7. Some Deficiencies and Their Remedies

Because of the relatively small size of the miniature entailment structures (Fig. 1.3 is typical), and the method of specifying both the exploration of a topic and the aim topic chosen, students were inclined to trivialise the aim transaction. Faced with the requirement of choosing some aim (as a precondition for goal selection and gaining access to demonstrations), the student may aim for a topic on grounds of layout, paying no obvious attention to the meaning of the descriptions. An aim of this kind is not (in the technical sense) an aim. The student cannot describe the topic for which he is aiming; that is, he cannot locate it in a space of descriptors such as "real/abstract" or "structural/metrical".

This disturbing manifestation was noted initially in the context of sparse exploration prior to aim selection. Consequently, we greatly enriched the exemplary material provided in response to an explore transaction. So far as possible, the enrichment was systematised in the spirit of Nelson's (1974) *hypertext* (Fig. 1.4).

An immediate (and apparently universal) result was a very marked increase (a factor of 5 to 10) in the number of explore transactions. Though gratifying, this result was not enough. For some students, though casting around by explore transactions, still had no pretence of a description of the aim topic (typical comments were, "it's at the top" or "it's the next one up"). We thus introduced a further procedure, Aim Validation, to ensure that before a student is allowed to instate an aim, he can describe the aim topic.

Aim Validation depends upon eliciting confidence estimates using a piece of equipment (Fig. 1.2) which is a scaled down version of the Belief and Opinion Sampling System (BOSS) described in the previous monograph. As in BOSS the student is presented with multiple choice questions (having one and only one "correct" answer). From his response (setting up meter readings that represent his belief that each of the alternatives may be "correct"), it is easy to calculate uncertainties and Shuford Scores. The questioning alternatives are constructed by specifying AltSets and an Alter^+ (previous monograph); they are inscribed on cards with electrical designating codes and inserted, as required, in the card reader (Fig. 1.2).

Suppose a student aims for *topic* i. He is questioned by cards that refer to the semantic descriptors of *topic* i. Notice that estimates of "look ahead uncertainty" and "belief" are obtained using question alternatives that refer to the syntactic and derivational coordinates of a topic. Here, the alternatives refer to the *descriptor* used to access, or point at, the topic. So, for example, if *topic* i is described by "material/structural" and by "real/abstract" the alternatives are formulated by citing objects or situations which (depending upon the nature of the topic) fill the cells in an array like:

	Metrical	Not Metrical (structure)
Real		X
Not Real (abstract)		

Suppose *Topic* i is described as structural and real. If so, any alternative set contains one "correct" object or situation (marked x in the array) and the alternative set is produced by citing four alternatives, of which one is "correct" and the others have divergent values of one or the other or both descriptors.

In general, descriptors are many (rather than two) valued so that even for a uniquely described topic, it is necessary to use a series of cards rather than one. To each card the student gives a confidence estimate response and his aim (*topic* i) is deemed valid if his Shuford score exceeds a threshold (conveniently, of 0.8).

If an aim is validated, the student is allowed to instate it. If not

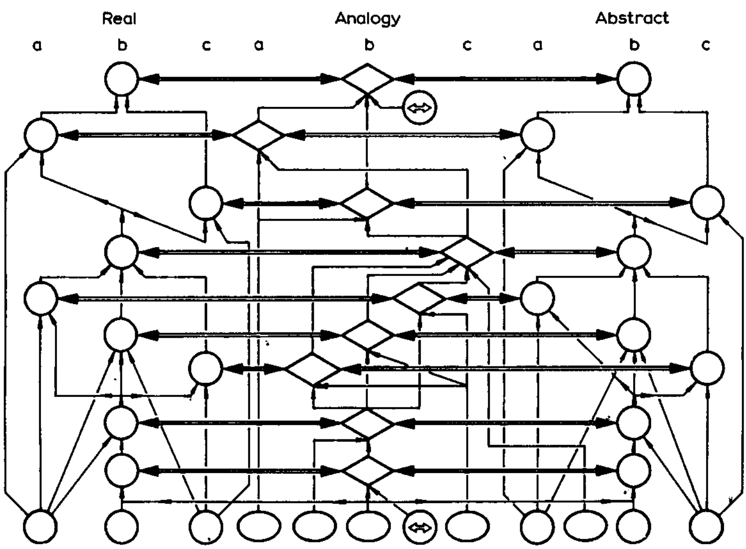


Fig. 1.3. Entailment structure (1 module only). Key: ○ = topic node; ◇ = analogical topic node; line arcs = derivations; double line arcs = analogues.

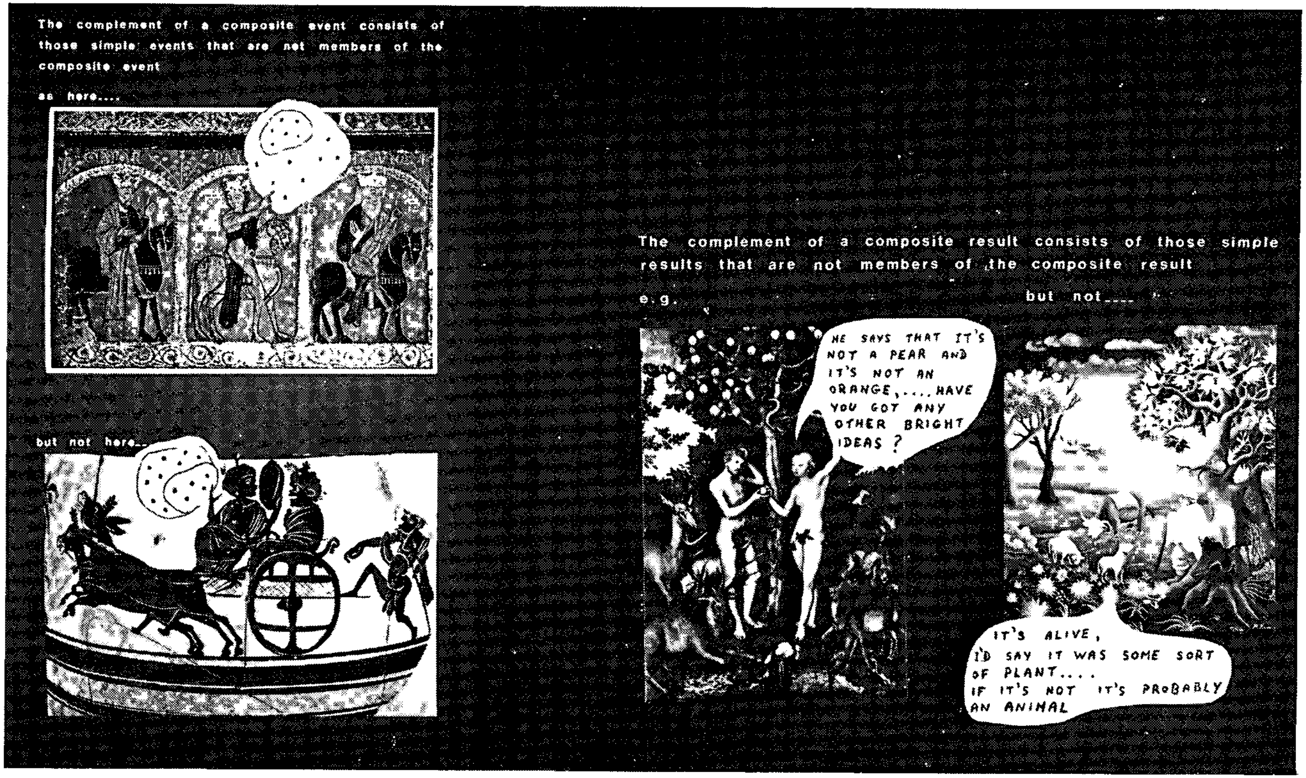


Fig. 1.4. A typical example and counterexample descriptor value slide.

(score less than the threshold), he is requested to continue explore transactions as a result of which he can gain further information about the meaning of the descriptors. Validation guarantees that the student can, at least, discriminate objects or situations in terms of the descriptors; the descriptors are to that extent meaningful.

The check secures the desired result. If there is only one way of completely describing the topic, then the routine goes on as already outlined. On the other hand, if there are several possible descriptors (redundancy based on many descriptors, all of which specify the topic), it is necessary to construct subsets of alternatives proper to each sufficient subset of descriptor values and to present the student with aim validation questions based upon the particular descriptors he chose to employ.

For INTUITION, where the aim transaction consists in plug insertion (an explore transaction, qualified and interpreted as an aim), there is no way of determining, directly, which descriptors the student actually chose and it is assumed that all are used. As a practical point, the glossing so introduced is not too damaging

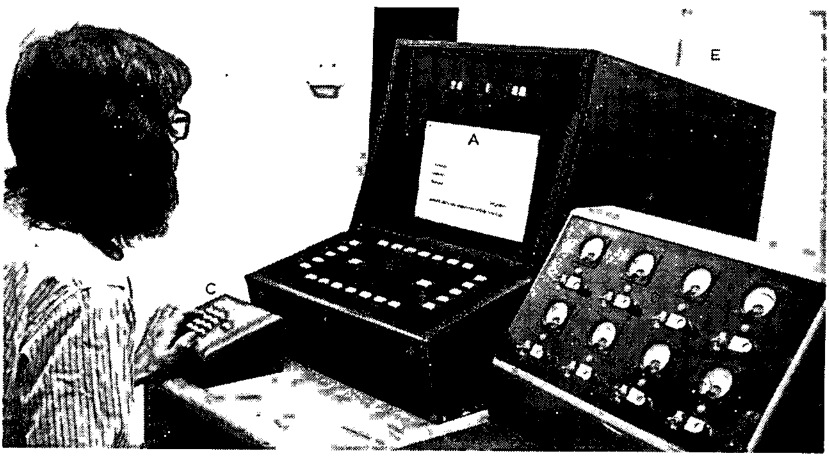


Fig. 1:5. Arrangements used in semi-mechanised free-learning and teachback experiments on learning style. A = Back projection unit for displaying examples. B = Selection buttons and signal lamps. C = Auxiliary indexing buttons. D = Confidence estimation equipment for determining look ahead uncertainty or topic uncertainty as numerical values. E = Lower point of the entailment structure (facing student but just visible in photograph).

since the descriptors are usually not very redundant (when they are, the student is asked which subset of descriptors he did employ). In CASTE (where aim validation is currently also implemented), the subset is specified as part of the transaction: for the student points at the topic by dialing a subset of descriptor values (indices) sufficient to uniquely ostend the topic. The aim validation procedure is thus implemented (but in essentially the same manner) with respect to whatever subset of descriptions is cited by a student.

2.8. Other Modifications

Recent work has shown the importance of analogical topic relations in learning. Hence, many of the entailment structures (for almost any subject matter) are replete with analogy relations. As a result the difficulties over modelling analogy (need for comparison of the topics related by the analogy) and the difficulties in accessing an analogy (that the existing routines do not generally allow the student to understand an analogy before the topics it relates) become obtrusive. These difficulties were mentioned in the previous monograph and were not completely surmounted.

In all of the present operating systems, the nodes of analogical topics are distinguished as requiring special accessing routines. These routines are fully implemented, but fairly complex. They can be much more meaningfully described in Chapters 4 and 6 when the characteristics of analogies have been discussed in detail. We thus note the existence of special routines and defer further discussion of them until later.

3. LESS RESTRICTIVE OPERATING SYSTEMS

An operating system (CASTE or INTUITION) secures a standard condition in which students who learn are required to understand each topic. Useful though it is as a standard, the condition is so stringent that it prohibits many interestingly defective methods of learning which deserve investigation. In order to study these (technical) misdemeanours, the standard condition is relaxed in various ways.

3.1. Less Rigid Regulations

Within a mechanised, heuristically regulated operating system, it is possible to systematically and selectively reduce the constraints which ensure understanding (for example, by replacing the requirement for non-verbal explanation with a correct response critéria, the analogue of ineffective as compared with effective teachback). It is also possible to withdraw the cooperative assistance provided either by stripping away part of the entailment structure, distorting the descriptive data (furnished in response to explore transactions) or by a stage by stage impoverishment of the demonstrations. All of these expedients have been adopted with the results described in Chapter 3. Several variations are possible.

3.2. Verbal Methods

On a different tack, the formalised conversational language L may be replaced by a (natural-language-speaking) participant experimenter, substituted for the regulating heuristic. Two variants upon this theme have been employed quite widely. One of the two is a combination of closely monitored free learning (with exploration of an indexed data base, founded on an entailment structure) and subsequent tape recorded teachback. The other is a mechanised form of the same procedure which is useful as a conversational test paradigm. Both methods are illustrated with reference to taxonomy learning but they can be employed for many different tasks.

3.2.1. Monitored Free Learning

Students are briefed about the task and the procedure to be followed. They are shown a graphical display of the indexed data base and examples of the kind of information available from it. No strict time limit is imposed; this is done to prevent undue haste or anxiety in performing the task, factors that might prevent students from exhibiting coherent behaviour. But the experimenter calls a halt to learning after 1—1½ hours work, by which time students have typically settled down to the task and are following a stable learning strategy.

The following cycle of events takes place.

(1) A student states his "aim". Aim statements are typically of the form, "I wish to learn about the official taxonomy based on

the categories A, B, C and D" (index description). Stating an aim does not restrict the student (hence, aim is much less strictly specified than it is in INTUITION). Other categories may be accessed and often are accessed if the student uses redundant or over specified information.

(2) The student requests access to cards in a data file by pointing out its indices (via a dialling arrangement, to allow for recording).

(3) For each card selection the student must state (into a tape recorder) his reason for requesting the card. Further, he must classify his intention under one of the following headings, by pressing selection buttons on his console (Fig. 1.5).

(a) Exploratory search: An intention to explore the categories in terms of the type of information available, without attending to specific content.

(b) General search: An intention to examine the content of cards with no commitment to its being relevant.

(c) Request for a particular item of information. Here the subject is asking a specific question in the form "How many legs does this kind of animal have", or "What distinguishes X animals from Y animals in terms of behavioural habits?"

(d) Requesting several particulars. Here the student is asking a complex question of the form "What are the several features that distinguish X animals from Y animals?" or "How many legs and how many heads has an X animal, and how is this related to the code name?"

(e) Testing a simple hypothesis. Here the student wishes to check a particular belief, for example, that "2" in a suffix refers to the number of heads.

(f) Testing a complex hypothesis. Here the student wishes to check a complex belief, for example, that an X animal has one head, three legs and a bushy tail.

When more than one card is selected, the cards may correspond to different intention classes or several cards may be subsumed under the same intention. In the latter case, the student is allowed access to the several cards simultaneously. Otherwise, cards are accessed one at a time.

(4) First, all the cards selected are moved from the data file to a card holder and arranged singly or in clusters. For each intention, the student takes out and reads the associated card or cards, making notes if he wishes. When finished, he returns the card(s) to

the data file and proceeds to deal with a further card or cluster of cards, repeating the cycle until his card holder is empty. The student is also required to give a commentary into the tape recorder of the results of his actions: whether he has been successful, what notes if any he has taken, and so on.

(5) If, during the cycle, the student wishes to modify his intention or request different cards he may do so, but first the card(s) being examined must be returned to the data file. This arrangement encourages the student to cluster his cards under intentions, veridically. Pilot studies showed that without this restriction a student is tempted to cluster *all* his requested cards together.

(6) When his card holder is empty, the student restates his aim (Step 1) and the cycle of events is repeated.

3.2.2. The Mechanised Procedure for Monitored Free Learning

The mechanised procedure has been used chiefly for learning theses about biological systems, typically using the menstrual cycle as a data base.

Information about the subject matter is partitioned into "chunks" each consisting of approximately 50 words. Each chunk stands alone as a statement but also cross refers to other chunks in which the meaning of terms is explicated.

A set of slides is prepared and used in a piece of equipment (Fig. 1.5) which incorporates a random access projector. Access to a particular slide is obtained by pressing one of 12 keys on a keyboard, whereupon the slide corresponding to the key pressed is projected. If no further key pressing occurs, after 25 seconds the screen goes blank. Key pressing must be repeated if the same slide is still required. Recording equipment records, on punched tape, which slide is requested on each occasion and the interval of its exposure (to the nearest 2.5 secs.).

The student's task is to learn about the menstrual cycle. He is permitted free access to all slides at all times and is given no time limit. He is told merely that the session ends when he feels ready to give a teachback account of what he has learned. The main restriction is that he is not permitted to take notes.

3.2.3. Teachback Method

Both types of free learning are followed by teachback, either "effective" (demanding explanations) or "ineffective" (correct response only, though teachback conditions are closely simulated). As noted earlier "teachback" (described in the previous monograph) is a specialised form of the Piaget/Vygotsky interview.

3.2.4. Main Use of the Methods

These relatively unrestricted conversational modes are chiefly used as discriminators of learning style (Chapter 3), and stylistic predictors are based both on the exploration/learning pattern and the form of teachback protocol subsequently obtained from the student.

For example, with respect to exploration and learning, the prediction is that a serialist will adopt a fairly rigid order of attending to the "chunks" and, further, will have a high frequency of consecutive repetitions of particular chunks within his rigid ordering. Conversely, a holist student will access chunks in a more "scattered" manner and have a low frequency of consecutive repetitions of particular chunks.

With respect to teachback protocols, the prediction is that the teachback of a serialist will follow the chunk ordering he has imposed. It will be as if he were recapitulating the frame ordering of a linear programmed text. Conversely, the teachback of a holist will give an account which has little regard for the original chunking. He will have constructed and organised his own set of richly interconnected chunks.

Chapter 2

Conversational Domains

In the previous monograph we described two basic procedures for constructing a conversational domain and its description ($D^1(R)$, $D^0(R)$) to represent a thesis about a subject matter.

One procedure is instrumented by a human interrogator/analyst who (given some mechanical "book-keeping" assistance) interviews a "source" or subject matter expert. The other procedure is a computer program, EXTEND, which performs a similar ritual. Operationally speaking, EXTEND replaces the interrogator/analyst but it does not "mechanise" the construction process. The fact is, only one human being, here the subject matter expert, is required. EXTEND uses him in an analytic role and provides the assistance needed to secure cyclicity and consistency (the essential properties of the relational network part of a conversational domain), as well as using him in the role of subject matter expert. This point was plainly exhibited by showing that EXTEND can be called as a routine by the tutorial operating system, CASTE, and *is* called whenever the student takes on the status of expert and (in an evolutionary system) enlarges the scope of subject matter by adding further topics.

Fig. 2.1 summarises the constituents of a conversational domain as it is produced by either of these methods. The labels *BG* (behavior graph) reflect the notation adopted to disambiguate the previous terminology (Task Structure, *TS*). Attached to each of the nodes, which stand for topics, there is a behavioural graph, *BG*(i), strictly a program graph. It is a class of programs of which any one, if executed in an appropriate modelling facility, will bring about and satisfy R_i the relation underlying this topic. Used

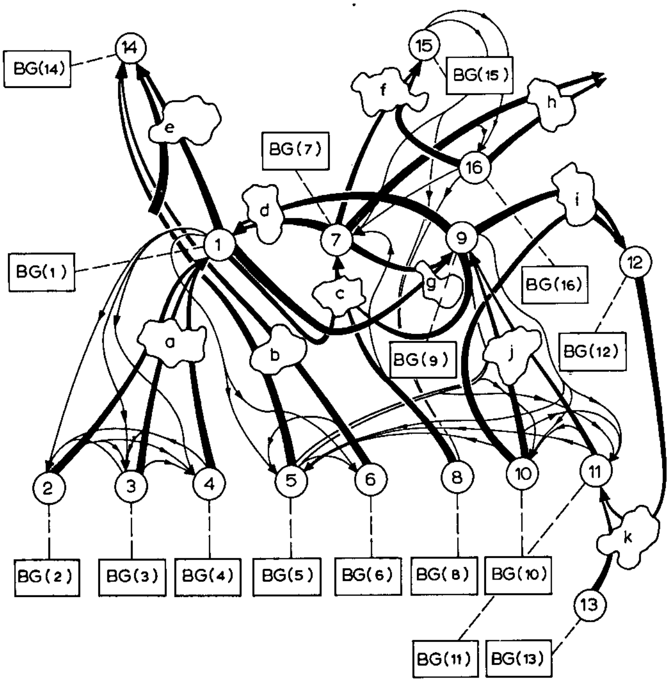


Fig. 2.1. Portion of a relational network. The nodes 1, 2, ... stand for topics. The arc bundles covered by a label represent a derivation of the topic on which the arcs are incident from the topics from which they emerge, by applying the relational operators specified in the labels a, b, ... The boxes attached by data links (not arcs) to each node specify the explanation of the topic in terms of a behavioural prescription or program graph (alias, Behaviour Graph, *BG*).

descriptively, *BG*(i) and its interpretation in the modelling facility *is* $D^0(R_i)$ of the previous monograph; used to prescribe a model-making behaviour which a student should carry out, it is *TS*(i). In either case, his (explanatory) model-making behaviour in the modelling facility ($Exec^0 i$) is compared for correctness with *BG*(i) and any correct model when executed in the facility also satisfies R_i.

It will be recalled that the relational network part of the conversational domain is processed to yield a structure such as Fig. 2.2 in which the relational operators, representing the derivation of one topic from others, are consigned to a data base, and the connective arcs depict entailment relations, i.e., derivations of any legitimate kind. The processing takes place (if and only if the original network is cyclic and consistent) at the point where the expert designates one topic or a cluster of strictly analogical topics as a head and specifies topics at a distance and direction from the head which he regards as subordinate to the head. Subsequently, the expert is required to *describe* the related nodes, using unary but many valued predicates, and the resulting mesh is embodied in a physical display, the *entailment structure*, in which each topic (or the node representing it) is associated with storage to accommodate tokens indicating its state during learning. It is possible to reduce the entailment structure to units of the type shown in Fig. 2.3 and it is important to notice that any legitimate network is an

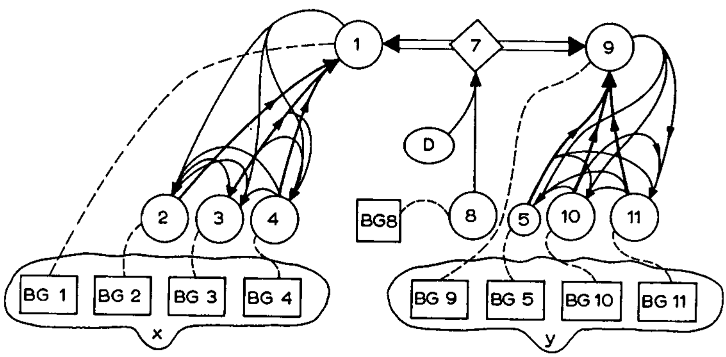
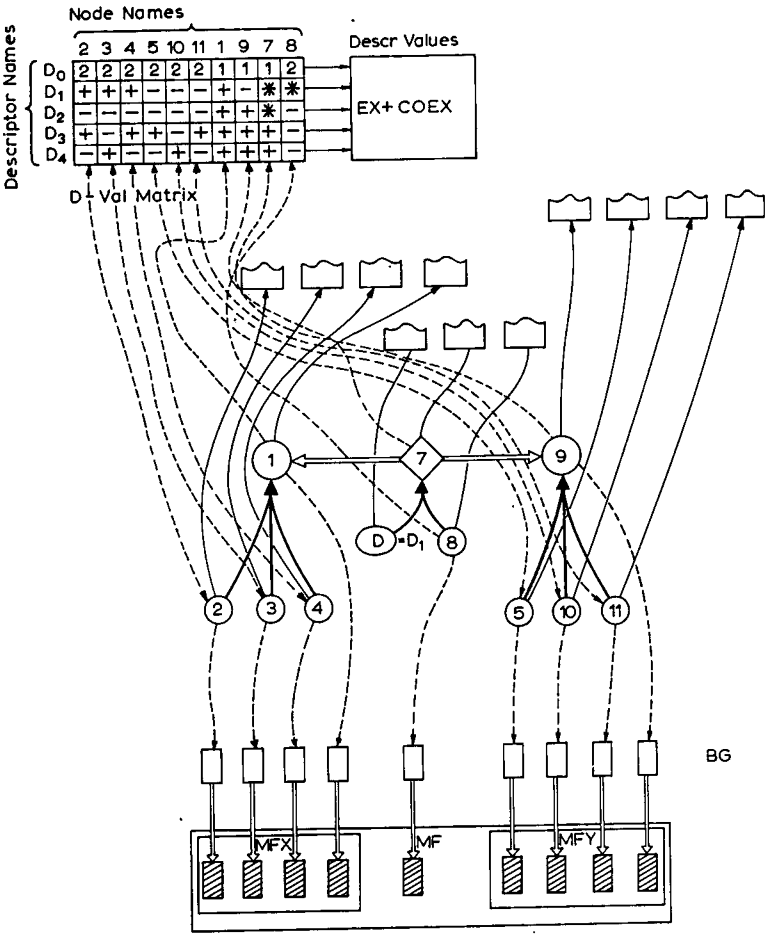


Fig. 2.2.

(a) The entailment mesh produced in preparation for pruning the network of Fig. 2.1, under the topic which is recognised as analogical, so that place holder node labelled D is introduced to accommodate the names of semantic descriptor(s), the values of which distinguish *topics* 1 and 9. Outgoing arcs from nodes 1, 7, and 9 are deleted except for those required to maintain the cyclicity of the structure (shown as thin arcs) and the cyclic component of the analogy relation (7) is represented by short hand ⇔ notation (*topic* 7 being itself distinguished as a ◊ node. The *BG* of *topics* 1, 2, 3, 4, are interpreted in a universe X, and *topics* 5, 9, 10, 11 are interpreted in a universe Y. X and Y are distinct, but as yet unspecified, and will be distinguished when D is named by the values of the D predicate.



(b) The entailment structure obtained if the mesh is pruned under the topic 7 (so that cyclic linkages are obscured *not* deleted), and its nodes are described by descriptors (unary many valued predicates or Fuzzy Predicates shown as $D_0D_1D_2D_3D_4$. Of these D_1 is D (the name of the distinguishing node) and D_0 is "depth from head in maximal arc distance". The D Val Matrix relates descriptor values to examples and counterexamples (the slide projected materials in INTUITION of Chapter 1) and to the name of nodes. ⌂ attached to each node represent storage for "node state" markers (explore or aim or valid aim, or goal or understood). The *BG* of *topics* 1, 2, 3, 4 give rise to models ▪ in a independent modelling facility *MFX*: the *BG* of *topics* 5, 9, 10, 11 in *MFY*. Both *MFX* and *MFY* are part of a Lumped Modelling Facility containing several independent processors (for example, STATLAB of Chapter 1). *Topic* 8 may be realised in either part of *MF*.

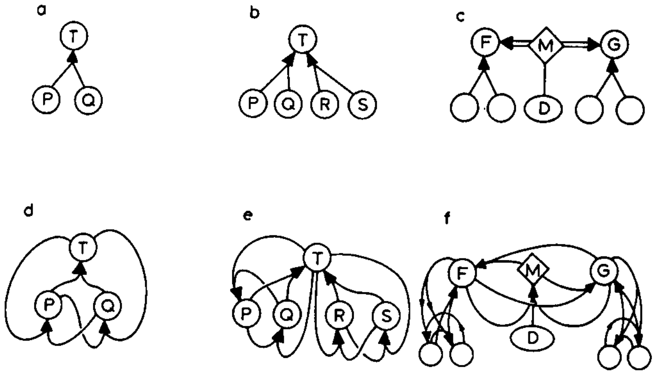


Fig. 2.3a, b, c, d, e, f. Typical structures. (a) Conjunctive, (b) Disjunctive, (c) Analogy relation with condensed symbolism, (d) Conjunctive, (e) Disjunctive, (f) Analogy meshes with cyclic derivations.

analogy relation in its own right. * Moreover, if the reconstructive derivation cycles of the original are reinstated, each substructural unit is cyclic (Fig. 2.3) unless it happens that it contains a node marked as primitive.

This essential property allows the mesh to be pruned under different head topics to yield completely different structures. For example, Fig. 2.4 shows a common construction in which a principle T, is reapplied to yield a topic relation A. On repruning in the most radical fashion, T is exhibited by examples (notice that these are *not* just aggregated under an arbitrary union. T is the join of A, B; or the join of B, C . . .; these topics may be *rederived*, as a result, from T). Other, intermediary prunings are illustrated.

These operations have been considerably refined since the previous monograph was written. Some of the refinements are of epistemological consequence and others of more pragmatic value; they will be described at appropriate points in the following discussion.

* Not any entailment structure. For example, an entailment structure can be, though seldom is, fully conjunctive. Even in that case, the network before processing contains cyclic derivations that are not eliminated by pruning (previous monograph).

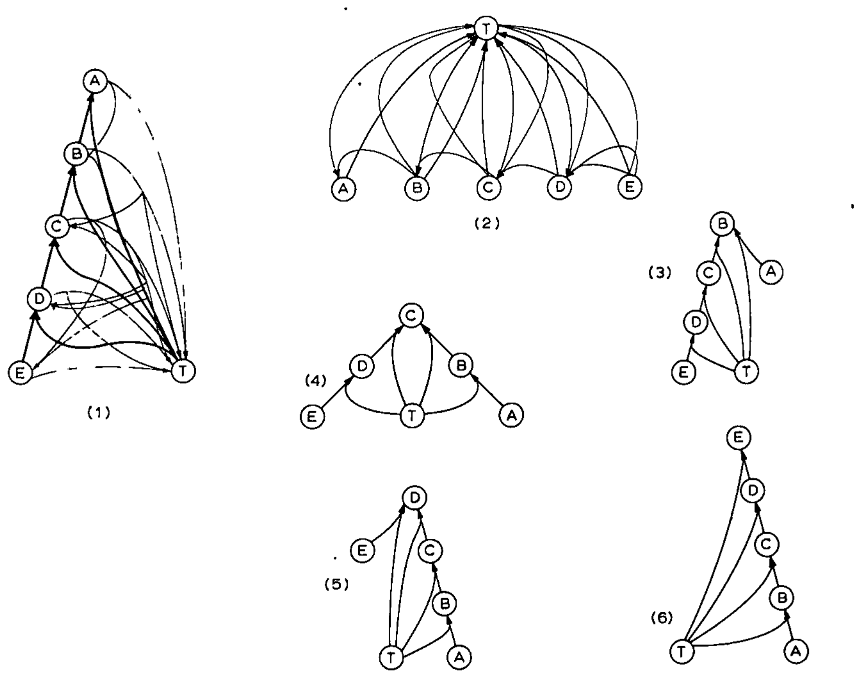


Fig. 2.4. Multiple head pruning. (1) Entailment mesh. Pruned under *topic* A (built up by reapplying a principle (T) with cyclic entailments shown as thin lines. (2) Converse pruning of the same mesh under the head topic of T. (3), (4), (5), (6) are other prunings (cyclic entailment connections are not shown).

1. SYNTACTIC AND SEMANTIC COMPONENTS OF A THESIS

It is expedient to discriminate between the syntactic, "5 is a prime number", and the semantic, "5 is a lucky number, or the numeral on your hotel room" aspects of a thesis and the structures representing it. The distinction is relative, "how do I know a purely syntactic entity, approximated by a logical text devoid of words?" But it is exceptionally useful.

Both of the construction procedures, and others introduced later in the book, are based on the idea that a thesis is a set of topics with syntactic relations between them and that a concept of a topic has a systemic (alias syntactic) core, roughly in Hartman's (1969) sense. Further, the syntactic component is output first, as a series of topic derivations, and later on is given semantic interpretation via the description. This is without prejudice to the fact that an expert or a student has a semantic interpretation in mind; it merely influences the order in which parts of his thesis are exteriorised.

For example, consider the unzipping (previous monograph) of the topic "efficiency" evoked by a question, "What does efficiency relate?" Clearly, the expert may be thinking of thermal/mechanical efficiency or some such interpretation, but the unzipping operation yields a syntactic derivation. For example, "Efficiency is a relation between work done and heat used, measured by a relation between source/sink temperature and the absolute temperature."

$$\text{Efficiency} = \frac{\Delta\text{Work}}{\Delta\text{Heat}} = \frac{T_{\text{high}} - T_{\text{low}}}{T_{\text{high}}}$$

All of the terms in this equation are discriminated upon syntactic grounds, as formally related symbols, and just this property renders them apposite as topics in a thesis which says, you can learn about *efficiency* if you understand "amount of work done and amount of heat" or "temperature difference and absolute temperature" or both. True, they also have semantic interpretations in a universe of heat engines, refrigerators, and the like; true also, these equipments are semantically related. But though the posited interpretations, or others, may be cognised, the mandatory feature of the derivation is a syntactic or formal relation.

Further, if the syntactic connection is pursued by successive unzipping until all of the subordinate topics are marked primitive, then these primitives are no more nor less than the constraints upon a modelling facility (a processor, not just a static entity) in which programs can be written to give imperative (temporally executed instruction) status to production rules. On execution, some of the possible programs, those that belong to *BG* (efficiency) and its subordinates, satisfy the "efficiency" relation and the relations "beneath" it.

Surely, any program, a syntactic entity, must be compiled as a model before it is executed; surely, also, the modelling facility (*MF*) in which it is compiled has a semantic description (it is a universe of possible actions). But this description appears later in the exposition of a thesis and it must do so in order to preserve the convenience of an "up to downwardly" directed derivation scheme (the thesis is the first and most global topic; further topics are differentiated as required), in contrast to the usual expedient of selecting sets of objects to begin with and using their members as building blocks (a "down to up" paradigm).

All this works satisfactorily except for analogy relations that are declared by the expert, in the simplest case, as isomorphisms. For an isomorphism (one to one correspondence) must be supported by a distinction between universes of interpretation (X, Y of Fig. 2.2), in practice, a distinction between modelling facilities designated *MF*(X) *MF*(Y). Lacking this support, the isomorphism would be confused with an identity and the derivation rejected as inconsistent. *

The topic that supports an analogy relation is one or more semantic predicate(s) (colour, texture, size, material, shape). The predicates supporting analogy relations (distinguishing *MF*(X) from *MF*(Y), for example) are the mandatory, and the only mandatory, semantic constituents of a thesis. The class of semantic properties named by these predicates includes time (execution time, order as determined by a processor clock). Recall from the previous monograph, that there are distinct clocks in the processors of *MF*(X), *MF*(Y).

One general point stressed in the previous monograph is worthy of repetition. Time, or precedence, is the least specific semantic interpretation given to syntactic productions, rewriting rules and implications. Moreover, any interpretation of such a (syntactic) sign involves time; though specific interpretations may entail specialised time orderings (realisable in the processor types of the previous monograph, L-Processors, the one clocked processors of modelling facilities, and so on).

The consequences of these observations ramify throughout the entire book. For example, they suggest a more systematic method

* *MF*(X) and *MF*(Y) figure as the "partitions of a modelling facility" in the previous monograph; for example, the "real" and "abstract" partitions of STATLAB. Henceforward, since analogy relations are considered in greater depth, we use the terminology *Lumped Modelling Facility* for the facility as a whole (for instance, all of STATLAB) and refer to its components or partitions, each with an a priori independent processor, as "modelling facilities" simpliciter: *MF*(X) or *MF*(Y) as the case may be.

for eliciting descriptions of the mesh depicting a thesis, which has been implemented and is described. They lie at the root of representing hypotheses/conditionals in a conversational domain. They are critical determinants of analogy relations. The class of analogies is far larger than isomorphisms (though the formal similarities can all be represented as morphisms of some kind). It includes, for instance, "analogies of analogies"; and the "of" ordering induces a hierarchy of descriptors. Finally, the distinction syntactic/semantic bears upon the issue of simplifying a thesis (a matter of practical consequence in course design).

2. DESCRIPTION METHODS AND THE SEMANTIC COMPONENT OF A THESIS

The expert's choice of a head topic and of a distance from the head at which topics are marked as primitive, is part of a description he gives to the entailment mesh. Any but specially contrived meshes permit the choice of several topics, and any such choice gives rise to a family of descriptors. Choice of a head topic extracts the thesis, under this head, from a potentially indefinite plexus of related knowables; it also imposes a quasi ordering (subordinate/superordinate) upon the structure which is isolated.

Under this ordering, the head topic(s) is (or are) *superordinate* to all others, and are assigned to a depth of zero. Several numbering algorithms may be used to convert entailment arc distances into values of the superordinate/subordinate descriptor. The algorithm currently employed in EXTEND (which is a refinement of the program in the previous monograph), is designed, so far as possible, to place the terms of all analogically related topics at the same superordinate/subordinate *depth* just as analogous head topics are at the same, zero depth.

2.1. *Forms of Analogy Relation*

Suppose that a depth numbering scheme exists (one scheme will be described in Section 2.2), it is possible within the framework of a depth numbering to examine the analogies, if any, at a particular depth. Let us also anticipate the argument and suppose that semantic descriptors are to be chosen and given values on the nodes of one or more analogy relations and the topics which it/they relate. Semantic descriptors are unary but many valued predicates; for simplicity it is much more convenient at this stage to regard them as having the possible values "+" (meaning "has the property") or "—" (meaning "does not have the property") and "*" (either "irrelevant" or "undetermined"). In Chapter 4, it is noted that the semantic descriptors are really "Fuzzy Predicates" with more complex value sets and that the assumption of convenience delineates a limiting case. Descriptors (the predicate names) are symbolised D, E ,...; their values D = + or D = — or D = *.

Fig. 2.5(a) shows a standard analogy relation (for example between the real/abstract universes of "probability theory", relating topics P and Q. The central node represents the syntactic similarity between P and Q, the common rule or formal relation these share. Suppose the expert is required to discriminate P and Q (using one or more descriptors D for this purpose) so that the differences which refer to the analogy are delineated. Whatever D he chooses for this purpose, it is obligatory that if D = + on P, then to secure the discrimination, D = — on Q, and it will be intuitively evident that D = * on the node of the analogy relation; an analogy between topics cannot have the semantic interpretation of the topics, since it exists in a distinct analogical universe.

The rational justification for this intuitive statement is shown in Fig. 2.6(a); the semantic descriptor D itself (not its value) enters the analogy relation as the distinguishing predicate which captures the semantic difference component of the analogy. In general, the distinguishing predicate is a subset of an ordered set of semantic descriptors, and (Fig. 2.6b, Fig. 2.6c) any analogy based upon a similarity, U, may be reduced to an isomorphism between restrictions of the U similar topics.

Thus an analogy relation induces an hierarchical ordering amongst predicates. It could be expressed by a hierarchy of logical types, but, looking ahead to Chapter 4, it is more parsimonious to employ a property of Fuzzy Sets; namely, the elements of a Fuzzy Set may be Fuzzy Sets. Whichever notation is used, the hierarchical structure is represented as a series of regions, the 0 region and the 1 region of Fig. 2.5(a), with any node in the mesh belonging to a region. If the topics related by an analogy belong to region r, then the node of the analogy relation belongs to region r + 1. It is important to avoid any possible shade of confusion between depth

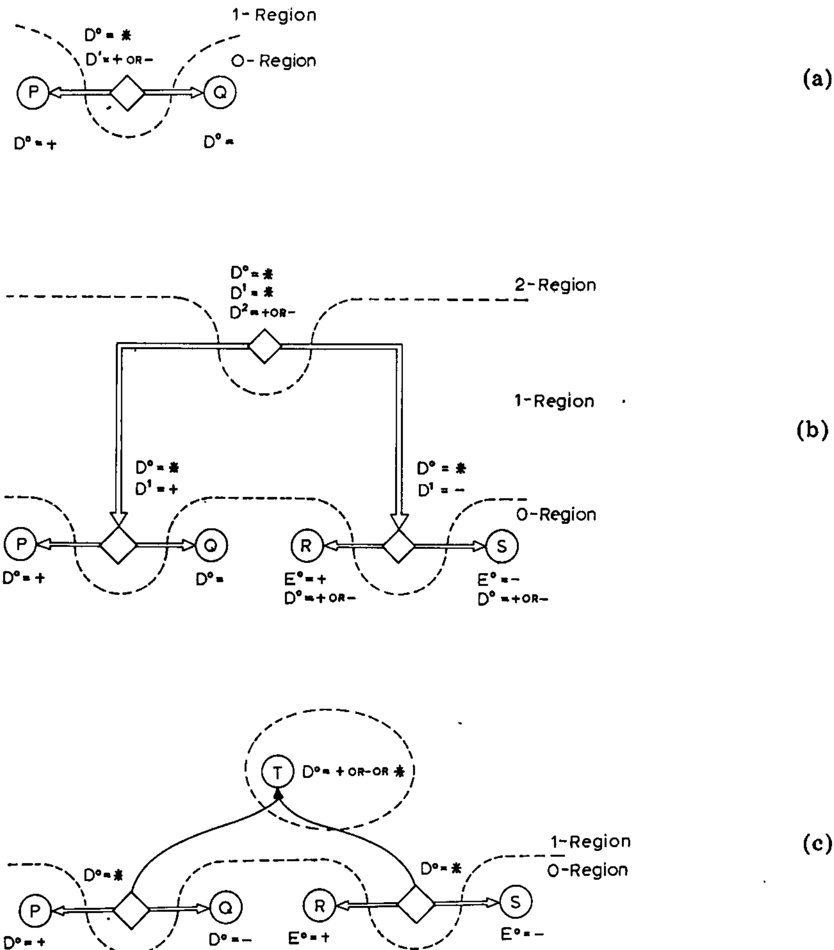


Fig. 2.5a, b, c. Analogy Relations, Descriptor Values, and Regions.

numberings, or levels, and the regions thus delineated. All nodes in Fig. 2.5(a) are at the same level and so are all nodes in Fig. 2.5(b), where the construction is iterated, as it may be indefinitely, by citing an analogy between analogies (alias, topics in Region 1 rather than Region 0) to generate a 2 region.

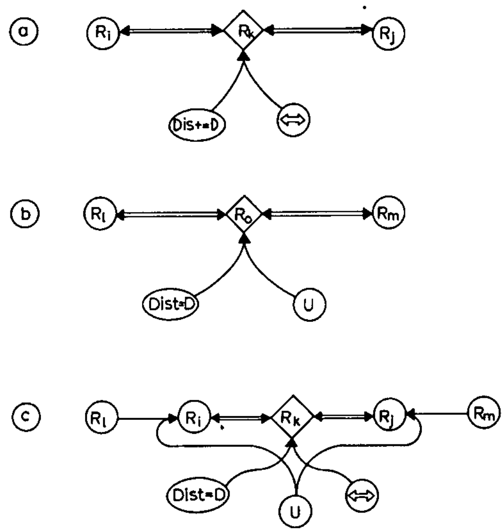


Fig. 2.6a, b, c. The distinguishing Predicate Dist on an analogy consists of one or an ordered set of predicate names that are used to indicate the *difference* between the analogous topics (here, R_i and R_j). The similarity of the analogical topic relation (R_k) is either an isomorphism (as shown in a) or a topic expressing the syntactic or systemic similarity (as shown in b) between R_l and R_m. This construction may always be reduced (as in c) to an isomorphism by restricting the analogous topic relations by U.

The region notation stems from the semantic descriptors and these are tagged by a superscript. For example, in Fig. 2.5(a), D^0 may have real values (+ or —) on nodes in the 0 region (and must have real values on the topics related by the analogy), but its value is $*$, by mandate, on nodes in the 1 region. Similarly, there is a descriptor, D^1, with real values (+ or —) in the 1 region and, in Fig. 2.5(b), a mandatory $*$ value on nodes in the 2 region.

Analogies between analogies are very common; especially so, it turns out, in physical science and other inherently compact subject matters. For example, the thesis on "energy conservation" used as a primary example in Chapter 7 is replete with them.

Another very common construction is a syntactic derivation involving the (syntactic parts of) two or more analogy relations of topic T in Fig. 2.5(c). The region convention clearly differs significantly; whereas an analogy between analogies with nodes in region r has a node in region r + 1, a derivation (like T) from analogies in region r has a node in region r. The model which is an interpretation of T exists in a distinct modelling facility. For example, in Fig. 2.5(c), if *topics* P and R are modelled in *MF*(X) and if topics Q and S are modelled in *MF*(Y), then *topic* T is modelled in *MF*(U) such that the models of T establish coupling relations between models built in *MF*(X) and in *MF*(Y). But notice that (though in the same region as the analogies) T is at a lesser depth.

Since the previous monograph was written, considerable effort has been devoted to analysing and representing analogies, motivated in part by the educational importance of analogies, properly used, as means for accelerating rapid comprehension of a subject matter. For example, though some analogies are isomorphisms (the type cited in Fig. 2.6) or isomorphisms valid for only some part of the related topics, others are generalisations. These varieties of analogy are amply discussed in subsequent chapters (notably Chapters 4, 6, 7 and 8) as they occupy a key role in innovative processes. Hence, generalisations are not examined at this juncture. It is, however, opportune to review one quite innocent complication which was mentioned in the previous monograph; namely, that analogy relations are not restricted to relating two topics.

Some of the more important many place analogies are shown in Fig. 2.7. Reading through the examples, Fig. 2.7(a) says that *topics* P, Q and R are analogous (their similarity represented in the central node, differences entering the central node as Dist). In Fig. 2.7(b) *topics* P, Q, and R are related by (possibly different) analogies. Fig. 2.7(c) asserts that the (different) analogies are themselves analogous. This construction is in register with Fig. 2.5(b), and Fig. 2.7(d), by the same token, is in register with Fig. 2.5(c). Fig. 2.7(e) expresses the existence of two analogies (x and y) between topics P, Q and R. For sensible discrimination x and y will be demarcated in terms of distinguishing properties that capture differences but also in terms of distinct (syntactic) rules (one to x and one to y). Even so, it often happens (Fig. 2.7(f)) that x and y have common features related by analogy between analogy relations (u). The constructions of Fig. 2.7(a), (b), (c) are all exemplified by the "real" department of "probability theory" (previous monograph) where P, Q, and R are topics in "games of chance", in "behavioural experiments" and in "genetics". The different con-

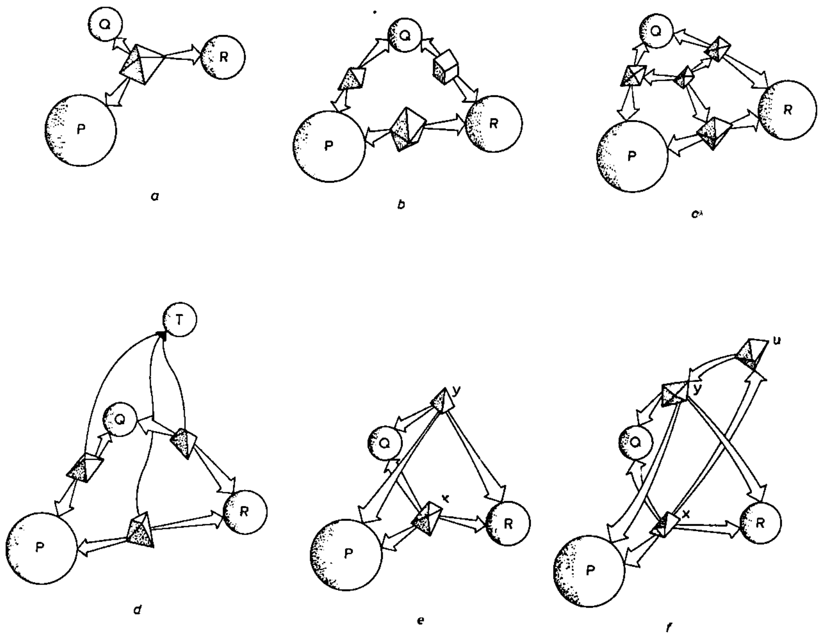


Fig. 2.7a, b, c, d, e, f. Complex Analogy Relations.

structions are appropriate to different levels and were deliberately glossed in the earlier treatment, as they may quite legitimately be, because the "real" nodes in this subject matter have the calibre of T in Fig. 2.7(d). The other constructions are more convincingly referred to generalised analogy relations of the kind we have promised to examine (in fact, any generalisation can be represented either in the fashion of Fig. 2.7(e) or else of Fig. 2.7(f)).

2.2. Depth Numbering

This preliminary discussion of analogy relations rested upon the idea of a depth numbering, the analogies being anchored to some depth. All depth numbering schemes rely upon the following types of process.

(a) A means for detecting the nodes of analogical topic relations.

(b) A means for determining the region of a node, using the 0 region nodes as a baseline.

(c) Some numbering arrangement that orders the nodes in a mesh from a head node (or a cluster of analogical head nodes), assigned a depth of 0, that are located in the 0 region.

Analogies are detected in syntactic terms by noting that they differ in establishing some kind of morphism. Hitherto, only the isomorphism operation was seriously employed; since the mechanics of generalisation have been studied, there is a general morphism (a mapping between relations that preserves some formal relation). In the scheme we employ, isomorphism $\Leftrightarrow$ appears as a relational operator; so, now, does a general morphism. If he employs the isomorphism, the expert is provided with a place holder node (Dist = ?) to accommodate the distinction between universes of interpretation required to maintain the integrity of isomorphism in contrast to equality; a similar distinction is needed if a general morphism is invoked. The nodes associated with these operators and placeholders (Dist = ?) are marked, mechanically, by an analogy detection algorithm. They are listed together with nodes, like T in Fig. 2.5(c), that represent derivations from analogy relations, *provided they are not part of a derivation re-entering nodes in the 0 region* (if the italicised condition is false, they will be numbered from their 0 region entailments). Call this list the *analogy list.*

A further algorithm is applied to the union of the original node list and the analogy list. Nodes that are not members of the analogy list are assigned to Region 0. The analogy list is now searched for analogies between nodes in Region 0 and these, together with nodes corresponding to immediate derivations (like T), are assigned to Region 1. The process is iterated, at the next stage finding analogies (between analogy nodes) in Region 1 which are assigned to Region 2, and continues until all the analogy list entries have been exhausted (for Regions 0, 1, ..., r, ..., r_{max}).

Finally, a depth numbering algorithm is applied to the original mesh and the distinguished (and region assigned) analogy list. This algorithm operates from the head downwards, first, with nodes in the 0 region. So far as possible, it satisfies the condition that the nodes related by an analogy and the analogical node itself are placed at the same depth. It is not always possible to satisfy this condition, and the expert is given the option of deleting an analogy he has previously inserted or of permitting analogies that cross between depths. Such analogy relations are not necessarily pathological and can be handled. As they are rare and because handling them greatly complicates the description process (to follow), it will be supposed that all 0 region topics related by one analogy are at the same depth. Having exhausted the 0 region nodes and assigned them depth numbers, the algorithm next addresses any nodes in the analogy list that are derivations from analogies and have no direct derivational link to 0 region nodes, again operating from depth 0 downwards.

2.3. Improved Method for Eliciting Descriptions and Their Values

Meshes are numbered as they are isolated from their surrounding (the pruning of the first monograph), and in practice pruning and depth assignment are carried out automatically before the current mesh is displayed to the expert. Since a mesh cannot be a simple chain of nodes, it is evident that the superordinate/subordinate descriptor does not uniquely name each node and the onus is placed upon the subject matter expert to select and assign values to further descriptors ("unary but many valued predicates of the nodes") so that:

(a) Statements of the conjoint values of the descriptors uniquely ostend one node (there is at most one node, standing for a topic, in each "cell" of a grid made up from descriptor values).

(b) Some "cells" are empty.

However, no restriction is placed upon the number of descriptors employed, and the description scheme may be as redundant as desired.

From the student's point of view, the descriptors, or some subset of them which he can show that he understands, furnish the means for exploring, gaining access to, and learning about the topics.

From the expert's point of view, it is useful to separate descriptors into the categories, syntactic and semantic.

The values of a syntactic descriptor, such as superordinate/subordinate, say nothing (except perhaps to the expert) about interpretation. They are properties (in this case a "depth" or "arc distance" property) of all the nodes in a mesh. The entire mesh could be described in these terms as an abstract graph and, for that matter, the syntactic component of this thesis, revealed in the derivation structure, could be described as an uninterpreted and formal system. Under these circumstances, however, it is difficult to see how a student could make sense of it; at any rate, since the incorporation of aim validation (Chapter 1), a student would not be allowed to use only syntactic descriptors when specifying his aim and starting to learn.

Semantic descriptions have values that refer to the universes of interpretation in which explanatory models for topics are realised as programs. One semantic descriptor is the head name (notice, this name is the value of a semantic descriptor, though the values, 0, 1, ... of subordinate/superordinate depth are values of a syntactic descriptor). Other semantic descriptors carve up the topics in various ways. For example, "steam engines" and "heat pumps", or "turbines" and "piston impulsion", in the "energy conversion" thesis of Chapter 7, or electrical/mechanical in physics. The current recommendation is that large numbers of semantic descriptors are specified.

Apart from the superordinate/subordinate descriptor, which is derived automatically once a head topic is chosen, the remaining descriptors are systematically elicited as "personal constructs" (Kelly 1955) using a modified repertory grid technique (Bannister and Mair 1968). The objects over which the personal constructs are elicited are the nodes in the mesh.

However, insofar as the expert is really evaluating interpreted explanations (models) of the topics which the nodes stand for, the constructs are semantic descriptors and convey substantive meaning. Even so, they are treated uniformly as unary (many valued) predicates of the nodes. For expository convenience we limited the values in the last section to +, —, and * (irrelevant). This limitation is inessential, but whatever values are permitted, the value * (irrelevant) must be preserved.

The names and values of the descriptors are elicited mechanically by a program akin to Thomas's (1971) DEMON. The chief peculiarity lies in the way that nodes are sorted and presented to the expert (as the objects having, or not having, a property).

The descriptor eliciting procedure is outline charted in Fig. 2.8. It accepts as an input a mesh with depth numbering (n) and regions (r) already specified, and its output is a described mesh to which is adjoined a set of primitive nodes representing the descriptors D, E, which figure as the distinguishing predicates (Dist) of analogy relations. The remaining descriptors, (d, e, ...) if any, that are eli-

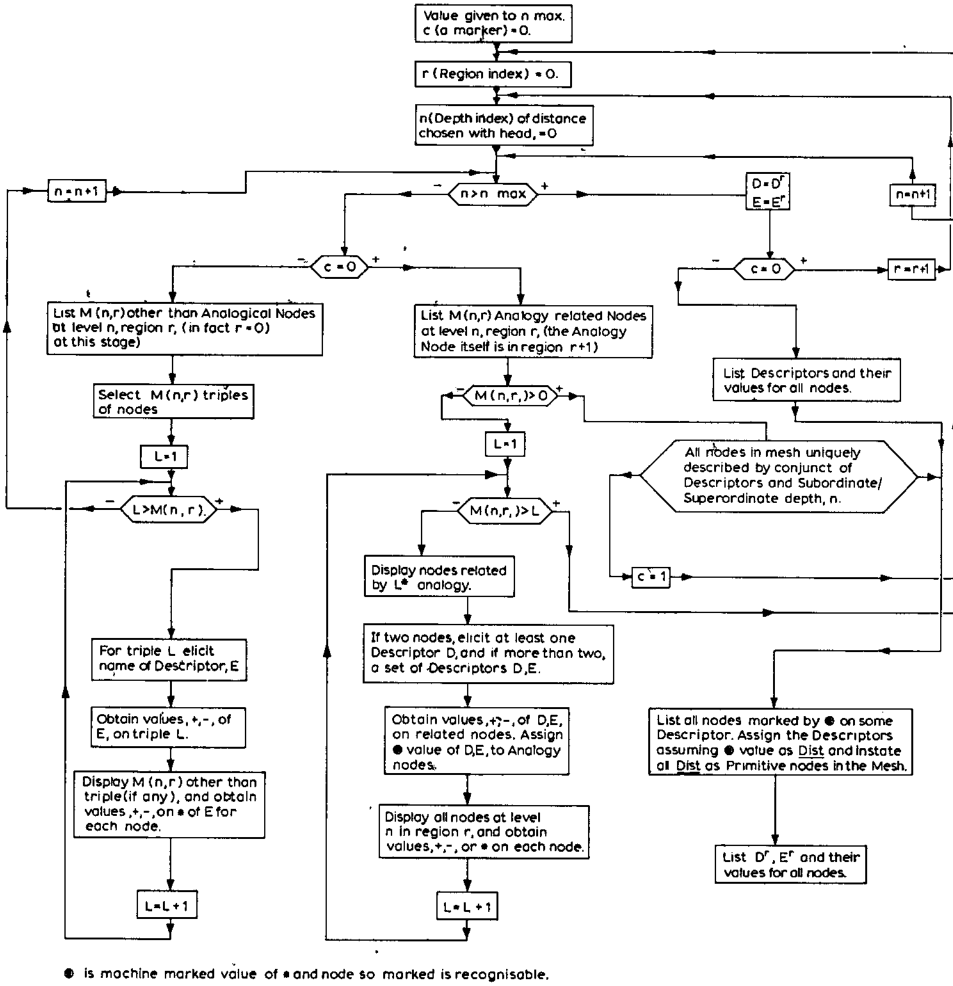


Fig. 2.8. Outline Flow Chart for Descriptor Elicidation Process.

cited to safety condition (a) and (b) for other than analogical topics are listed but are not represented by nodes.

Several points are usefully kept in mind whilst reading this flow chart. First, when the expert is asked to choose the name and values of a descriptor (alias a personal construct, or a property) with respect to a set of nodes, he is really being asked to contemplate the models which will, on execution in an appropriate modelling facility, satisfy the topic relation. Semantic descriptors are properties of this interpretation.

Next, the "model" of an analogy relation between two (or more) topics is a coupling between two (or more) models, distinguished by execution in a priori (without the coupling) independently clocked processors and by the distinguishing predicate (Dist) which is specified by way of the selected descriptors.

Finally, although the program which realises this flow chart can be interfaced with the expert using a teletypewriter terminal, this expedient is completely impracticable except for the simplest meshes. All practical systems employ a display of the mesh which is continually accessible to the expert and an "interrupt" which provides the expert with the displayed values of the descriptors he has so far chosen, superimposed upon the nodes in the mesh. One interface of this kind is described in Chapter 7, but most graphic consoles will provide the required facilities.

2.4. Tutorial Materials

The described and pruned mesh is transformed into an entailment structure (Fig. 2.2) by encoding (either in computer storage or the hard wired form of Chapter 1), each node being associated with storage locations to indicate its state as learning proceeds.

Tutorial materials are based upon demonstrations constructed from the *BG*(i) as task structures *TS*(i) (previous monograph), together with the "How" questions (EQuest^0 and Comm^0 and their qualified forms). "What" questions (PQuest^0) span the topic relations, again as described in the previous monograph.

In Chapter 1, we noted that experience with both operating systems, CASTE and INTUITION, has underscored the necessity of providing rich semantic data in response to explore transactions, and shown, also, that an *aim* must be validated before it is accepted by the system. The data provided when a topic is explored (by citing a conjunct of descriptor values that ostends and uniquely identifies the topic) consist in one or more slides. The artwork is important (some examples are shown in Chapter 1), but is generated systematically as a series of illustrations that exemplify the topic and a series of counterexamples that differ in one or more descriptor values.

Aim validation questions (of the form PQuest[1] in the previous monograph, since they refer to subsets of nodes) are multiple choice questions having one and only one (correct) response alternative that illustrates the descriptor values conjoined to identify the topic. The remaining response alternatives (incorrect) show counterexamples differing in one or more descriptor value.

3. SOME USEFUL OPERATIONS UPON ENTAILMENT MESHES

Insofar as the derivations of a thesis are retrievable, it is always possible to generate a binary decomposition of any conjunctive or disjunctive (but not analogical) structure in a given mesh. Each kernel in the binary decomposition has exactly two members.

Labelled clusters of relational operators (Fig. 2.9), reduced to a kernel in the entailment mesh, are replaced by sequences of the complete subset (Natural Join, Projection, Union) of operations. These sequences are arranged in order and further nodes are introduced (Fig. 2.9). These nodes stand for topics which were not made explicit in the original thesis (and which in general need not be made explicit), but which are needed to satisfy the requirement that each kernel has two members.

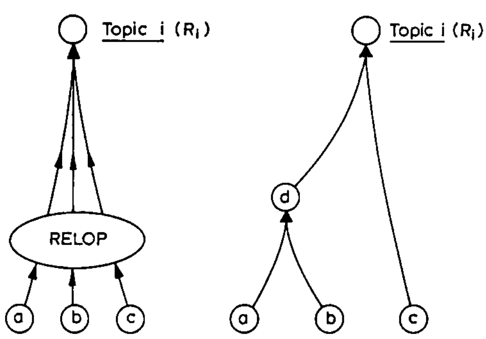


Fig. 2.9a, b. Binary decomposition. (a) A conjunctive substructure (kernel) in which *topic* i with formal relation R_i is obtained from a, b, and c. In the original thesis the derivation was labelled by a complex of relational operators Relop. (b) One Binary Decomposition. The components of Relop are replaced by sequences of {Natural Join, Projection, Union} and nodes, such as d are introduced to represent intermediary relations.

3.1. Trade Off Methods

The binary decomposition of a structure showing the derivation of a topic relation R_i (at its head) together with all Behaviour Graphs, *BG*, of its primitive nodes (*BG*(a), *BG*(b), *BG*(c)) has as much information or specificity as the relation R_i and its Behavioural Graphs *BG*(i).

It is also true that an undecomposed structure representing the same topic, R_i, with the same task structures attached satisfies this condition; in fact, if B (Fig. 2.10) is a binary decomposition of A (Fig. 2.10), then A and B contain the same amount of information or specificity.

The information or specificity is differently arranged. In A, it is relatively localised, since most of it is packed into the Behaviour Graph or, tutorially speaking, the task structure, *TS* of R_i. In B, it

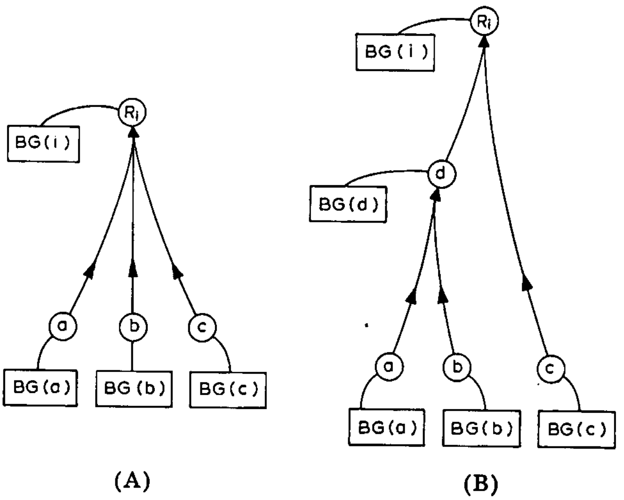


Fig. 2.10A, B. Trade off and the distribution of specificity or information between the entailment mesh and the Behaviour Graphs/Task Structures, connected to its nodes. The redundancy in any conversational domain (even with purely conjunctive mesh) should not be confused with redundancy in disjunctive mesh representing alternative derivations of the same topic. Equalities: If Sp = Specificity, then Sp(*BG*(i), R_i) = Sp(*BG*(a), *BG*(b), *BG*(c), Derivation R_i from a, b, c) = Sp(*BG*(d), *BG*(c), Derivation R_i from c, d).

is distributed over the network. We comment that a trade off is always possible. Though a behavioural specification *BG* or *TS*, and a cognitive (relational network) specification are distinct, and though they are both needed in a tutorial system, their combination is also fundamentally redundant.

Hence, within limits, there is a systematic method for deploying the information in a thesis in an educationally desirable manner. It may be conveyed primarily by demonstrations and the tutorial materials attached to them, or primarily by an entailment structure display, or, redundantly, in both ways.

There are restrictions upon the kind of information which is traded off in this manner and upon the amount of trade off which is possible; namely:

(1) Kind. The traded off information is in the syntactic (not in the semantic) content of a thesis; the semantic information is conveyed by descriptor values and in exemplary data, accessed by explore transactions.

(2) Amount. The distribution which maximises the information in the entailment structure is obtained by constructing and displaying a binary decomposition of the underlying relational network (as in B of Fig. 2.10). The distribution which minimises the information in the entailment structure is obtained by maximising the number of arcs that contribute to the derivation of a topic (as in A of Fig. 2.10). The limit is set by the following rule: "no essential precedence ordering may be omitted." Thus, in A, there is only one precedence requirement (a, b, c must all be understood before R_i is understood, but a, b, c may be studied in any order, or simultaneously). In general, this is not the case, though it *is* possible to eliminate precedence orderings that are not required on syntactic or computational grounds.

Binary decomposition and trade off work for disjunctive structures, but some care is needed to avoid confusion. Any disjunctive structure represents the fact that the same topic may be derived in several ways, or that the thesis is redundant. This redundancy is quite distinct from the redundancy immanent even in conjunctive structures, due to the fact that the entailment structure and the task structure have information in common. So long as this distinction is appreciated, disjunctive structures may be reduced to the set of all possible conjunctive components and dealt with as before.

3.2. Simplification

A locally cyclic (conjunctive or disjunctive) structure of topic relations stands as an understandable topic. This is emphasised by drawing a line around a structure headed by the topic in question; for example, R_j in Fig. 2.11(a).

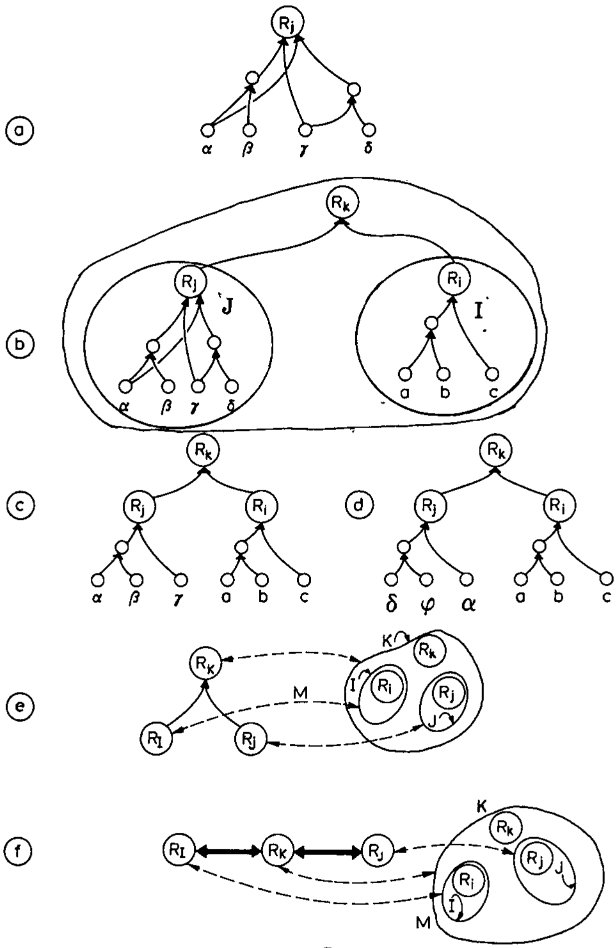


Fig. 2.11a, b, c, d, e, f. Simplifications. The circular regions in (e) and in (f) are those delineated in (b).

Call the circumscribed region J (since it is headed by R_j). J forms part of a system, insofar as the circumscribing lines are nested with respect of superordinate topics naming hierarchically arranged subclasses such as I (headed by R_i), J (headed by R_j), and K (headed by R_k), in Fig. 2.11(b).

"What is the simplification of R_i (or of R_j) in the context of R_k?"

One answer to this question is that a simplification is any irredundant or conjunctive structure, compatible with the original, and yielding the same derivation. For example, the structures in Fig. 2.11(c) and Fig. 2.11(d) are simplifications (in this sense) of R_j; there is no simplification (in this sense) of R_i. This sort of simplification (by "selection") implies that since there is less content to a course representing an irredundant thesis than there is to a course representing a redundant thesis, the "selected" irredundant representation is "simpler". Though of dubious utility (since the irredundant representations are rarely easier to learn), there is an algorithm for extracting all such "simplifications" from a given structure.

A very different kind of simplification (by consistent "smudging") maps the circumscribed regions I, J, and K of the original picture onto points representing nodes in a distinct network (Fig. 2.11(e)).

The mapping (M in Fig. 2.11(e)) is plausible enough. What must be ascertained is the precautions needed to ensure that M gives rise to a coherent simplification rather than a mess.

There is no difficulty in convincing oneself that simplifications exist, that they are widely employed in practice, and that they are used to good effect. For example, let R_k represent a statement of the gas law $P^* \times V = \text{Const} \times T^*$ as conceived by an elementary student for whom P^* is pressure and T^* is temperature, taken as matters of experience (how much "push" there is, how "hot" it is), though being, of course, susceptible to measurement. V, the volume of a container, and Const (the gas constant) are understood as thoroughly as required at any point in the course of studies for which the entailment structures have been devised.

Conversely, let R_k represent the gas law $P \times V = \text{Const} \times T$, as conceived by a fairly sophisticated student, for whom P and T are known in terms of the motion of idealised molecules and the mean kinetic energy of these idealised molecules, the volume V having the meaning it has for the elementary student. If Boltzman's constant is S, the P and T terms are defined for the advanced student by equations such as:

$$P = \frac{1}{3}\frac{N \times m \times Z^2}{V}$$

and

$$T = \frac{2}{3S}\left(\tfrac{1}{2}m \times z^2\right)$$

where $(\frac{1}{2}m \times z^2)$ = Mean Kinetic Energy, m = Mass of an idealised molecule, N = Number of idealised molecules in gas, Z = the mean velocity of idealised molecules.

The mapping M is legitimate since it may be maintained (by a physics master, for example) that if the elementary student used the prescribed measuring methods on objective reality to reach (an obviously simple minded) understanding, it would still be the case that (numerically) $P = P^*$ and $T = T^*$.

The relevant psychological requirement is that in the context of a course up to R_k (which determines, for example, the uniform connotation of volume V), no statement made in teaching R_K and understanding R_I and R_J as its prerequisites shall contradict or falsify any statement made later (when more complex material is presented) in teaching R_k and understanding its prerequisites, R_i and R_j. Of course, more "true" statements appear in understanding the "enriched" or detailed course materials.

Mappings, M, that satisfy these requirements exist if the primitives of R_k, R_i and R_j belong to (are modelled in) the same universe of interpretations, say U. It is also *possible* that topic R_K is an analogy and that its separate terms are modelled in distinct universes of interpretation, R_I in X and R_J in Y (Fig. 2.11(f)).

In general, analogy relations cannot be simplified by consistent smudging, though all of the conjunctive or disjunctive subtheses that are analogically related may be simplified. The particular example of Fig. 2.11(f) is exceptional insofar as there is a thesis containing some conjunction, to which the analogy is subordinate. Such a structure unifies the distinct universes X and Y.

For example, let I stand for elementary physics and X for a universe where Temperature (T^*) is "hotness" and pressure (P^*) is "push". Let J stand for advanced physics and Y for a universe where temperature and pressure have the other meanings T and P. There is a thesis about science which unifies X and Y in the sense that all X measurements or actions are open to expression as clusters (homomorphic images) of measurements or actions in Y. The analogy, in this case, is a cognitive reality but is not epistemologically essential.

3.3. Discussion

The educational uses of trade off and of simplification by consistent smudging are fairly obvious, though the merit of simplification becomes most obtrusive for really large scale subject matters. The following notes are an attempt to augment the concept and to exhibit the advantages in terms (as usual) of realisable operating systems. It is not too difficult to bridge the gap between quasi mechanical (but definite) realisations and classroom practice.

Just as a topic is described, so may a class of topics be afforded a coarser grained description. For example, the class named I is described by subsets of the values of the descriptors of the topics within class I, and such subsets are readily pointed out more economically by the values of additional descriptive predicates; call them *attributes*, for reference.

Using explore transactions in the coarse grained attribute space (in contrast to the fine grained descriptor space), a student can locate I or J and determine its properties. Moreover, he can establish his aim on I; meaning "on the head node of R_i in I".

At this point, supposing the operating system accommodates the underlying fine grained structure, he can mechanically "zoom in" on the detail; for example, to engage in a fine grained exploration or to relocate his aim at some node (other than the head node).

A coarse grained display of a large structure in an attribute description, circumscribing regions like I and J, is generally desirable, provided it is possible to retrieve the underlying fine grained structure and its descriptors. Practical implementation involves an interactive graphic display, the structures in question being represented in computer storage.

Under these circumstances, there is no objection to storing the entire derivation as a relational network together with its cyclic components, and it is possible, as a result, to realise an identity

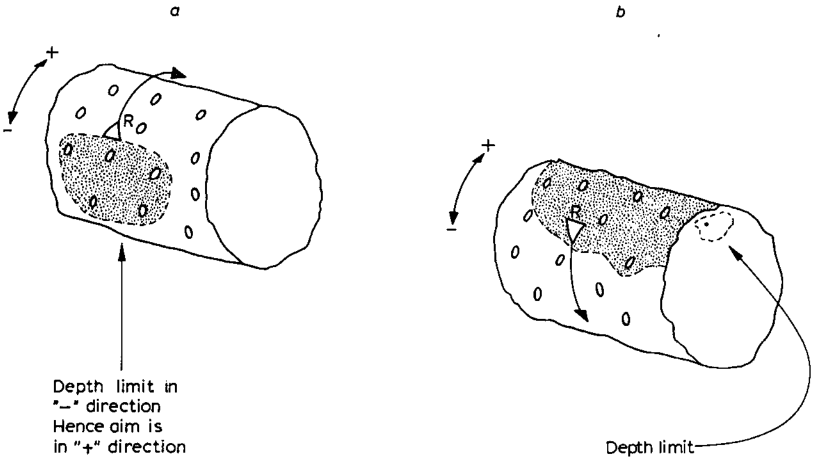


Fig. 2.12a, b. The student's aim as an oriented or directional marker on a par with the expert's head and depth. For simplicity the cyclic mesh of the stored derivation is wrapped round a cylinder (primitive topics are thus at the edges) (a) shows an aim oriented from node of topic R in one direction (+); (b) in the other direction (—). Only nodes in shaded region are displayed to the student, but he may vary area or depth.

between the aim topic chosen by a student and the head topic chosen by a subject matter expert. The student's aim of necessity becomes a vector, corresponding to the expert's "head and depth", naming the aim topic itself and a lower boundary, which may be established in several directions.

Fig. 2.12 shows two such directional aims which reverse the orientation of the syntactic depth descriptor (subordinate/superordinate). For all that, the underlying derivation is unchanged and the values attached to semantic descriptors are unchanged whichever of the two (or more) aims is selected.

4. DERIVATIONS

As noted at length in the previous monograph, the syntactic or derivational component of a thesis is represented in terms of formal topic relations (subsets of a product set) and relational operators that transform relations into other relations. The calculus of relational operators was introduced into data base design by Codd (1970) and the originality, if any, of the present approach resides in how the topic relations and derivations are specified ("from up downwards" rather than from "basic unit upwards"), certainly not in how the relations are manipulated.

Even in the field of education, other researchers have independently developed comparable schemes with their own peculiar advantages; the differences are chiefly notational. For example, Scandura's (1973) "Structural Learning" Techniques represent topics (Scandura calls them "Concepts") as sets and functions rules and "higher order" rules. Bunderson and Merrill (1973), together with their colleagues working on the TICCIT computer aided instruction system, have much the same approach. The topics appear as sets, functions and relations abutted by compositions and set theoretic combinations that either are, or are equivalent to, relational operators.

These and similar spirited schemes referenced in the previous monograph have proved useful and flexible. The present work deviates only in respect of how the topic relations and derivations are elicited (as noted already) and in the emphasis placed upon analogy relations. Though very comprehensive in most respects, the other schemes are not primarily intended to uncover the structure of analogies (as this scheme *is*).

There is nothing sacred about the choice of relational operators as a canonical means for representing derivations. The calculus is used metalinguistically and by programs like EXTEND which sort out derivation paths and determine legality. Any other competent calculus would serve just as well. In particular, the "axiomatic" schemes due to Steltzer and Kingsley (1974) are more appropriate, more amenable to manipulation by a subject matter expert, and more clearly exhibit the distinction between the syntactic (formal, axiomatic, derivational) part of a thesis and its semantic content. A good deal of our recent work has employed this axiomatic scheme in place of our augmented relational operator scheme.

As in the present discussion, Stelzer and Kingsley distinguish between what may be known (the theses represented in a GCN or General Cognitive Net) and what may be done (a set of *BG*s or Task Structures). Only the derivational component (the GCN) will be discussed.

An axiomatisation of a thesis about a subject matter (represented as a GCN) is rooted upon the following categories of objects called constituents: primary notions, derived notions, basic principles (axioms), and established principles (theories). The constituents x, y enter into two relations F (x, y) ("y is formulated in terms of x") and E(x, y) ("y is established in terms of x"), and these relations may hold as follows:

F(x, y)
Possible constituents in x, y

x	y
Primary Notion	Derived Notion
Derived Notion	Basic Principle

E(x, y)
Possible constituents in x, y

<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>Primary Notion</td><td rowspan="4">Established Principle</td></tr>
<tr><td>Derived Notion</td></tr>
<tr><td>Basic Principle</td></tr>
<tr><td>Established Principle</td></tr>
</table>

The GCN may be expressed as the complex of relations type F and E holding between a set of constituents. Since the intention is to obtain an axiomatisation, the GCN will be minimally redundant, but there is no necessary restriction upon the order in which parts of the complex relation are spelled out, nor upon the order in which the final constituents are chosen. It is evident, on inspecting Steltzer and Kingsley's examples, that GCNs correspond to generally conjunctive derivations which exhibit the kernel structure of a subject matter.

The GCN rules (for using F, E, and so on) are designed to prohibit loops; hence, analogy relations (which surely hold between the task structures; for example, the course on photography, one instance in the 1974 paper, has several universes of interpretation) are not made explicit. The prohibition is computationally convenient as well as axiomatically defensible but is unacceptable (on psychological grounds) from the present point of view.

Several kinds of compromise are possible. Our present approach is to obtain conjunctive substructures as GCNs, to adjoin an extra-axiomatic postulate that any established notion or principle is cyclic (consistency is guaranteed), to form disjuncts of GCNs after they are constructed, and to add on analogies between the F, E relations of the GCNs by an independent process. In other words, GCN rules are used locally in course assembly and the local products (GCNs or conjunctive structures) are unified by the methods already outlined or to be described.

5. THE SIGNIFICANCE OF ANALOGY RELATIONS

In hindsight, it was fortunate that conversational domains were first constructed for theses dealing with applied science. As a result, we were forced to take the representation of analogies seriously from the beginning.

In particular, analogies are non-verbally explained by executing two or more models that are built in two or more a-priori-independent processors or universes of interpretation (MF(X) and MF(Y)) together with a coupling that establishes their dependency. Though at first sight this looks like an overly complicated technique, and at the next glance seems to be a statement of the obvious, it turns out to be one starting point for a theory of innovation.

Any thesis represented in an entailment mesh is a justifiable hypothesis expounded by someone, the subject matter expert. He may remain anonymous until more than one thesis is represented in the mesh, for example, more than one scientific theory or an overall thesis about several rival hypotheses. In this case, it is necessary to name the advocates or protagonists as people, schools of thought or whatever. Call them A and B. Now A's thesis is justified insofar as A can model it in some universe and B's insofar as B can do the same in another universe, and there is a sense (to be developed in chapter 4) in which these universes are a-priori-independent.

The basic transaction between A and B, regarded as dynamic entities in conversation, is an agreement over their theses, including an agreement to differ. This agreement may sometimes be founded upon an additional act (a constituent of verification and falsification methods) whereby A's thesis and B's thesis are modelled in a common or reference universe. But, prior to that, A and B must agree upon or accept a reference universe (as a student does when he subscribes to an experimental contract). In either case, the microstructure of agreement may be complex as it will entail A's hypotheses about B (and B's hypotheses about A), in addition to the theses, alias hypotheses, to which they overtly adhere. If the act of agreement is frozen and the result inscribed in an entailment mesh, then it is an analogy relation.

Conversely, any analogy relation represented in a mesh is the inscription of a petrified agreement between people, and there is a sense in which the dormant and possibly unnamed participants (A, B) are resuscitated when the analogy is understood. This may illuminate the obscure, even cryptic, remark in the previous monograph that the basic utterances in an L Conversation are agreements, the basic statements are L Metaphors designating analogy relations. Any thesis contains such a unit, explicit or not. Most theses of interest contain many.

Chapter 3

Some Educationally Relevant Studies of Learning

The salient facts about learning, reported in the previous monograph, are as follows: (1) If a condition of *understanding* (evidenced by an explanation and a derivation of each topic addressed) is secured during the learning process then the concepts learned are stable and reliably retained. (2) Either of the expedients (teachback or the CASTE operating system) employed to exteriorise normally concealed cognitive processes as stretches of dialogue in a strict conversation also guarantee that any topic learned is understood. Moreover, these arrangements promote understanding. (3) Students may learn on their own account, adopting an autonomously generated learning strategy. Alternatively, their learning may be guided by a teaching strategy imposed by an instructor or a mechanism. (4) In either case, a student has certain natural learning strategies. These may be used or may be dominated by a teaching strategy. The natural learning strategies belong to mutually exclusive classes named holist and serialist, as do teaching strategies. (5) If a teaching strategy and a learning strategy are *mismatched* (belonging to exclusive classes), learning and retention are impaired; understanding is difficult to achieve, or even unachievable. (6) Conversely, a matched situation enhances learning and retention.

The main conclusions are summarised in Table 3.1. The differences between matched and mismatched learning, reflected in these gross figures, are more poignantly exhibited by specific differences in the tutorial dialogue, the form of explanations and derivations.

This chapter is concerned with recent findings, and it is expedient to start the discussion from a less specialised and theoretically committed point of view. The cognitive process which goes on in a conversation has certain uncontentious characteristics;

TABLE 3.1

Differences Between Understanding/Not Understanding and Matched/Mismatched Learning: (I) Detailed Data (II) Gross Data from Study Using Different Students.*.

Student Number	Clobbit Task (free learning) Test Scores		Student Classified as Serialist (S) or Holist (H)	Gandlemuller Task (Programmed text: either Serialist type or Holist type)	
	1st session (max. 30)	2 session (max. 30)		Program type	Test score (max. 30)
TB 1	13	30	S	H	9
TB 2	25	27	H	S	21
3	25	17	S	S	29
TB 4	10	28	S	H	7
5	27	23	H	H	30
6	15	5	S	H	9
TB 7	27	30	H	H	30
8	27	18	S	S	28
TB 9	28	30	H	H	30
10	17	13	S	S	30
11	23	19	H	S	19
TB 12	18	30	S	H	9
13	21	25	H	H	30
14	26	17	H	S	16
TB 15	21	28	H	S	20
16	22	17	S	S	29

* Students classified as holist/serialist on Clobbits task. Difference between 1st sessions/2nd session shows effect of teachback or simulated teachback (teachback test scores > simulated test scores significant 0.01 > p). Same students later learned from matched/mismatched programmed text about Gandlemuller taxonomy (similar in form but not in content to Clobbits). Subsequent retention test on Gandlemuller material shows matched scores > mismatched scores difference, significant at 0.001 > p. All comparisons use Mann—Whitney U-Test.

TABLE 3.1 (continued)

	Number	Mean % Test	S.D.
Matched	8	61.0	4.8
Mismatched	8	31.5	6.1
Any Understanding Conversational	10	96.0	5.6

Difference Matched > Mismatched significant at $0.001 > p$; difference Conversational > Matched significant at $0.001 > p$; Experiment used CASTE as operating system. (In part from Pask and Scott, 1972.)

nearly all theoretical formulations point them out, using different terminology. The characteristics of immediate concern are *embedding* (a neologism) and *style.*

1. EMBEDDING AND FIXITY

Embedding is an omnibus name for the conservative aspects of cognition. Not only concepts that are officially learned, but peripheral and possibly extraneous concepts become entrenched in a student's repertoire due to ubiquitous trapping and entrainment phenomena. Amongst the peripheral concepts that become entrenched is the *style* of learning about the officially relevant concepts. Regarding these "official" components as the "figures" in a psychological test, *style* is the "ground" against which the "figures" are displayed.

For Piaget (1921 to 1968) and his school, for example, the embedding operations are the general cycle of accommodation and assimilation subsumed by adaptive transformation and modulated by group development, *decalage* and the like. For Bartlett (1932) embedding is the conservation and invariance of schemata. Harlow (1959), at quite a different level, invokes mechanisms related to learning set; Helson (1964) an adaptation level; and the information-processing psychologists an irreversible component — for example, transfer from short-term to long-term storage (Atkinson and Shiffrin 1967) or distributed retention (Simon and Feigenbaum 1964). Our own theory predicts embedding as a consequence of executing the procedures (or compiled programs) called concepts and memories. These theories and a legion of others differ, with respect to embedding, chiefly in the form of wording employed.

If it happens that exclusion principles can be formulated so that one class of concepts is incompatible with another class of concepts, at least in the sense that members of these classes cannot be conjointly assimilated into a cognitive repertoire, then it is possible to strengthen embedding, and to speak of fixity. Again, all the theories do so. Perhaps the most general expression for incompatibility is *interference*, as this term is used by the eclectic information-processing psychologies, notably, Broadbent (1963, 1971), Entwistle (1975), and Welford (1968). Combining interference with embedding leads to the prediction of fixed states, either deep rooted concepts or deep rooted habits of searching for an integrating concept. Festinger's (1956) "Cognitive Dissonance" is a special case of fixity observed in such contexts as adherence to social beliefs (the original study) or hypotheses leading to a decision, for example, to purchase a product. "Cognitive Dissonance" is a specific mechanism for fixing concepts. Under these circumstances, whoever exhibits the fixity will reject or pervert to affirmative form evidence denying the accepted belief or hypothesis. In the present theory, "cognitive fixity" is employed as a noncommittal name for the result of processes that may be identical with Festinger's "dissonance" or which depend upon more general interference effects. The phenomenon of fixity is so well and widely evidenced that it counts as a basic fact of cognitive psychology.

We return to the question of fixity very soon, but before we do, some exclusion principles will be stipulated.

2. STYLE OF LEARNING

Style is the other salient characteristic of cognition. Since the previous monograph was written, several recent studies vouch for the reality of distinct and idiosyncratic learning styles.

For example, there is a body of work due to Daniel (1975), Dirkzwager (1974), Beishuizen (1974), and their colleagues on style in logical problem solving; by Klix (1973) on concept acquisition; by Strubb and Levitt (1974) on decision style; by Hankins (1974) on styles exhibited by engineering design students. Also, Landa's (1971) major work on logic and language learning has appeared in an English translation, edited by Kopstein, and apart from demonstrating the value of concepts that delineate rules, the protocol data clearly exhibit distinct (and often ineffective) indigenous styles. Mulling over the last few issues of *Instructional Science*, about one fifth of the papers are devoted to this topic, for example, Bree (1974) or Allen (1974); and about half of the *Structural Learning Proceedings* (Scandura, Ed. 1974).

Serious quantification of style is presaged and to some extent anticipated in Newell and Simon's (1972) account of thinking. But the notion of style (in contrast to its empirical exhibition) goes back to antiquity; see, for example, Yates (1966) scholarly account of the memorial and combinatory systems employed by the ancients, by the mediaeval rhetoric schools, and others. Moreover, differences in style are reliably detectable outside the laboratory, most dramatically perhaps in the way people explore, learn about, and image their environment (Lynch 1960; Glanville 1975). There is little need to labour an obvious point; style is one of the commonest psychological observables; it has always been recognised by tutors, priests and actors; it is nowadays a respectable topic for overt discussion.

The conversational situations we employ reveal quite definite learning styles, several of which were described in the previous monograph. There, we mainly stressed two styles which are exhibited in a strict conversation as classes of learning strategy, holist and serialist. Shortly, it will be appropriate to recall and buttress the holist/serialist distinction, but before doing so, it is worth considering the styles manifest under less rigidly controlled conditions, in conversations maintained by "Free Learning" and "Teachback" for example (Chapter 1 and the previous monograph).

2.1. Comprehension Learning and Operation Learning

When a complex subject matter is learned by a student (for example, statistics, the menstrual cycle, various taxonomies) and when pains are taken to exteriorise his mental activities, it is possible to distinguish between comprehension learning and operation learning as dominant learning styles. The distinction is clearcut but not dichotomous. The styles are as follows.

Comprehension learners pick up an overall picture of the subject matter; for example, in a taxonomy the number of classes, the type and number of items in a class, redundancies in the taxonomic scheme, relations between the distinguished classes, a clear picture of where information about items can be discovered. These learners may or (significantly) may not be able to perform the operations required to use the subject matter information (here, to classify specimens). Often enough, comprehension of a layout or framework exists in the absence of rules or operational meaning and perhaps in ignorance of details that have to be filled in if the taxonomy (or whatever) is to be used in practice.

Conversely, operation learners pick up rules, methods and details but are often unaware of *how* they fit together, still less of *why they do* fit together. Typically, operation learners have at most a sparse mental picture of the material. Their recall of the way they originally learned (insofar as they learned at all) is guided by arbitrary numbering schemes or accidental features of the tutorial information frames.

2.1.1. Multi Purpose Experiments

A series of experiments (called the "multi purpose" experiments for reference) were carried out to investigate: (a) Means of determining style and their reliability, (b) the effect of securing or not securing understanding of each topic (by comparing effective teachback with simulated "ineffective" teachback, (c) the stability of a style-determined learning strategy over different subject matters, and (d) the influence of a matched as against a mismatched mode of tuition (teaching strategies either matched to a student or mismatched are built into programmed instruction materials). The subject matters used for learning and for style assignment were the two taxonomies (Gandlemuller and Clobbit) of the earlier studies, (Pask and Scott 1972), two biological subjects "The Operon" and "The Menstrual Cycle", and an inductive inference task.

The experimental design is shown in Fig. 3.1. The 62 students were from Kingston and Chiswick Polytechnics. Two batches were processed. Batch 1 (32 students) was exposed to the "Clobbit" taxonomy free learning task and each student was classified as a holist or serialist. At approximately two week intervals, subjects returned first for exposure to the "Gandlemuller" taxonomy task in a matched or mismatched condition, and second, for exposure to the operon cycle task in a matched or mismatched condition. They returned later for a final session of retention tests and teachbacks.

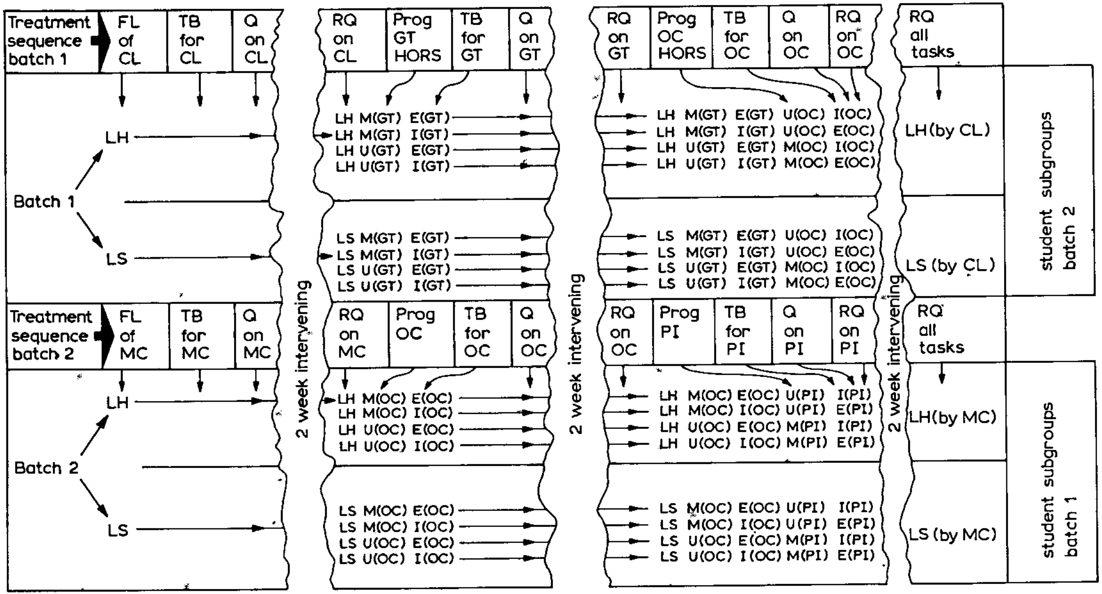


Fig. 3.1. Experimental design securing a balanced combination of matched (M) and mismatched (U) instruction and of effective (E) and ineffective (I) teachback (TB), for students classified as like holist (LH) or like serialist (LS) after free learning (FL), different initial subject matters (the Clobbits taxonomy CL and a menstrual cycle MC); other abbreviations are GT for Gandlemuller taxonomy, OC for operon cycle and PI for probabilistic inference. Q means "Questions Test" (including explanatory questions) and RQ is test questionnaire for long term recall.

TABLE 3.2

Summary of Data From Free Learning Session *

Student Group		Frequency of Intention types						Mean No. of Cards/Cluster	Mean Values of Uncertainty (H), Correct Belief (Θ), Look Ahead Uncertainty (H*), and Look Ahead Correct Belief (Θ*)			
		I	II	III	IV	V	VI		H	Θ	H*	Θ*
Batch 1 Operation Learners (n = 18) "Like Serialist"	Means	1.6	2.8	9.3	3.5	2.2	0.2	1.74	1.29	0.38	0.71	0.06
	SDs	1.7	1.9	5.3	1.9	2.5	0.03	0.36	0.80	0.31	0.87	0.28
Batch 1 Comprehension Learners (n = 14) "Like Holist"	Means	1.2	1.4	3.9	7.9	3.8	3.7	2.95	1.19	−0.05	1.32	0.28
	SDs	0.5	0.8	3.4	1.8	1.1	0.42	0.42	0.60	0.40	0.42	0.66
Batch 2 Operation Learners (n = 17) "Like Serialist"	Means	1.7	2.4	8.6	1.4	1.9	0.3	1.72	1.09	0.37	0.43	0.36
	SDs	1.0	1.3	1.9	1.0	1.5	0.5	0.30	0.68	0.21	6.47	0.32
Batch 2 Comprehension Learners (n = 13) "Like Holist"	Means	1.8	2.6	2.8	8.6	2.0	3.6	2.98	1.34	0.30	1.76	0.38
	SDs	1.0	1.2	1.7	2.0	0.9	1.0	0.36	0.89	0.26	0.64	0.32

* Printed with the permission from *British Journal of Educational Psychology* (Pask, et al., 1976).

Statistical summary:

1. Frequency of intention classes IV and VI: comprehension learners > operation learners ($p < 0.001$).
2. Mean no. of cards/cluster: comprehension learners > operation learners ($p < 0.001$).
3. Operation learners have significantly higher means for θ ($p < 0.01$).
4. Comprehension learners provided values for H* and θ* on significantly more occasions ($p < 0.01$). (All comparisons by Mann-Whitney U-Test.)

Key for intention types:

I = exploratory perusal of cards
II = general search for information
III = looking for a particular piece of information
IV = looking for several pieces of information
V = testing a single predicate hypothesis
VI = testing a multi predicate hypothesis

TABLE 3.3

Content Analysis of Teachback Protocols

Student Group	Program Type	Subgroup Matched/ Mismatched		Classification of Statements 1	2	3	4	5	6	7
Batch 1 Operation Learners (n = 18) "Like Serialist"	Gandlemuller	matched (n = 9)	Means	1.5/0.5	0.2	0.8	0.3	0	0	34
			SDs	1.0/0.5	0.5	0.7	0.6	0	0	3.8
		mismatched (n = 9)	Means	1.0/0.1	0.2	0.3	0.8	7.0	1.5	30
			SDs	0.5/0.3	0.5	0.6	0.7	3.6	0.9	4.2
	Operon	matched (n = 9)	Means	0.9/0.2	0.2	0.5	0.2	0	0	35
			SDs	0.4/0.5	0.5	0.4	0.5	0	0	3.6
		mismatched (n = 9)	Means	1.5/0.5	0.2	0.5	0.2	8	2.2	32
			SDs	1.0/0.5	0.5	0.4	0.5	2.6	2.4	5.1
Batch 1 Comprehension Learners (n = 14) "Like Holist"	Gandlemuller	matched (n = 7)	Means	0.5/0.3	2.3	6.0	5.0	4.5	4.0	49
			SDs	0.5/0.4	1.9	4.2	4.0	4.2	3.8	14
		mismatched (n = 7)	Means	0.8/0.4	3.0	3.7	3.8	4.4	1.1	38
			SDs	0.1/0.3	2.6	1.8	2.2	2.6	2.4	6.3
	Operon	matched (n = 7)	Means	0.4/0.3	1.9	8.1	5.7	2.7	12	48
			SDs	0.5/0.4	2.2	3.6	2.8	3.3	9	10
		mismatched (n = 7)	Means	1.0/0	0.4	4.5	0.6	6.1	0.2	36
			SDs	0.5/0	0.5	2.4	0.5	3.1	0.4	5.0
Batch 2 Operation Learners (n = 17) "Like Serialist"	Operon	matched (n = 9)	Means	0.5/0.3	0.1	0.1	0.1	0	0	34
			SDs	0.3/0.3	0.1	0.1	0.1	0	0	4.5
		mismatched (n = 8)	Means	0.8/0.1	0.5	0.5	0.2	7.2	2.2	28
			SDs	0.7/0.1	0.7	0.7	0.1	2.4	1.8	3.6

	Probabilistic Inference	matched (n = 8)	Means	0.3/0.2	6.3	6.1	0.4	0	0	32
			SDs	0.1/0.1	0.1	0.1	0.2	0	0	3.4
		mismatched (n = 9)	Means	1.3/0.2	0.2	0.5	0	9.1	2.1	33
			SDs	1.1/0.1	0.1	0.2	0	2.6	1.6	5.4
Batch 2 Comprehension Learners (n = 13) "Like Holist"	Operon	matched (n = 7)	Means	0.1/0.1	2.6	6.1	8.0	6.3	5.3	52
			SDs	0.1/0.1	1.0	1.9	2.0	2.0	1.8	12
		mismatched (n = 6)	Means	0.5/0.2	3.5	3.2	2.5	4.0	0.3	40
			SDs	0.1/0.1	1.2	0.9	0.7	1.4	0.1	3.8
	Probabilistic inference	matched (n = 6)	Means	0.2/0	1.9	7.1	4.5	0.6	6.1	61
			SDs	0.1/0	0.4	1.5	1.0	0.2	2.0	13
		mismatched (n = 7)	Means	0.7/0.1	0.9	4.7	0.2	5.5	1.7	34
			SDs	0.2/0.1	0.2	1.6	0.1	2.8	1.8	3.1

* Printed with permission from *British Journal of Educational Psychology* (Pask, 1976)

Key:
1 = falsehoods/corrected falsehoods
2 = inventions
3 = statements of information to come or delivered
4 = statements deduced
5 = irrelevant
6 = redundant
7 = total no. of statements (excludes interjections, repetitions and corrections)

Statistical Summary:
1. Comprehension learners type 2 statements > operation learners type 2 statements. ($p < 0.001$)
2. Comprehension learners type 3 statements > operation learners type 3 statements. ($p < 0.001$)
3. Comprehension learners type 4 statements > operation learners type 4 statements. ($p < 0.001$)
4. Mismatched operation learners type 5 and 6 statements > matched operation learners type 5 and 6 statements. ($p < 0.001$)
5. Mismatched students uncorrected falsehoods > matched students uncorrected falsehoods. ($p < 0.001$) (All comparisons by Mann-Whitney U-Test.)

Students given ineffective teachback on Clobbits were given effective teachback on Gandlemullers and ineffective teachback on the operon cycle. Students given effective teachback on Clobbits were given ineffective teachback on Gandlemullers, and effective teachback on the operon cycle. At each stage, half the students assigned to effective teachback had been classified as holists, and of those, half were in the matched condition. Retention tests were given and teachback protocols elicited for all tasks completed on earlier sessions, each time a student returned for further sessions.

Batch 2 (30 subjects) was treated similarly, but for them the free learning task was the menstrual cycle and the two programmed text tasks were the operon cycle and probabilistic inference. The mechanised version of the menstrual cycle task was introduced into the design during the latter part of the project; 18 of the Batch 2 students were classified as holist or serialist on the basis of their performance on the mechanised task.

2.1.2. Main Results

Predicted style assignment is based on the indices of Table 3.2. The most reliable quasi objective method of assignment depends upon the intentions that students expressed (by edict) when accessing data items during free learning (the intention categories of Chapter 1). An independent determination is possible by means of confidence estimates over response alternatives to questions about items lying ahead of those currently addressed (Table 3.2). A more readily quantified though less discriminating prediction is obtainable from the mechanically monitored free learning situation described in Chapter 1. Two indices, shown in Table 3.3, are the frequency of repetitious explorations and the extent to which immediate teachback order recapitulates the order in which items are addressed during free learning.

Retrospective determinations of style were carried out after learning in those phases of the design devoted to teachback and recall under interrogation. It is again possible to classify the students as comprehension learners or operation learners by the content analysis of teachback protocols (Table 3.3); for example, by ascertaining the extent to which the students do or do not have a picture of how they learned the subject, the topics they regarded as pivotal and whether or not ordered segments of learning became fragmented upon recall. In this particular study the retrospective

TABLE 3.4

Scores on Tests for Other Cognitive Traits *

		"Logical Meaning"	"Embedded Figures"	"Analogies"	Perceptual Discrimination	"Circles" Test Number of Items/ Number of Circles Used
Serialist Batch 1 (n = 18)	Mean	3.9	34.4	11.8	35.2	7.2/ 3.0
	SD	2.8	10.4	4.6	7.9	2.3/ 2.6
Serialist Batch 2 (n = 17)	Mean	3.8	34.2	12.8	35.5	6.6/ 7.8
	SD	2.6	11.4	3.9	8.9	1.9/ 2.2
Holist Batch 2 (n = 14)	Mean	4.1	36.7	18.8	30	10.5/12.6
	SD	2.5	12.1	3.8	6.2	2.9/ 3.1
Holist Batch 1 (n = 13)	Mean	4.5	38.4	18.3	36	9.9/12
	SD	2.3	9.4	3.3	8.6	2.4/ 2.4

* Printed with permission from the *British Journal of Educational Psychology*. (Pask, 1976)
Statistical summary:
Holist Scores on the "analogies" test are generally higher than serialist scores ($p < 0.05$).
Holist scores on the "circles" test for "divergence" are generally higher than Serialist scores ($p < 0.05$).

indices are influenced by a variation in the teachback conditions interpolated during earlier phases (either "effective teachback" or an "ineffective teachback" which plausibly imitates the genuine teachback conditions but elicits indices of correct response that are not tied to giving an explanation, i.e., no explanation is encouraged or obtained.

Various tests (embedded figures, logical word-problem solving, analogy-completion) were administered to the same students in an attempt to discriminate styles. Modest and marginally significant correlations exist (Table 3.4).

2.2. The Spy Ring History Test

Style may be predicted quite reliably as a function of performance in conversationally administered tests (the Clobbits and Gandlemullers test of the previous monograph come into this category). One test which has proved extremely informative is called the "Spy Ring History" test. It has been administered to 5th and 6th formers at Henley Grammar School (65), students at the Architectural Association School of Architecture (40), and at various colleges and Polytechnics (50 or more).

The Spy Ring History test permits a student to learn a fairly complex subject matter by synoptic methods, particulate methods, or both, and the performance indices pick up the extent to which he has made use of a synoptic or particulate approach. (Either approach is useful and has its merits; full scoring is most easily achieved if a student has exercised and relied upon both methods, though it is possible to give correct replies on all of the test questions by adopting only one method.)

The test is based upon paired associate lists which indirectly specify a communication network linking spies, who (earlier version) are identified by alphabetic characters or (later version) memorable code names ("Abel" and "Boris" and so on). Ostensibly list-learning tasks of this type were employed by Hayes (1965) and later by Michon (1966). The serially presented list actually specifies a graph which can be recalled quite easily and which could be apprehended at a glance if it was (instead) displayed as a visual image. Some typical lists and networks are collected together in Fig. 3.2. The students are told that the lists determine pathways of communication between members of a spy ring during the last

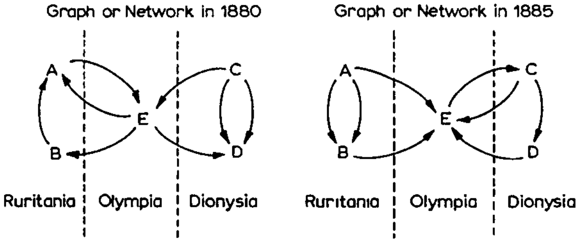


Fig. 3.2. Lists and graphs for Two Historical Epochs in the Spy Ring History test for Competence and Style. The lists are presented on tape. The network graph is not shown to the student.

century and the development of the ring is sampled at years 1880, 1885, 1890, 1895, 1900 (one network to each period in history). The networks all contain the same spies (in the same roles or positions) and are further described by a "countries" predicate, assigning each spy position to a "country" (Ruritania, Dionysia, Olympia, as imaginary European States). The several spy network graphs form a graph-product or "Cartoon" (Winner 1973). Parenthetically, the mathematical properties of Cartoons have been investigated by Winner. For example, some Cartoons are periodic (so that the morphism which relates one graph to the next in an indexed sequence leads to repetition of the same graph after so many cycles of iteration); some Cartoons are aperiodic. The five graphs in the Spy Ring History test belong to a Cartoon which becomes periodic after six repetitions (Fig. 3.3). This property, though convenient in designing the question format of the test, is not essential.

A student is required to learn, and later to explain, various features of the spy system history. The input he receives is in the form of paired associate lists, each specifying one period's spy ring configuration (for 1880, 1885, 1890, 1895, 1900), each list qua list, being learned to a criterion of faultless repetition, before the next is presented.

After learning, students are questioned in various ways. The object of the interrogation is to elicit complete information about the entire history including the minutae of each era or epoch (details of the questions and replies are given in Pask, Scott, et al. (Tech. Rep. 1974). Students can seldom provide all the informa-

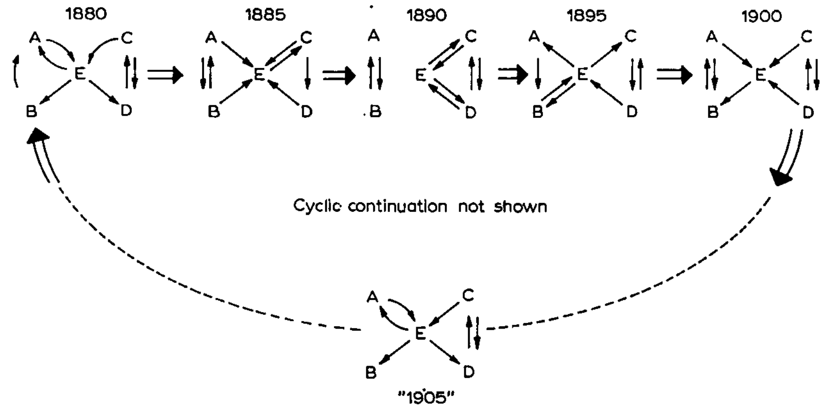


Fig. 3.3. The six graph cyclic cartoon and the five graphs that are presented for Historical Epochs 1880, 1885, 1890, 1895, 1900. The sequence is completed by "1905" at which point the process returns to 1880.

tion required, but typically give what they can in one of two patterns. Some students, classified as comprehension learners and potential holists, can answer broad questions like, "What went wrong with the spy system around 1885?" or even predictive questions relying for cogency upon the cyclic character of the six graph Cartoon from which the five graph Cartoon used in the test is extracted. For example, "Do you think that outstanding events are likely to be repeated in 1905; if so, why?" Other students, classified as operation learners and potential serialists, focus upon the individual networks or even the paired associate lists. For example, "How could Abel communicate with Boris in 1890; by how many paths, what are they?" or even "Draw the spy network of 1890". It should be emphasised that all students are required, if possible, to answer each kind of question as well as intermediary enquiries like, "Draw the boundaries of Dionysia, Olympia and Ruritania on a map", and "Say which of the agents belong to each country". The point is that comprehension learners will, if successful in this pursuit, work out the particulars by inference within the framework of global properties, whereas operation learners, again if successful, work out the answers to global questions by piecing together their local knowledge of particulars. These tendencies are reliably exhibited providing the overall score is high enough to provide a discrimination. If method of learning and success are both taken into account, the scoring categories are as follows.

(a) Operation Learner, Successful (in deriving global properties).
(b) Operation Learner, Unsuccessful (in deriving global properties).
(c) Comprehension Learner, Successful (in deriving local properties).
(d) Comprehension Learner, Unsuccessful (in deriving local properties).
(e) Both styles used successfully, called Versatile.
(f) Both styles and unsuccessful performance, or equally, neither style (that is, low overall score on the test as a result of which no discrimination is possible).

Certain qualifications and extrapolations are in order.

First, the test is "officially" biassed by the requirement of fully learning the original lists to favour recall of particulars even by the comprehension learner. Probably due to gross interference between the lists (which occurs if a student fails to assimilate them as the network graphs of historical epochs), the "official" bias is not, in fact, obtrusive.

Next, although the test is effective when personally (and conversationally) administered, it has not been possible to use it successfully in mass administration. Students treated in various ways as mass recipients do not achieve a high enough overall score. If they learn at all, interference dominates their recall.

Finally, it is extremely important to present a fairly rich semantic interpretation of the syntactic or formal structure. If the graphs are baldly interpreted as communication networks and the predicates as country boundaries, successful comprehension learners clothe the structure in further (redundant) properties of their own invention (a gambit previously observed amongst redundant holists) and use the imposed description scheme as a means for accessing the necessary data. Though we cannot prevent invention, and do not wish to do so, it is easier to quantify and discuss what goes on if the invention is tied to a known, rich and redundant account, which is open to scrutiny (anecdotes about the spies, pictures characterising the countries, and so on). We noted a similar requirement in the context of mechanical operating systems; it is necessary to ensure by aim validation that a student gives meaning to his aim topic, that he does not merely select an uninterpreted node in the entailment structure because of its index number or position.

One of the reasons why rich interpretation is crucially important emerged very much in retrospect and is discussed more fully in the sequel. At the least provocation, tasks like Spy Ring History are construed as "academic": as just another mental test or examination. The material *is* so construed during mass administration, and the construing is not altogether perverse. However, the result is crippling for it seems that institutions, the general nature of curricula, and subject matter presentation bias many students in their approach to the task concerned. They feel impelled to treat learning serially/operationally. To do so is a prerequisite for success; it is part of the task specification and regardless of their aptitude in the matter, they *do* tackle the task serially.

The belief has a large grain of truth in it so far as examinations are concerned, though no doubt the degree of restriction is over stressed, but obviously, the existence of the serial/operational bias defeats the attempt to discover which style a student is best able to use.

If these precautions are taken, the Spy Ring History test is a creditable predictor of style. Although the test was developed and piloted during the multi purpose study, it was seriously employed in later experiments involving the operating system INTUITION (Chapter 1) and, as judged by the subsequently observed learning strategies, the comprehension learners appear (in the operating system) as holists and the operation learners as serialists. The data are shown in Table 3.5 (notice, these students are drawn from a different population; the students in Tables 3.2 to 3.4 have no connection with those of Table 3.5.

2.3. Analogy Learning

How and what do the successful students learn?

It is argued that comprehension learning *must* involve valid analogy relations * and that operation learning *may* do so (recall

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* "Analogy Relation" is used with more than usual rigour to designate a morphism between interpreted topic relations. The simplest morphism is a one to one correspondence, or isomorphism.

TABLE 3.5

Spy Ring History Test Scores for Students Whose Learning Strategy on INTUITION was Holist or Serialist *

Student Number	Holist or Serialist on INTUITION	Comprehension (A) (INTUITION serialist prediction)	Operation (B) (INTUITION holist prediction	Score on Neutral Question(s)	Success = $\frac{A + B + C}{300}$	Operation (+) Comprehension (—) bias = (A — B)/100	Versatility
1	S	66	20	60	0.49	0.46	0.01
2	S	80	0	80	0.53	0.80	0.001
3	S	54	15	54	0.41	0.39	0.04
4	S	76	5	80	0.53	0.71	0.03
5	S	72	0	68	0.47	0.72	0.005
6	S	60	25	60	0.48	0.35	0.09
7	S	83	5	76	0.55	0.78	0.03
8	S	79	0	70	0.50	0.79	0.006
9	H	70	90	76	0.79	—0.20	0.48
10	H	22	72	68	0.54	—0.50	0.11
11	H	16	85	76	0.59	—0.69	0.10
12	H	64	95	64	0.74	—0.31	0.39
13	H	92	82	94	0.89	0.10	0.71
14	S	100	5	100	0.68	0.95	0.05
15	S	56	0	60	0.39	0.56	0.003
16	S	62	10	66	0.46	0.52	0.04
17	S	84	15	84	0.61	0.69	0.11
18	S	78	15	80	0.58	0.63	0.09
19	H	88	90	86	0.88	—0.02	0.68
20	H	36	72	88	0.65	—0.26	0.23
21	H	18	74	68	0.53	—0.56	0.09
22	H	18	70	68	0.52	—0.52	0.09
23	H	82	72	84	0.79	0.10	0.50

* In Part From *British Journal of Educational Psychology* (Pask, 1976)

parenthetically the correlations of Table 3.4). Further, successful learning is an admixture of comprehension and operation learning in which one style or the other may be predominant.

Where are the analogies in the Spy Ring History test?

The different spy ring networks (for 1880, 1885, 1890, 1895, 1900) are held together by an analogy between the "graphs", each considered as an interpreted formal relation. Moreover, there is a very sound sense in which the entire Cartoon must represent an analogy. Without prejudicing this point, there are other optional analogies; for example, the "countries" may be regarded as analogous and so may subgraphs of a given graph. Any successful student must learn, and learn to use, certain analogy relations; he may, as a matter of choice, learn others. The successful comprehension learner places a great deal of reliance upon analogies; the successful operation learner makes less use of analogical inference and integration. A versatile student does all of this.

2.4. Cursory Globetrotting, Improvident Learning, and Versatility

Turn now to the less successful learners and consider their deficiencies, which are summed up by comparison with versatile behaviour in Table 3.6.

On inspecting the records and student replies to deeper interrogation, there appears to be a consistent trend. The comprehension learners who fail to make the grade (but have a high enough

TABLE 3.6

Relation of Operation Learning and Comprehension Learning to the Commonly Observed Pathologies; Globetrotting and Improvidence

		Comprehension learning	
		Yes	No
Operation Learning	Yes	Versatile	Improvidence
	No	Globetrotting	Failure

Versatile students, showing neither pathology, are successful as comprehension learners and operation learners. Although the defects are clearcut, the dichotomies in this scheme represent "dominances" or "biasses"; for example, "Failure" students do not lack all comprehension/operation learning capacity, but execution of either process runs into difficulties.

overall score to merit comment upon their performance) use the purely descriptive components of an analogy and prove unable to grasp or transfer an underlying principle. The 1885 epoch is "like" the 1895 epoch because of some vaguely perceived similarity or just because they are a decade apart. The student fails to use or appreciate the genuine similarity of process which is common to the *different periods in historical development.* For the difference component of the analogy relation it would be possible to substitute all manner of given or invented distinctions. But there is, in this case, only one genuine similarity (in general, there are many legitimate similarities, but the class boundaries are strictly drawn). To phrase the matter so that it fits the idiom employed in the rest of the argument, it may be said that unsuccessful comprehension learners are able to describe a topic relation and thereby to derive its description from others, but they fail because they are unable to complete the derivation and build a concept. As a result, they are also unable to explain whatever is described. They comprehend only in the sense of making descriptions. They do not augment their comprehension by the operations needed to form a concept.

The less successful operation learners show signs of a converse difficulty. As a rule they are quite able to explain anything they know, using partial complementation ("there is a missing link" or "this spy must communicate with the others because I know the network is fully connected and the parts I can recall are disjoint"). Their stumbling block is inability to describe analogical relations between distinct entities, and it is usually manifest in an attempt to learn and recall the spy network of each epoch as a distinct graph. It is virtually impossible to learn and store five separate spy networks without destructive interference, and the problem is particularly acute if the student attempts to regard them piecemeal, ultimately, in terms of the original paired associate lists unmodified by any further structure. It looks as though the students in question are adept at concept building operations but are embarrassed by inability to comprehend descriptions.

Now the difficulties experienced by unsuccessful comprehension learners and operation learners parody two pathologies of learning which are quite generally recognised. I shall call these pathologies *Globetrotting* and *Improvidence.*

In its most pronounced and pernicious form Globetrotting leads to chains of tautologous constructions such as "a city is like an ant hill is like a beehive and that in turn is like a city". Unfortunately, the student is frequently unable to explain (and has no concept for) either a city or a beehive or an anthill; moreover, even if he does have a grasp on one of the concepts cited, he cannot say *how* ant hills and beehives are similar, so that he cannot validly derive the form of the remaining relations.

Such vacuous constructions are generally and rightly frowned upon. But it is important to realise that analogical reasoning is not in itself improper; on the contrary, it is essential to effective learning. Moreover, provided that a firm similarity is recognised, the analogical argument can proceed by way of many different descriptions, having the similarity in common but distinguished by employing various differences (period in history, character of the agents, social and political atmosphere). Finally, it is possible to base an analogical derivation upon a very terse description invoking but one difference, or upon a redundant description involving many related differences. Both redundant and irredundant descriptions are justifiable, though particular students have a preference for one or the other.

Improvidence is just as counterproductive as Globetrotting and is the reverse of it; namely, operation learning in the absence of comprehension learning. The pathology is clearly exhibited in connection with subject matter that is artificially (though perhaps usefully) carved up by traditional demarcation lines or established disciplines. For example, it is common practice to divide physics into neatly specified compartments such as "heat" and "light" and "electricity" and "mechanisms" and "magnetism"; to divide psychology into departments like "perception" and "motivation" and "learning". It would be stupid to reject these divisions; some description is required as a guide around the subject matter and these divisions are probably more defensible than most. But the existence of any divisions (and some divisions are surely essential) encourages the profligate deployment of cognitive resources manifest as Improvidence: failure to use the valid analogy relations that exist. Science, in particular, is replete with valid analogy relations, denoted by metaphors. Their formal similarities are captured in such notions as "Field" and "Dual" and "Equilibrium" and "Conservation of Quantity" or less widely applicable notions like "Conjugate" and "Valency" and "Inertial Frame".

Suppose an improvident learner is coming to grips with a general physical concept. For example, one concept we examine later in the book is "Oscillator". The student learns first about a mechanical oscillator, made from a spring, an attached mass, a frictional component and a forcing displacement. Probably at, or before, this juncture he learns a formal relation (the 2nd order differential equation governing all harmonic oscillators). Next starting from scratch, he proceeds to learn about an electrical oscillator made from a capacitance, an inductance, a resistive component, and a forcing potential variation. Again, the equation is pointed out, and it may be noted that the same equation covers the behaviour of mechanical and electrical oscillators. However, this fact, which establishes a strict analogy between the electrical and the mechanical departments, was not used in learning about electrical oscillators; nor will it be used in addressing oscillators in different departments.

An improvident learner wastes effort. It is quite unnecessary to learn and relearn the same formal relation in different universes of interpetation. Not surprisingly, the reconstruction of many ostensibly unrelated concepts gives rise to considerable interference. Topics in different departments (mechanics and electricity say) are treated like disparate lists. Without recognising the valid correspondences (mass ↔ inductance, friction ↔ resistance, and so on) that relate the departments, there is little positive transfer (as there is when the analogy is appreciated), and any transfer that takes place becomes negative if the correspondences are distorted.

For these reasons, improvidence is culpable, though, because of the curricular/academic bias noted in Section 2.2, students are less often blamed for it. We comment that an improvident student who does make progress must be an outstandingly good operation learner; otherwise, he would proceed at a snail's pace. Further, his success depends upon regarding the departments as rigid categories. Without comprehension learning the valid correspondences, this is the only way to avoid negative transfer founded upon arbitrary and usually false similarities.

Globetrotting and Improvidence are both well recognised by practicing teachers, and it is probably gratuitous to quantify evidence supporting their existence. Data on their frequency of occurrence are available from recent studies of examination essays (Pask, Scott, et al., Tech. Report, 1974).

2.5. A Classification of Learning Styles

On the basis of introspection and commonsensical observation, it seems that any coherent act of learning involves at least two processes. First, a concept is described in terms of other descriptions. This operation, dubbed *Description Building* or *DB*, is equated with appreciating a topic. So, for example, a student able to appreciate an aim topic (chapter 1) builds a description of it; in general, people can describe whatever occupies their attention. Secondly, there is a concept building (or according to our formulation) a *Procedure Building* operation, for short *PB*, as a result of which a concept is constructed to realise the description.

Tentatively, a "coherent act of learning" means an *understanding* (again in the technical sense), and we posit that both the first and the second operations (*DB* and *PB*) are involved in achieving an understanding. These loosely stated speculations are backed up by a more detailed and well-grounded discussion in Chapter 5, and in Section 3 of this chapter. But this statement is sufficient for the immediate purpose.

Again, introspection, supported by common observation, suggests that descriptions of concepts may be global or local. A global description is typically redundant, but an irredundant description spanning many other concepts or based upon ancestors that are united by an analogy relation will also count as global. Learning strategies that rely upon global descriptions tend to be holistic. In contrast, a local description is parsimonious; it rests upon a minimal set of supporting topics. Learning strategies relying upon local descriptions are serialistic.

The global/local distinction was introduced after completing the multi purpose experiments, though it was suggested by the results obtained. The distinction was first actively employed during the current experimental series using the INTUITION operating system installed in schools and colleges (Henley Grammar School, Twickenham Polytechnic, AA School of Architecture, Furzedowne College, Streatham).

By combining a bias to *DB*, a bias to *PB* or both with the global/local distinction, we obtain the categories of learning style shown (and related to operation/comprehension learning) in Table 3.7. Although the *DB* process is related to comprehension learning and the *PB* process to operation learning an adequate discrimination also relies upon a test for global and local orientation.

Various criteria are used as global/local discriminators. When students have completed learning they are asked to recall how they learned. Amongst other things, students are required to classify cards labelled with names of the topics they have encountered, using personal construct descriptors elicited by the Repertory Grid technique. Such descriptions are reliably global or local and the distinction tallies with a discrimination based upon the adicity or complexity of topic configurations dealt with during learning (high adicity = global, low adicity = local). Finally, the adicity measures correlate with four tests for personality traits which were administered during the earlier part of the study: a test for "Cognitive Complexity", Bieri et al. (1966); a test for "Attention Deployment", Mendelsohn and Griswold (1966); a test of cognitive "Flexibility", Robertson (1974); and a test for "Self-Consistency", Gergen and Morse (1967). Summary results are shown in Table 3.8.

One notable feature of the global/local propensity is that it is not confined to situations in which concepts are understood (though it is manifest as an aspect of understanding). A global/local orientation also pervades learning where understanding is not elicited, such as adaptation, problem solving and probably the

TABLE 3.7

A Proposed Classification of Learning Styles and Its Resolution in Terms of Versatility, Comprehension Learning and Operation Learning

	DB + PB	*DB* Bias	*PB* Bias
Global	Versatile or Comprehension Learning	Comprehension Learning	Operation Learning
Local	Versatile or Operation Learning	Operation Learning	Operation Learning

Assignment as Versatile depends upon the conditions of observation. All behaviours in "*DB + PB*" category are deemed "Versatile" but the globally oriented versatile learner appears to have a bias to comprehension learning and the locally versatile learner a bias to operation learning.

TABLE 3.8

Results from Tests of Cognitive Style for Low Adicity and High Adicity Learners *

	Score on Bieri Test for Cognitive Complexity	Score on Test for Attention Deployment	Score on Flexibility Test	Score on Self-Consistency Test	
Mean	155	0.36	10	38	"Low Adicity" Learners (n = 5)
SD	22	0.06	4.9	27	
Mean	108	0.44	6	45	"High Adicity" Learners (n = 5)
SD	14	0.12	1.7	8.3	

* Printed with permission from the *British Journal of Educational Psychology*. Significant correlations (0.05 > p) are as follows. Attention deployment and Bieri (0.63), attention deployment and flexibility (0.65), flexibility and Bieri (0.55), self-consistency and attention deployment (0.65), and self-consistency and Bieri (0.79).

exercise of perceptual motor skills. We conjecture that global or local orientation is a property of the brain regarded as a processor rather than the cognitive processes executed in the brain.

3. DISPOSITION COMPETENCE AND LEARNING STYLE

Style is a convenient but general rubric which conceals two quite different structural distinctions. On the one hand, style encompasses gross differences like comprehension/operation learning and the global/local orientation, as well as relatively precise characterisations of learning strategy, for example, holist/serialist and the subcategories redundant/irredundant holist.

On the other hand, style encompasses both a student's disposition to adopt a given type of learning strategy, as well as his competence to execute a strategy of the chosen type.

3.1. Refinement of Style as Holistic or Serialistic

To refine the grain of characterisation it is necessary to control the learning situation with greater stringency either by insisting upon effective teachback, or by employing an operating system (CASTE or INTUITION) and the subject matter representations (entailment structure and task structures) that support its regulatory action. It will be recalled that an operating system secures two basic conditions, namely:

(a) All topics that are learned are also understood (it is, of course, possible that a student may not be able to "learn" under these circumstances and opts out).

(b) Cooperative assistance is provided in measured quantity so that, so far as possible, understanding is enabled. Further, the minimum amount of cooperation is provided in order to obtain this result.

Teachback approximates these conditions with much of the subject matter representation held in the participant experimenter's head. For small subject matter areas, the approximation is close and teachback using verbal rather than non verbal explanation is more flexible. For large subject matter areas, such as the extended probability theory material used in Henley and London, teachback becomes impracticable.

There is ample evidence that Condition (a) Understanding is satisfied and that it predictably gives rise to a permanent body of concepts. Table 3.9 shows various differences between effective and ineffective (or simulated) teachback obtained in the multi purpose experiments; Table 3.10 shows comparative retention scores for these two conditions. The data in Table 3.11 tell a similar story, in this case, for the operating system INTUITION and the subject matter of probability (students from Henley and London). Learning and the incidence of defects are compared for the case when the full operating system is in action and the case when the understanding requirement is replaced by a demand for correct response but no explanation (the parallel to ineffective teachback).

The effect of Condition (a) and Condition (b) (or simply of "experience" in the INTUITION operating system) is a reliable positive transfer. Records of time per topic and unsuccessful explana-

TABLE 3.9

Summary of Results for Study of Effects of Matching/Mismatching *

Student Group		Matched with Gandlemuller program	Mismatched with Gandlemuller program	Matched with Operon program	Mismatched with Operon program
Batch 1 Operation Learners (like serialist)	Means SDs	90.9 3 7 (n = 9)	33.0 8.4 (n = 9)	90.8 3.8 (n = 9)	34.9 6.1 (n = 9)
Batch 1 Comprehension Learners (like holist)	Means SDs	91.0 3.6 (n = 7)	45.6 5.4 (n = 7)	47.0 6.3 (n = 7)	90.0 1.8 (n = 7)
		Matched with Operon Program	Mismatched with Operon Program	Matched with Probabilistic Inference Program	Mismatched with Probabilistic Inference Program
Batch 2 Operation Learners (like serialist)	Means SDs	92.0 3.7 (n = 9)	35.3 8.8 (n = 8)	98.0 20.1 (n = 8)	32.8 9.8 (n = 9)
Batch 2 Comprehension Learners (like holist)	Means SDs	92.9 4.0 (n = 7)	38.8 8.0 (n = 6)	93.7 3.4 (n = 6)	43.0 11.1 (n = 7)

* Printed with permission from *British Journal of Educational Psychology* (Pask, et al.).

Statistical summary:

1. Each student's matched performance > mismatched performance ($p < 0.001$, Wilcoxon Matched Pairs Signed Ranks Test).
2. Aggregate difference: matched task scores > mismatched task scores ($p < 0.001$, Mann-Whitney U-Test).

TABLE 3.10

Summary of Results for Study of Retention Using Effective and Ineffective Teachback (post-teachback score is represented as a % of pre-test score) *

Student Group		Effective Teachback on Gandlemuller Program	Ineffective Teachback on Operon Program
Batch 1 Operation Learners (n = 9) "like serialist"	Means SDs	99.0 5.4	38.0 12.2
Batch 1 Comprehension Learners (n = 7) "like holist"	Means SDs	101.1 7.1	56.0 8.8
Batch 1 Operation Learners (n = 9) "like serialist"		Ineffective Teach-back on Gandle-muller Program	Effective Teachback Operon Program
	Means SDs	51.0 12.1	109.0 5.2
Batch 1 Comprehension Learners (n = 7) "like holist"	Means SDs	57.0 9.8	104.0 6.4
Batch 1 Operation Learners (n = 9) "like serialist"		Effective Teachback on Operon Program	Ineffective Teachback on Probabilistic Inference
	Means SDs	84.1 26.4	47.9 19.9
Batch 2 Comprehension Learners (n = 7) "like holist"	Means SDs	98.7 23.9	71.0 16.1
Batch 2 Operation Learners (n = 8) "like serialist"		Ineffective Teach-back on Operon Program	Effective Teachback on Probabilistic Inference
	Means SDs	40.1 13.9	103.5 13.8

TABLE 3.10 (continued)

		Ineffective Teach-back on Operon Program	Effective Teachback on Probabilistic Inference
Batch 2 Comprehension Learners (n = 6) "like holist"	Means SDs	62.0 13.1	116.0 30.8

* Printed with permission from the *British Journal of Educational Psychology*.
Statistical summary:
Each student's effective teachback results > ineffective teachback results. Differences are significant for all students (and for all operation learners and comprehension learners treated as separate subgroups) ($p < 0.001$, Wilcoxon Matched Pairs Signed Ranks Test).

tions of topics for two of the modules in "Extended Probability Theory" are shown in Table 3.12. They belong to 11 closely studied 6th form students from Henley grammar school. Assistance given during work on Module 2 is no more than (and in most cases very much less than) the assistance furnished during work on Module 1. A similar effect was observed using CASTE in terms of mean uncertainty and mean correct belief which are continually sampled in this operating system. These data are also shown in Table 3.12. Positive Transfer is beneficial, if the operating system is viewed as a training device for making students aware of how they learn. However, we suspect that transfer involves an increase in versatility, at any rate for some students, and this blurs some of the predictions essayed in the sequel.

The cooperation provided in pursuit of Condition (b) is of two kinds: first, provision of a description of the topics to be learned (from the entailment structure and the explore/aim transactions), and secondly, provision of demonstrations which specify how a concept should be built from other concepts that are already understood. Clearly, the first kind of assistance is an external surrogate for description building (*DB*) operations the student would otherwise have to perform, and the second kind is an external surrogate for procedure building (*PB*) operations.

In other words, instead of introducing the *DB* and *PB* compo-

TABLE 3.11

Learning by Students of Differing Competence Profile in INTUITION System

	Student No	Test Score	Mean Time per topic (mins)	Index of Overall Success	Bias for Operation (—) or Comprehension (+) Learning	Versatility Index	Subjective Estimate of Incidence of Globetrotting (% of no. of topics tackled)
Entailment	1	100	6	0.68	0.95	0.05	
Structure	2	95	11	0.39	0.56	0.003	
Full	3	95	11	0.46	0.52	0.04	
Explanation	4	100	8	0.62	0.69	0.11	
Demanded	5	100	13	0.58	0.63	0.10	
	6	100	5	0.88	—0.02	0.68	
	7	90	9	0.65	—0.36	0.23	
	8	95	8	0.53	—0.56	0.09	
	9	100	10	0.52	—0.52	0.09	
	10	95	10	0.79	0.10	0.50	
Means		96.5	9.3	0.56	0.10	0.19	
SDs		4.0	2.1	0.12	0.35	0.21	
Entailment	11	72	9	0.60	—0.48	0.17	1
Structure	12	80	15	0.54	0.66	0.06	6
Multiple	13	38	12	0.57	0.70	0.07	14
Choice	14	52	7	0.47	0.61	0.02	8
Questions	17	60	18	0.57	0.72	0.05	10
Only	16	90	8	0.67	0.64	0.19	7
	17	100	14	0.77	0.16	0.46	5
	18	70	14	0.59	0.65	0.10	0
	19	65	13	0.55	0.66	0.06	15
	20	50	12	0.51	0.49	0.05	15

Means		67.7	12.2	0.52	0.48	0.12	Subjective Estimate of Incidence of Improvidence (% of topics tackled)
SDs		21	2.6	0.07	0.18	0.07	
No Entail-	21	50	16	0.56	−0.38	0.12	15
ment	22	46	14	0.73	−0.08	0.38	5
Structure	23	65	10	0.62	0.62	0.16	0
Multiple	24	90	12	0.62	−0.42	0.18	20
Choice	25	70	15	0.65	−0.18	0.26	10
Questions	26	65	15	0.56	−0.60	0.11	12
Only	27	40	13	0.50	−0.47	0.07	21
	28	72	11	0.53	−0.34	0.13	17
	29	80	8	0.74	−0.12	0.39	2
	30	90	14	0.55	0.22	0.14	8
Means		67	12.8	0.60	−0.16	0.19	
SDs		17	2.8	0.07	0.22	0.06	

Statistical summary:
Group 1: test scores greater than Groups 2 and 3 ($p < 0.001$, Mann Whitney U-Test) Mean times per topic are significantly less than those of groups 2 and 3 ($p < 0.5$, Mann-Whitney U-test). *Group 2*: mean times per topic are less than Group 3, the difference is not significant.
For *Group 2* there is a significant negative rank correlation ($p < 0.01$) between test scores and estimated incidence of globetrotting.
For *Group 3* there is a significant negative rank correlation ($p < 0.05$) between scores on "holist" questions of the Cartoons Test and estimated incidence of improvidence.

TABLE 3.12

Gross Transfer of Learning Skill Connected With the Use of the INTUITION and CASTE Operating Systems. Study of Different Groups of 12 and 10 Students

	INTUITION			
	Mean Time/Topic (mins)		Frequency of Unsuccessful Explanation	
	Module 1	Module 2	Module 1	Module 2
Means	9.7	6.6	0.12	0.02
SDs	2.9	1.4	0.05	0.02
(n = 12)	CASTE			
	Mean Δ H_i		Mean Δ θ_i	
	1st $\frac{1}{2}$	2nd $\frac{1}{2}$	1st $\frac{1}{2}$	2nd $\frac{1}{2}$
Means	0.97	0.22	−0.72	−0.22
SDs	0.49	0.23	−0.22	−0.06
(n = 10)				

Statistical summary:
INTUITION mean time/topic module 1 > mean time/topic module 2 (0.01 > p, sign test). Unsuccessful explanation module 1 > unsuccessful explanation module 2 (0.01 > p, sign test).
CASTE system Mean ΔH(1st) > Mean ΔH(2nd) (0.005 > p, Mann-Whitney U-Test) and Mean $\Delta\theta$(1st) > Mean $\Delta\theta$(2nd) (0.005 > p, Mann Whitney U-test).

nents of learning on intuitive grounds (Section 2.5), it would have been possible to argue that *DB* and *PB* are genuinely distinct *because*, in an operating system, it is possible (and necessary if the system works) to furnish differential *DB* and *PB* assistance to the student.

Thus augmented (by *DB* or *PB* as needed) and thus restricted (to understand each topic, if necessary with assistance given), students who learn at all adopt a learning strategy which may be classified as holist or serialist. Moreover, their tendency to adopt one type of learning strategy or the other is predictable from indices of learning style.

Specifically, the refined categories of holist and serialist are manifestations in an operating system (or in teachback regulated conversations) of the more general characteristics of style. Our hypothesis is crystallised in Table 3.13 and 3.14. Of these, Table 3.13 posits the combinations (of *DB/PB*, global/local) yielding categories of learning style, and Table 3.14 shows the behaviour predicted if a student of a given stylistic category learns in an operating system. The behaviours are resolved as holistic or serialistic, and we emphasise that this distinction is established unequivocally in terms of marker distributions on the entailment structure and transaction records. The prediction of a versatile student is that he may adopt either a holistic or serialistic learning strategy, by instruction or on whim. Moreover, he may change strategy if the subject matter is changed. But, having once adopted a holist/serialist strategy, cognitive fixity will make him stick with it whilst he is learning in the same conversational domain. The stylistic categories "G-Null" and "L-Null" are predicted "not to learn"; that is, augmentation is insufficient to induce understanding.

TABLE 3.13

Stylistic Categories

	Versatile Students Comprehension and Operation Learning	Comprehension Learners	Operation Learners	Failures
	DB and *PB*	*DB* not *PB*	*PB* not *DB*	Neither *DB* nor *PB*
Global	*GDB* + *GPB*	*GDB*	*GPB*	G-Null
Local	*LDB* + *LPB*	*LDB*	*LPB*	L-Null

The categories are shown as dichotomous in the interest of clarity. They are supposed, in fact, to represent polarities; for example, "*DB* not *PB*" means "Dominantly *DB*" and "*PB* not *DB*" means "Dominantly *PB*". With the caveats noted in the text, stylistic categories are *Competence Profiles* and are, henceforward, referred to as *Competence Profiles*.

Apart from this last prediction, there is ample evidence that exactly these behaviour patterns do occur and are related, as proposed, to indices of comprehension learning, operation learning, and the global/local propensity (previous tables). Typical student records are shown in Fig. 3.4 and Fig. 3.5 to indicate the level of detail at which these patterns are identified.

It would be inappropriate to cite pattern frequencies until more work has been done; the case is made just as convincingly by noting that all the patterns of Table 3.14 have been observed with varying frequency (minimum 4 patterns) over more than 50 students (run during the ongoing field studies) and that the undetermined entries (apart from G-Null or L-Null) can be resolved by resource to the way that analogical topic relations are learned. We

TABLE 3.14

Behaviours Predicted in an Operating System

Competence Profile on Stylistic Category	Predicted Learning Strategy		Exploration Predicted	Demonstration Required
GDB + *GPB*	Versatile	Redundant Holist	Many	Few
		Serialist	Few	
LDB + *LPB*	Versatile	Irredundant Holist	Undetermined	Few
		Serialist	Few	
GDB	Redundant Holist		Many	Many
LDB	Irredundant Holist		Undetermined	Many
GPB	Serialist		Many	Few
LPB	Serialist		Few	Few
G-Null	No Understanding		Undetermined	Many
L-Null	No Understanding		Undetermined	Many

return to this matter in Chapters 4, 5 and 6, when the theoretical backbone has been discussed.

The power of this explanatory scheme is increased by extending Table 3.14 to accommodate situations in which the cooperation offered by the operating system, Condition (b), is systematically cut down and/or the demand for understanding, Condition (a), is systematically relaxed. Behavioural predictions for the wider range of situations are shown in Table 3.15.

The corresponding experimental conditions are realised as follows (A, B, C, D, E refer to the columns in Table 3.15).

A. Strict conversation, Fully Fledged Operating System, as in the upper group of Table 3.11.

B. The requirement for non verbal explanation (model building) on each topic marked as understood is replaced by a correct response criterion on a test made up from questions spanning the relevant topic. The overall impact of this modification is shown in Table 3.11. The more intimate results are those predicted in Table 3.15; versatile learners (*GPB* + *GDB* or *LPB* + *LDB*) are not significantly affected, for they explain topics whether or not they are required to do so. Much the same is true of operation learners (*GPB* or *LPB*) who build concepts but do not easily build descriptions. However, the comprehension learners (*GDB* or *LDB*) are strongly influenced by Globetrotting (prohibited in a fully fledged operating system that calls for non verbal explanation) which becomes a common occurrence and accounts for most of the deterioration in performance revealed by the middle lower group in Table 3.11.

C. Explanations are required, but the surrogate *DB* operations are no longer made available under these circumstances. Any understanding (and each topic must be understood) depends upon a *DB* operation performed by the student himself. To realise this condition, the entailment structure is denuded; all indications of analogy relations are deleted, as are the corresponding explore transactions. As a result, learning is slowed down. Versatile performance is least impaired, comprehension learners (*GDB* or *LDB*) are not greatly influenced; both kinds of learner can build their own descriptions. In contrast, the operation learners (*GPB* or *LPB*) become improvident and the deceleration of learning shown (lowest group) in Table 3.11 is mainly due to this fact.

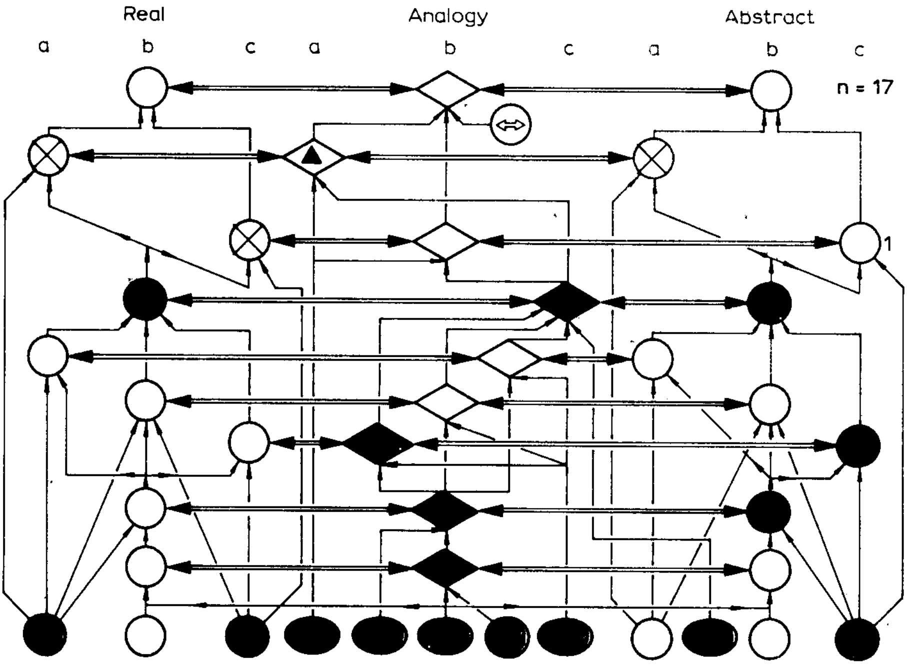


Fig. 3.4a. (Figs. 3.4b,c and d are on the following pages.) Occasions n = 17, n = 18, n = 19, n = 20 in typical holist learning strategy (▲ = aim, ○ = goal, ● = understood). Numbers index "explore" transactions.

D. To provide the condition "No *PB* assistance" it is necessary to preserve the explanation requirement but to abrade the demonstrations. Experiments are in progress and do not deny the predictions of Table 3.15, given the caveat that the table is based on the (false) simplifying assumption that students *do* obtain demonstration aid, when in fact they *may do* so. Our main prediction is substantiated; namely, that comprehension learners (*GDB* or *LDB*), break down completely. The prediction is definitive compared to the others, since learners of this class must have recourse to demonstrations and fail to achieve understanding if this assistance is withdrawn.

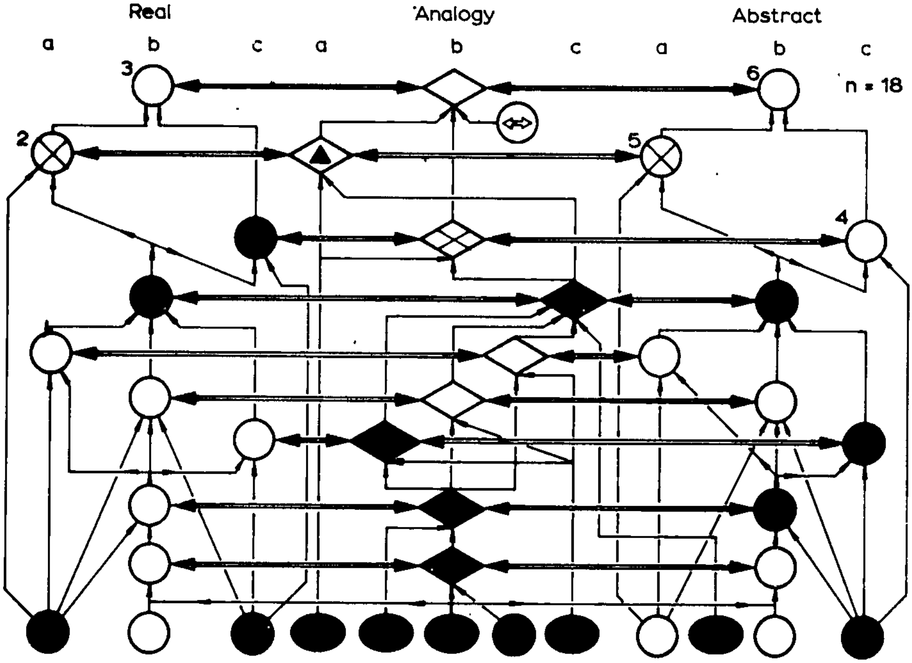


Fig.3.4b.

E. Trivially, explanation is not demanded and no assistance is explicitly furnished. The condition is approximated by a free learning situation or by various unmonitored learning situations.

It is probably legitimate to extrapolate the categories of holist and serialist, which are strictly defined for an *operating system* to yield a characterisation of learning strategies with respect to *any conversational domain*. The characterisation is illuminating since it exhibits a distinction between the "comprehension learning/operation learning" dichotomy and the "holist/serialist" dichotomy in a way that secures a place in the overall scheme for "redundant holists" as compared to "irredundant holists". Further, the present characterisation, though worded differently, is in close agree-

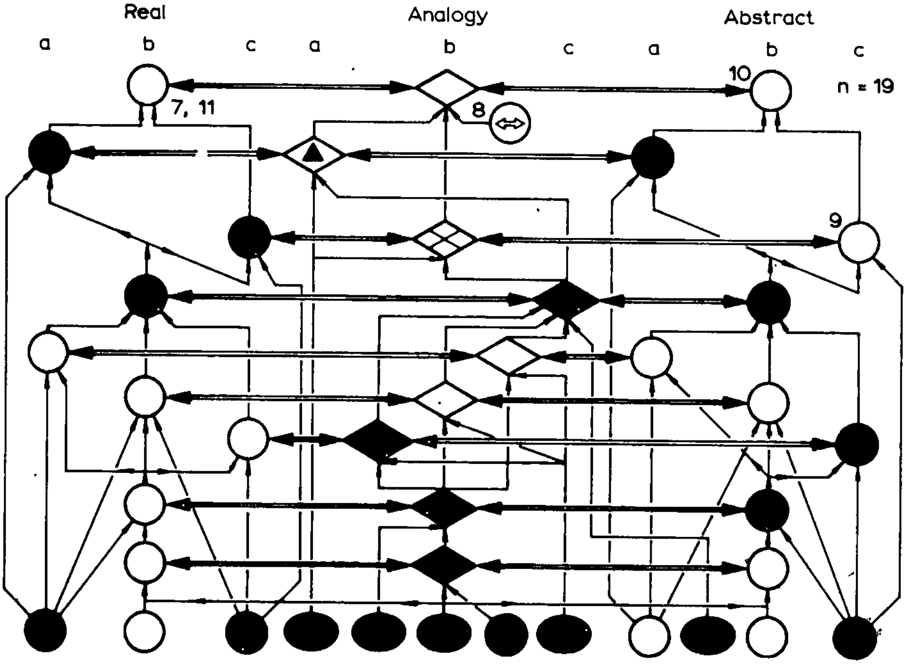


Fig. 3.4c.

ment with the operational definitions of a "holist strategy" and a "serialist strategy" as given in the previous monograph.

Seen in the context of a conversational domain, with cyclic derivations exhibited, the holist/serialist strategies differ as follows: Students employing either learning strategy come (of necessity) *to understand some cyclic and reconstructible substructure,* a Gestalt. The substructure is a syntactic (derivational) entity. Whereas the holist chooses as large a cyclic substructure as possible, the serialist chooses the smallest possible cyclic substructure. If the mesh is pruned, as it is before inscription upon an entailment structure, the "size maximising" case appears as the aim, goal, understood marker distribution of Fig. 3.4, which tallies with the diagnostic criterion for holist learning in an operating system. Similarly, the "size minimising" configuration gives rise to the

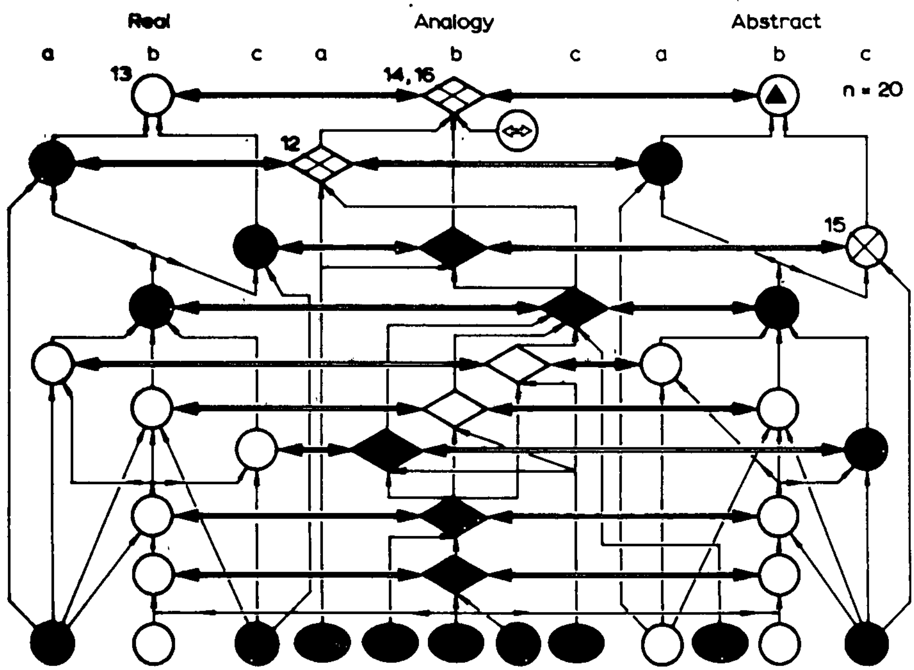


Fig. 3.4d.

marker distribution of Fig. 3.5, which tallies with the diagnostic criterion for serialist learning in an operating system. In either case, the "sizes" are syntactically specified. "Distance" is measured in terms of the length of derivations (entailment chains) and do not take into account the number or diversity of semantic descriptors which are evaluated or assimilated as a result of learning. This latter and unaccounted index is particularly important for those cyclic substructures which represent analogy relations. For example, someone acting as a holist but also anxious to adumbrate many distinctions established on semantic grounds would, of necessity, aim for analogical topic relations; and conversely, someone equally holist who is not anxious to deal with many semantic distinctions would avoid analogy relations though he could not eschew them completely. We posit that a redundant holist is a

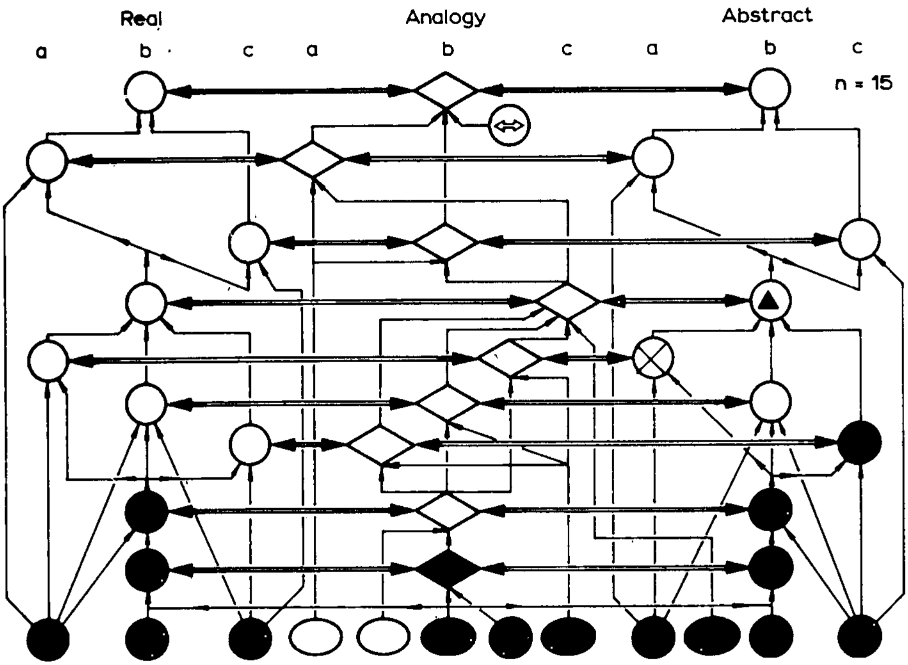


Fig. 3.5a. (Figs. 3.5b,c and d are on the following pages.) Occasions n = 15, n = 16, n = 17, n = 18 in typical serialist learning strategy (▲ = aim, ○ = goal, ● = understood). Numbers index "explore" transactions.

holist who does process many semantic distinctions, either those of exhibited analogy relations or, failing that, analogies of his own invention. Conversely, an irredundant holist either steers clear of analogies (when possible) or uses only a minimal number of semantic distinctions in order to make sense of the syntactic or formal similarity expressed by an analogical topic relation.

The distinction "comprehension learning/operation learning" is subtly different. Comprehension learners, with their bias to *DB* operation, tend to use analogies as the scaffolding of knowledge whenever possible. As holists, they are inclined to be redundant holists. At any rate if the bias to comprehension learning is extreme. In intermediary cases (and, a fortiori, for versatile students),

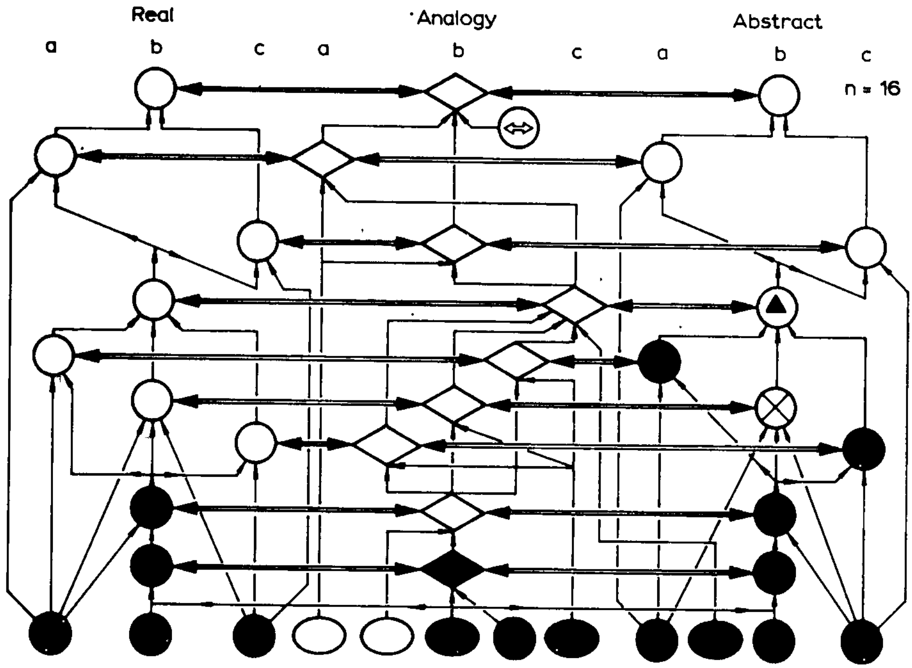


Fig. 3.5b.

they may figure either as redundant or irredundant holists. On the other side of the coin, operation learners with their bias to *PB* operations tend to learn other-than-analogical substructures and to stick them together with minimal recourse to analogy relations. In extreme cases they are in register with serialists. But the intermediary cases (a fortiori, versatile students with an operation learning bias) may be either serialists or irredundant holists.

3.2. The Distinction Between Disposition and Competence in Execution

Style predisposes a student to a learning strategy; once the learning strategy is adopted it is stabilised as a result of cognitive fixity. This dogma is supported by data from all the experiments, but is most dramatically evidenced by the multi purpose experi-

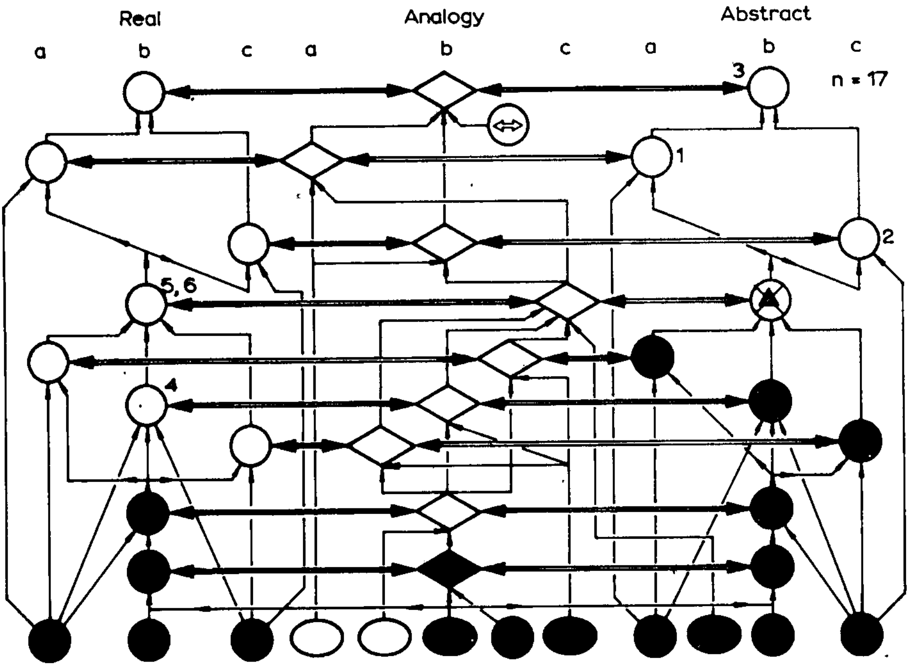


Fig. 3.5c.

ments where students characterised as holist or serialist with respect to learning one subject matter were found to exhibit the same learning strategy when mastering quite different subject matter. The stability is exhibited by choice of learning strategy and the effect of instruction which is matched or mismatched with respect to the original assignment as holistic or serialistic (Tables 3.1, 2, 3, 9, 10). Such a marked stability is surprising, for it is only possible to predict on theoretical grounds that cognitive fixity will stabilise an originally selected learning strategy whilst the student is attending to the *same* conversational domain.

The overall result of mismatching a teaching strategy (imposed upon a student) and a learning strategy (which he has adopted) produces a high magnitude impairment; in this respect the data from the multi purpose experiments are in accord with the previous

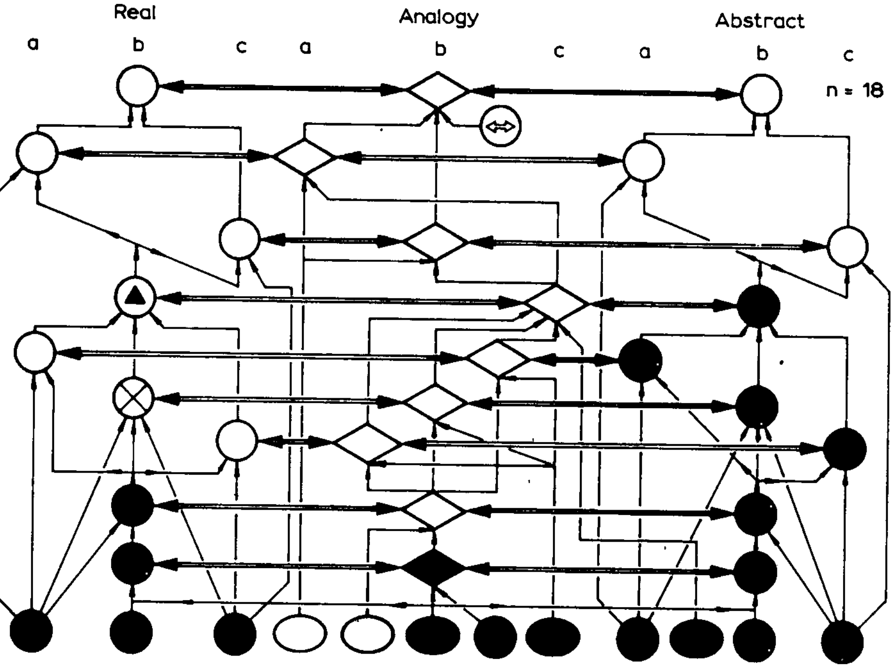


Fig. 3.5d.

monograph and provide valuable confirmation of the result using a larger sample of students. But the stability data say little about *why* the learning strategy was chosen (this choice is non committally attributed to style).

Can we dissect style into a conative part, reflecting a student's desire or disposition to adopt a learning strategy (later stabilised by cognitive fixity), and a part to do with his competence in executing this strategy? Only if the chosen learning strategy is one which the student is fitted to execute would it be legitimate to relabel the stylistic categories of Table 3.3 and Table 3.14 as "competence profiles". If it were the case that students are only disposed to do whatever they are competent to do, then dissection would be fruitless. Contrariwise, if a dissection is meaningful then

TABLE 3.15

Predictions Arising from Combinations of Competence Profiles and Constraints upon an Operating System in Which Learning Takes Place

	A	B	C	D	E
	(Exp. Full *ES*.)	(CE no Exp. Full *ES*)	(Exp. Denuded *ES*.)	(Exp. Full *ES*. RD)	(CE No Exp. Denuded *ES*)
L-Null	Flounders about. Often eventually succeeds but experience probably changes the competence profile.	Serialist. Aim = Goal. Path dictated by displayed *ES*. Rote learns. Recall (if any) depends upon path.	Uses denuded *ES* display like headings in a book. Rote learns topics. No content recalled	Fails to learn.	Usually flounders Only learns if told what to do, if one topic at once, and if concept construction is spelled out.
LPB	Takes aim topics from displayed *ES*, preferring serial path. Gives explanations. Few demonstrations.	Serialist. Aim dictated by displayed *ES*. Takes little notice of analogies between topics unless forced to do so. Can explain topics even though explanation not demanded.	Improvident and repetitious learning. Takes no account of analogies (not displayed in denuded *ES* but might be inferred). Can explain topics learned.	Serialist. If any topics learned can explain them.	Learns algorithmically, or sequences of chained conditional responses.

TABLE 3.15 (continued)

	A	B	C	D	E
	(Exp. Full *ES*.)	(CE no Exp. Full *ES*)	(Exp. denuded *ES*.)	(Exp. Full *ES*. RD)	(CE No Exp. Denuded *ES*)
LDB	Strongly serialist. As far as possible learns in departmental manner. Hesitates over, but gives, explanation. Many demonstrations	Serialist. So far as possible learns in departmental manner. Fails to explain.	Improvident and repetitious learning. Takes no account of analogies (not displayed in denuded *ES* but might be inferred). Hesitates over, but gives, explanations.	Fails to learn.	Learns formal pattern of data structure, but not content. List-like recall.
LDB + *LPB*	Uses all entailment relations but bias to serialist learning. Can explain topics learned. Few demonstrations.	Serialist. Can explain topics even though explanation not demanded. Few demonstrations.	Effective. Constructs missing derivations with bias to axiomatic or formal extrapolations. Gives explanations.	Serialist. Explains topics learned.	Versatile. May have "learning to learn" ability. Predicted "convergent" bias to innovation.
G-Null	Flounders about. Often eventually succeeds but experience probably changes the competence profile.	aim = goal. Topics scattered. Rote learns. Recall (if any) depends upon path taken.	Uses the denuded *ES* display as pattern. May learn this pattern but no content recalled.	Fails to learn.	Usually flounders. Only learns if told what to do in broad (perhaps pictorial) manner and if concept construction is spelled out.

GPB	Takes aim topic from displayed *ES*, preferring departmental path, if permitted. Gives explanations Few demonstrations.	Holistic. But next aim dictated by displayed *ES*. Topics scattered. Can explain topics even though explanation not demanded	Fills in missing analogies between topics by valid relations. Can explain topics learned	Globetrotting over constrained path in *ES*. Explains topics learned.	Does intuitive problem solving or learning. May learn perceptual motor skills.
GDB	Strongly holistic. So far as possible relies on analogical relations. Hesitates over, but gives, explanations.	Holistic. Cursory globetrotting. Learns usually vaccuous quasi analogies or similarities. Fails to explain.	Fills in missing analogies by often vaccuous similarity relations. Globetrotting. Fails to explain.	Globetrotting in *ES* Fails to explain.	Learns meshlike (often pictorial) formal patten of data structure but not content. Broad associative recall.
GDB + GPB	Uses all types of entailment relations. Bias to holist learning. Can explain. Few demonstrations	Holistic. Can explain topics even though explanation not demanded. Few demonstrations	Effective. Constructs missing derivations with bias to valid analogical relations. Can explain.	Holistic. Explains topics learned.	Versatile, may have "learning to learn" ability. Predicted "divergent" bias to innovation.

there is an issue of "internal matching" that deserves consideration; matching or mismatching between the learning strategy chosen and the student's competence (in contrast to matching between style and an "externally imposed" teaching strategy).

There is an abundance of hard-to-quantify evidence in favour of the latter point of view. Unless special precautions are instituted a student's choice of learning strategy does not necessarily reflect his mental competence.

It happens that proper safeguards have been incorporated, more by luck than foresight, in most of our experiments. During strategy determination, the student is taught to appreciate his own learning process and its deficiencies; if he shows signs of ineffective learning, an internal mismatch is suspected and the student is encouraged to try a converse strategy. In the Spy Ring History test the subject matter is richly described and rendered unlike test or examination material for which the student is likely to entertain beliefs and convictions about the officially good way of learning. Finally, and crucially, the experimental work is backed up by rather detailed and prolonged individual interviews (most of the comments in the next section stem from interview data).

Because of this, we are fairly confident that the stylistic categories act as "competence profiles", and that terminology is henceforward employed.

4. NOTES ON THE CHARACTER OF COMPETENCE MECHANISMS

Because of the precautionary measures, it is also possible to detect the existence of internal mismatching, engendered by belief or indoctrination.

As noted, counterproductive dispositions are quite common and seem to generalise over more subject matters than suspected, in fact, over all academic or institutional subject matters. (Just as a learning strategy is stable over the diverse subject matters used in the multi purpose experiments.) It is true that exactly the same induced dispositions are productive if they tally with the student's ability to execute the class of learning strategy he is disposed to adopt. The case of mismatching between disposition and competence is more easily observed: the student adopts a definitive learning strategy without encouragement; he is manifestly unable to execute the learning strategy he so readily adopted. At that point the experimenter becomes alive to a difficulty and probes the issue of disposition and competence in greater depth.

The converse (productive) form of induced disposition is noticeable amongst science-stream 6th formers. Due to the subject matter load and a certain preference for unfolding scientific discoveries in a historical sequence, these students receive markedly serial instruction, and an incidental premium is often placed upon operation learning. Some 30 percent to 50 percent of the students in this group are aware of having a disposition to adopt one class of learning strategy before the requirement to exteriorise such a thing, explicitly, is forced upon them by contact with the operating system INTUITION. Of the students who *do* make a definitive choice, nearly all state quite dogmatically that the study habits which determine their disposition were induced by the teaching and the content or arrangement of the subject matter. Further, they are satisfied with their disposition and are, in fact, expert in adopting serially biassed learning strategies (though, as a rule, these students are outstanding learners and have the versatile competence profile *LDB* + *LPB*).

The only holist students in such a class seem to belong to the group who do not *have* an initial disposition. On scrutinising records and reports they do not exhibit the excellence (in science subjects) of their colleagues. However, there is a very appreciable improvement in their performance, when, after competence testing, they are advised to adopt a particular learning strategy *; sometimes the recommendation is holist, and if so, the students often turn out to be versatile with the profile *GDB* + *GPB*. Data from individual interviews indicate that the students who "had no disposition" but "turned out to have a holist bias" have actively rejected the serial/operational mores of the science course; just as their peers actively accepted a serial/operational disposition.

The real difficulties begin only with less sophisticated students; either those who are less likely to be versatile or those who are more inclined to accept conventions in an unquestioning manner. Students from technical colleges and some students in the 5th

* Performance with respect to learning in the experimental system. We do not yet know how long the improvement lasts or how well it generalises to other school subjects.

form of the same school come under this denomination, and again judging from the interview data, members of this population are the people most likely to have dispositions out of kilter with their competence and to have the disposition just because they are told, directly or indirectly, to do so. The commonest mismatch between disposition and competence is (for the reasons already stated) a serialist disposition unfitted to a holistic competence, which for profiles other than versatile is a major impediment. An interesting concomitant is that such students often learn extracurricular subject matter in a competence-suited manner and learn it with far greater efficiency. The reader who is sceptical on this point is invited to compare the (often arcane) extracurricular knowledge of a representative student with his academic knowledge. Using any reasonable measure of the amount known, academic knowledge forms a small proportion of the total, and the difference is enhanced by weighting this ratio with the time spent since becoming acquainted with (say) astrology or anthropology, and the time spent in learning (say) computer science.

All this highlights the question, "where is competence found?" According to our theory, one aspect of competence is part of a cognitive organisation (the student as a P-Individual) which has a collection of useful and stable concepts. This is the competence ingrained by cognitive fixity and induced by social interaction. At this level, there is no difference in kind between disposition and competence. Though they need not run in the same direction, they often do so, and if not, remedial action can be taken to bring them into accord. Moreover, the result of this action should also be perpetuated by cognitive fixity. On the other hand, there appears to be a further and substantially immutable factor in competence which often runs counter to induced disposition. As a conjecture, this is a property of the student's brain as a processor, not of the student as a cognitive organisation (a P-Individual). The evidence to hand, though still scanty, does not deny the hypothesis that this factor is the "global or local" orientation, tapped by measures of "adicity" and "recall".

5. RATE OF LEARNING, ANALOGY RELATIONS, AND VERSATILITY

There is appreciable variation in the interval required to master a subject matter. To some extent this variation can be accounted for in terms of competence and the existence of previously acquired concepts. To some extent the rate of progress can be modulated by processor parameters (physiological changes and specific conditioning, for example).

However, scrutiny of the records, either in the multi purpose or the school based experiments, discloses an interesting fact. Students who learn rapidly are students who use (and understand) valid analogy relations.

Like many of our findings, this one states the obvious (at any rate with hindsight). The only way to change mastery rate by many orders of magnitude is to employ analogical relations between disparate topics. It is a truism of education, the entertainment industry, and journalism alike. Hence, Improvident learning is slow but sure and may be the norm; Globetrotting is hazardous if not self defeating; understanding analogies is the only way to get on.

The analogy relations may be discovered by an expert and displayed (in an entailment structure or some other subject matter representation); if so, the student learns them straight forwardly, and for this purpose, either of the versatile competence profiles is sufficient. There is some evidence that the training effect of experience in an operating system is chiefly due to inducing versatility (from other profiles) and exercising it in this manner.

On the other hand, the analogy relations may be discovered or invented by a student, as they must be if he is coping with an unstructured environment and structuring it on his own account.

Students who have an art of learning in general, who have learned (or been taught) to learn are able to play the discovery and invention trick. Certainly, versatility is a prerequisite for the art of learning. Moreover, insofar as it fosters versatility, experience in an operating system teaches people to learn. But it will be argued that a further prerequisite for the art of learning in general is an ability to compare descriptions and concepts built under different perspectives and that the proper training ground for this ability is a many-aim operating system (discussed in Chapters 4 and 6; described in Chapter 7 in the context of course assembly and innovation).

Of all the structures that might be imposed upon an unstructured environment by someone who has learned to learn, the most important are formally expressed as analogy relations. Just as these analogy relations shortcut the tedium and repetition of Improvident learning and lead to rapid mastery of a predigested subject matter, so also, analogy relations are the glue required to make sense of an otherwise chaotic reality and to stick together theories (and, a fortiori, scientific theories) which otherwise are disparate essays bringing order only to small regions of what may be known.

Chapter 4

Theoretical Developments

Only in recent years have mathematical logicians seriously considered systems that are constructed from the point of view of a participant; that are, in the non vicious sense of the first monograph, "subjective" and "reflective". The most comprehensive development is due to Andreka, Gergeley and Nemeti (1973a,b); but several, seemingly quite different pieces of work complement the picture. In toto, these mathematical systems lend credibility to the "string and sealing wax" formulation of conversation theory. These additions could be advanced independently, as systemic propositions which are supported by empirical data. But, since their otherwise peculiar form fits the larger and axiomatically respectable framework, a more convincing case is made if they are viewed as instances of this general and well-formulated system.

The logico-mathematical advances bearing upon reflective systems belong to the following areas of study:

(a) A non classical and model theoretic treatment of languages and logics (Andreka, Gergeley, Nemeti 1973a,b).
(b) A general formulation of Fuzzy Predicates (Goguen 1968).
(c) A theory of Fuzzy Algorithms, and Fuzzy Sets (Zadeh 1971).
(d) Coherence Theory (Rescher, 1973). *

* Since this book went to press, certain important theoretical developments have taken place and the list should be updated by two additions, namely, (e) Varela's logic of self reference and (f) work, chiefly due to Goguen, on the category theoretic foundations of General System Theory. These recent developments are briefly outlined in a footnote at the end of the chapter on p. 162.

These developments are discussed in the context of conversation theory (not always in the order listed above), to provide the general framework promised in the last paragraph.

In this chapter we review (a), (b), (c), and (d) as they apply to conversation theory and, where necessary, generalise the dicta and definitions of the first monograph. At this juncture, attention is still concentrated upon strict and *one-aim-at-once* conversations (though the underlying mental operations are often more liberally conceived). Even within this framework, it is possible to advance the notion of a *common meaning* reached by an agreement having a syntactic and a semantic component. In particular, we develop the idea that topics in a conversational domain which stand for analogy relations between other topics, are static inscriptions of a *common meaning.*

Analogy relations have a curiously central position because of their educational significance (only by using them accurately, can the student genuinely accelerate his learning of a subject matter), and because the appreciation of analogy relations is at the root of innovation and discovery. In order to introduce these ideas, it is convenient to describe certain augmentations of the transactions permitted in a conversational operating system such as CASTE or INTUITION. The augmented transactions were mentioned in the first monograph, but have been incorporated since it was written (in 1973). The understanding of analogy relations is discussed in these terms, and some ideas about the construction of analogy relations are sketched by way of introduction to the next chapter. Little more can be done until the "one aim at once" condition is relaxed.

1. GENERAL AND NON-CLASSICAL LANGUAGE, LOGIC AND MODELS

A language has a semantic (or interpretative) as well as a syntactic (or formal) status. The conversational language L is necessarily of this type, so are the languages in Barralt-Torrijos and Chiaraviglio's formulation, referenced in the first monograph. But the notion contrasts quite sharply with an "uninterpreted formal language", a purely syntactic construct of symbolic logic. Notationally, a language is a triple:

Language $\triangleq$ ⟨Set of Sentences, Interpretation Function, Universe of Interpretation⟩

or, for brevity

$\mathcal{L} \triangleq \langle S, I/F, \underline{\mathrm{Univ}} \rangle$

These entities and their relations are shown in Fig. 4.1. There are indefinitely many languages. Labelling any one of them by an index i:

$\mathcal{L}_i \triangleq \langle S_i, I/F_i, \underline{\mathrm{Univ}_i} \rangle$

and they may differ in any or all, of their component terms.

A logic is conceived as any pair:

Logic $\triangleq \langle \mathcal{L}_i, \mathrm{Calculus} \rangle$

where the calculus is capable of expressing algorithms or programs, themselves syntactic entities which generate sentences in a set, S.

Clearly, a calculus could degenerate to one program or a certain class of programs (<u>Prog</u>). Generally, we equate the notion of calculus with this degenerate form.

$\langle \mathcal{L}_i, \underline{\mathrm{Prog}} \rangle = \langle\langle S, I/F, \underline{\mathrm{Univ}} \rangle, \underline{\mathrm{Prog}} \rangle$

where <u>Prog</u> produces some, or all, members of S.

Further, the conversational language L is held in mind as $\mathcal{L}_i$ or a class of $\mathcal{L}_i$ accommodating full (or degenerate) logics.

At the cost of stratification (as in the conversational language $L = L^1, L^0$), or any other trick which discriminates between a description and an instruction to bring about whatever is described, it is possible to incorporate goal descriptions. Thereby, a pragmatic of goal satisfaction is adjoined to the syntactic and semantic system to form a semiotic system.

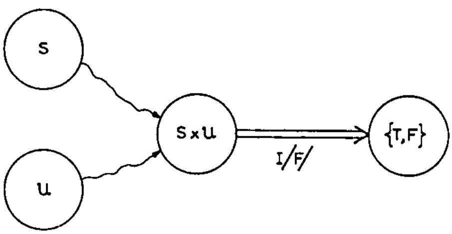


Fig. 4.1. Graphical representation of a language L, considered as a triple, consisting in a set S of syntactically admissible statements, a universe of interpretation U, and an interpretation function which maps the product of S and the universe onto values in a truth set (here $\{T, F\}$).

The motivation behind this development is that a sentient being (unspecified, but aware) is able to have indefinitely many languages, $\mathcal{L}_i$, and in them to imagine as possibilities and contemplate an indefinitely large number of realities (universes). In any universe, the interpretation of those sentences of S that are true (in this universe) is a model.

For a classical logic, the truth set is {True, False }(Fig. 4.2). But the overall scheme is designed to accommodate non classical logics. For example, a logic of action or of command execution (Rescher 1966), as sketched in the first monograph, is non classical. Statements prescribe operations. The statement of a procedure is true in a certain universe, if this procedure satisfies a mooted goal in this universe. For another example, a logic of Fuzzy Predicates in a non classical logic: the truth set is values in {Interval 0, 1; Meaningless}. A logic of Fuzzy Programs that compute the values of Fuzzy Predicates is also non classical and is of special interest.

One noteworthy aspect of the logic and language under discussion is its systemic orientation. Most treatments of model theory are constructed according to canons of parsimony and are directly applicable only to the simplest situations. So, for example, a universe is generally regarded as a set of elements, objects, or at the most, events. In the present case the restriction is waived, as it must be in the interpretation of a logic of action and operation. The universe can have the characteristics of a processor. Using the terminology of the first monograph, universes of interpretation are M-Individuals (one or other sort of processor). The *model* for an action engendering statement (Prog, for example) is a compilation of Prog. Moreover, time is implicit in the universe (perhaps only in

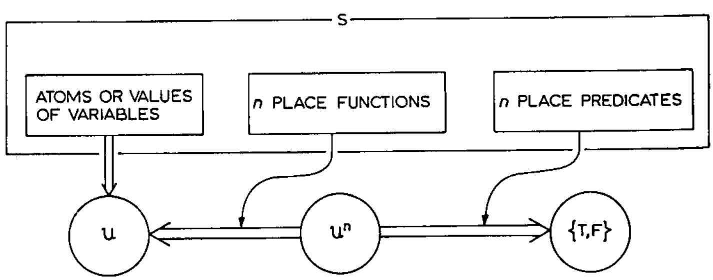


Fig. 4.2. Standard interpretation of the 1st order predicate calculus using the conventions adopted in Fig. 4.1. All of the variable values, predicates, functions, etc. (together with the connectives and quantifier symbols) are part of language S, i.e., the first order predicate calculus language.

the weak sense of order and the injection of negentropy to set a process in motion). But it is neither necessarily nor usually the case that time is uniform, so that different parts of the processor are a priori synchronised. If synchronicity exists, this is a special constraint built in as part of the syntactic statement which is given an interpretation either as a compiled and executable program, or as a result of productions manifest when instructions are taken in the imperative form, or as a special condition (for example, in the first monograph, the characterisation of modelling facilities, as "one clocked" or "many clocked" processors).

It is possible to view a scheme or system of this type from two equally legitimate perspectives, by considering the various processes that are licenced by the scheme from either an outward or an inward looking stance.

Of these two perspectives, the outward looking is less familiar and more definitely subjective or reflective (though in a sense, both of them have a reflective component). The notion underlying the outward looking perspective is that languages and, a fortiori, universes can be generated constructively in some medium which will be identified (as suggested already) with a *processor.* That is, the interpretation function I/F is regardable not only as a mapping (Fig. 4.1) between truth values and the product of statements and universes, but also, given certain statements and a truth criterion, as a process in the stipulated *processor* which constructs universes as imagined possibilities. Under these circumstances, the interpretation function I/F is itself a constructive process and it will be distinguished as such by writing (with processor given).

$\mathcal{L}_i \triangleq \langle S_i, \underline{\text{Inter}}_i \rangle$; *where* $\underline{\text{Inter}}$ i is a compiler that produces a specific universe, $\underline{\text{Univ}}$ i, as an interpretation in the processor.

Logic $\triangleq \langle \mathcal{L}_i$, Calculus$\rangle$; or, degenerately, $\langle \mathcal{L}_i, \underline{\text{Prog}}\ i \rangle$ where $\underline{\text{Inter}}$ i produces a compilation of $\underline{\text{Prog}}$ i (and an interpretation of its input and output domain) in the processor.

We shall identify the processor with an L-Processor, the most general kind of M-Individual considered in the first monograph. At least, an L-Processor is an indefinitely sized ("inexhaustible") collection of a priori independent and asynchronous, programmable machines; of course, these machines are brought into local dependency and synchronicity when a program is executed.

The psychological interpretation of this construction is obvious in experience. If the L-Processor is identified as a human brain, then the compilation and subsequent execution of any program gives rise to an imagined world in which the input and output variable of the program range over sets of imagined objects. These may be abstract objects (for example, the set of real numbers) or they may be concrete objects (as in the case of a visual image, or the apparitions of any other sense modality) or they may have an undetermined status in this respect (the "imageless thought" of the Würzburg School, or simply unclassifiable impressions). In any case, these universes of interpretation (the input and output sets of the compilation of a program) are dubbed "imaginary", because they are constructed in a processor which the participant has described, for tenable but all the same arbitrary reasons, as *his own*. Apart from this, and to a lesser extent, the peculiarities of compilation in an L-Processor, the objects are no more "imaginary" and no more nor less "real" than the objects of everyday sensation and perception. *

* If an L-Processor is equated with a human brain, then this proposal is no more outrageous than Muller's 19th century doctrine of "specific nerve energy"; the notion that modalities of sensation, and ultimately of perception, are determined by patterns (of "specific nerve energies" to sustain the archaism), rather than being direct consequences of physically distinctive stimulation. Conversation theory takes "L-Processor" more generally (though a human brain is an L-Processor, so are many other systems).

Judging by everyday experience, internal compilations exist for different sense modalities and compilations that are not identifiable with any known sensory organ. Such introspections are well supported experimentally; for example, in the work of Wallach and Averbach (1955) or Posner (1966) on the existence of distinct visual and verbal memory traces, the informational value of which is stressed in Atkinson and Shiffrin (1965) "copy trace" scheme. As a matter of fact, there is no serious dispute about the existence of dedicated sensory buffer stores (which is of little immediate concern) or of distinct internal compilations of whatever process represents a memory in the theory at issue. It is also empirically obvious that sensory traces are translated from one modality to another at the least provocation either in short-term storage or long-term storage (Atkinson and Shiffrin (1967), so that only under special circumstances will different compilations of the same process (or a process engendered by the same stimuli) remain unrelated. But these special circumstances can be engineered (as witnessed by the results referred to above). The resulting interactions and occasional independences are clearly compatible with the present theory under the caveat that we refer to "what is remembered" as a concept (compiled procedure) and reserve the name

A fortiori, L-Processors are able to accommodate several languages, $\mathcal{L}_i$, $\mathcal{L}_j$, simultaneously, or as a special but important case, several degenerate logical systems (with "Calculus" set equal to "Prog"), for example, the systems:

$\langle \mathcal{L}_i, \text{Prog}\ \alpha \rangle = \langle\langle S_a, \text{Inter x} \rangle, \text{Prog}\ \alpha \rangle$

$\langle \mathcal{L}_j, \text{Prog}\ \beta \rangle = \langle\langle S_b, \text{Inter y} \rangle, \text{Prog}\ \beta \rangle$

where S_a contains goal descriptions proper to Prog α (relations satisfiable if Prog α is compiled and executed), S_b contains goal descriptions for Prog β. Inter x, Inter y are processes that realise models, generally distinct models, that are compilations in the L-Processor of Prog i, Prog j that *do* satisfy the goals described and, in this sense, are *true* valued.

"Truth" in this internal organisation need only refer to the possibilities of compiling and successfully executing a certain class of programs (all with an associated goal description) in an L-Processor, the existence of which is surely affirmed. These possibilities depend indirectly upon the program classes, Prog, already compiled and under potential or current execution. Hence, "truth" is tantamount to a statement that a system of inferences, hypotheses or beliefs is *coherent*, that it "sticks together" and (first monograph) is "conflict free". Contradiction is not excluded, provided it is conditional and thus hypothetical; for example, the system may contain programs that are modelled and interpreted in *distinct* parts of the universe which compute statements that *would be* contradictory, *if* the distinction were obliterated.

There is nothing in the outward looking perspective, sketched in the last paragraph, to preclude an inward looking perspective. From this latter point of view, certain universes of interpretation *exist*, usually outside the boundaries of an L-Processor, each with its own structure; for example, a molecular view of chemistry, a wave mechanical view of chemistry, the mechanics of a quite different part of the real world. If so, it is possible to reinstate the interpretation function as a mapping (I/F) between existing universes, truth sets and program statements. This is an external observer's image of things. Or, as a more pertinent alternative, interpretation

memory for "a procedure that compiles and reconstructs this concept". In other formulations (chiefly directed towards laboratory sized tasks) our "memory" is more often a "retrieval search".

may still be regarded constructively except that it leads to an identification with some pre-existent reality whereby, for instance, an imagined and "coherent" model is "tested" empirically to establish "correspondence truth" or veridicality.

2. FUZZY PREDICATES

Just as an ordinary predicate, or adjective, names a set of entities having the named property, so a Fuzzy Predicate names a Fuzzy Set. A Fuzzy Set is a function from a universe (its own particular universe) to a truth set. Though several possibilities exist, our immediate concern is with Fuzzy J Sets (Goguen 1968) for which the truth set is $\{0, 1, *\}$ or, verbally, "The interval $\{0, 1:$ meaningless$\}$. Some Fuzzy Sets F, G, ... are shown in Fig. 4.3, named by Fuzzy Predicates. It is crucial to notice that the Fuzzy Set itself *is* the *function*. However, someone in a position to select an element x in the domain of F may refer to the value picked out in F's range (the truth set) as x's "grade of membership" in F; written, in Zadeh's (1973) notation, as μ_F/x. Similarly, x (of Fig. 4.2) has a grade of membership μ_G/x in G and the pair $\langle x, y\rangle$ has a

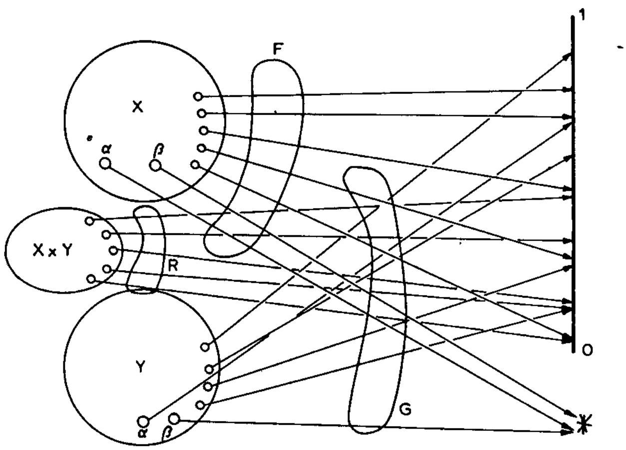


Fig. 4.3. Fuzzy Sets F, G and a Fuzzy Relation R, named by Fuzzy Predicates considered as functions from universes, X, Y and their Cartesian product X × Y onto a many valued truth set.

grade membership $\mu_R/\langle x, y\rangle$ in the Fuzzy Relation, R. In general. just as l-ary Fuzzy Predicates name Fuzzy Sets or properties, so also, n-ary $(n > 1)$ Fuzzy Predicates name relations; as usual, a property is a unary relation. Quite possibly, the elements in the domain of a Fuzzy Set are Fuzzy Sets; so hierarchical organisations are perfectly permissible.

The algebra of Fuzzy Predicates differs somewhat from the algebra of Non Fuzzy Predicates (see, for example, Goguen 1968). Union and intersection are defined, so are *various* forms of complementation. But the behaviour of subsets of Fuzzy Sets is atypical and interesting.

Goguen, explicitly in the 1968 paper, has proposed Fuzzy Sets as the semantical or interpretative images of inexact concepts; that is, real concepts as entertained by minds beset by ambiguity and vagueness to a greater or lesser extent. This point of view is consonant with the position taken in this book and in the first monograph, but it is not identical with it. We maintained previously that a concept is a procedure under execution in an L-Processor which does in fact compute some property or relation named by an L-Predicate. In the generalisation, a procedure is identified with a Fuzzy Program (to be specified below; but a term which encompasses the various programs and non deterministic programs of the first monograph). Surely, the Fuzzy Program (alias procedure) will, if it undergoes execution, produce (stabilise, compute the values of) a Fuzzy Relation or property. This relation or property is given, in extenso, by a Fuzzy Set named by a Fuzzy Predicate. Our (entirely compatible) usage remains: that the concept *is* a procedure undergoing actual or potential execution. The *Fuzzy Predicate* is identified (in proper context) with a *topic* designating a (generally fuzzy) *topic relation.*

Goguen's major insight (which is used in Section 11) is that the universe of interpretation for a *natural* language consists in a set of Fuzzy Sets and that *natural* languages are distinguished from other languages primarily because this is so. The proposal is compatible with conversation theory. Natural language interpretations, especially the analogy relations that are the interpretations of natural language metaphors, serve as a peculiarly flexible modelling facility. The degree of freedom so obtainable may, in principle, be approximated in a physical modelling facility, made in the metal external to the user, and would be a processor able to accept and execute Fuzzy Programs. Any L-Processor is such a thing, paradigmatised by a brain which, we argue, is the (internal to the user) modelling facility for thought.

3. FUZZY PROGRAMS

Just as an ordinary program may be represented by a series of "instructions" which reduce to assignment statements and conditional imperative statements, so a Fuzzy Program (Zadeh 1973) may be represented as a series of Fuzzy or deterministic "instructions" * which reduce to assignments and Fuzzy Conditional Imperatives. A Fuzzy Conditional Imperative specifies a Fuzzy Relation, and that the execution of such a step (for example, using Zadeh's 1973 rule of compositional inference) usually results in a range of values or elements.

Fuzzy Programs have been characterised as algorithms by Santos (1970) and by Zadeh. But they yield Fuzzy or "approximate" results within a certain "tolerance" (see, for example, Cin Dal 1974). Broadly, a Fuzzy Program is a "heuristic". This amounts to slightly more than a mating of nomenclature; something is added to the idea usually conveyed by "heuristic" (even used carefully, as in the context of problem manipulation, by Polya (1954) and others). In fact, the multiplicity of values (or elements of sets pointed out by values of a variable) which generally results from an execution step may either be *perpetuated* or *resolved.* For execution on a serial machine, resolution is almost mandatory. Of the several values generated by execution, one is selected as the representative value to be carried forward into subsequent stages in the computation. Any defensible resolution rule can be employed for this purpose; for example, to select the maximum value or the numerically mean value as the representative. On the other hand, there is nothing in the formulation of a Fuzzy Program to suggest resolution, and given an other than serial processor (notably, an L-Processor accommodating several a priori asynchronous operations), the program itself calls for operations that may either be parallel or, in the sense of the first monograph, concurrent and only local-

* Generally, *Go To,* and *Start* and *Stop* are deterministic instructions, but these structures may be relaxed.

ly synchronised. In the sequel, perpetuation (no resolution) and parallel or concurrent execution are taken for granted.

Mechanically speaking, nothing remarkable is involved but the resulting computation (a heuristic operation) is far richer than the serially resolved process which merely simulates it. The heuristic or Fuzzy Program is, if permitted this fuller meaning, a class of programs for achieving the same Fuzzy Result (for computing values of a Fuzzy Predicate), together with those communicative or locally synchronising interactions required for the execution of these programs.

4. SPECIALISATIONS AND NOTATION

Henceforward, Prog stands for Fuzzy Program.

The term S Prog is reserved for a serial representation of Prog which computes the same relation as Prog but may be compiled in a serial processor.

Similarly Inter (given an L-Processor) stands for a (Fuzzy) Program that compiles a Prog in the processor and assigns the values needed if it is to be executed.

Further, as a special case, Inter is the generation of the mapping in Fig. 4.2 (the constructive realisation of an interpretation, as in Fig. 4.1). *

* The price is that we are committed to a view of the world of conceivable realisations; namely, an L-Processor containing any required number of asynchronous programmable machines (the loci of control of the first monograph) in which an indefinite number of independent dynamic systems may be specified and brought into local synchronicity and/or dependency by instruction, and in which the least element is a system. Realisations of a more restrictive kind are characterised, as they are needed, by the expedients of the first monograph; for example, by *stipulating* that a modelling facility is a one clocked processor, or a collection of one clocked processors permitting, as the case may be, serial or parallel execution of programs. Precisely the same commitment is fairly characteristic of general system theory though it is differently voiced. For example, Beer (1966) refers to the richness of fabric (nature, the unrestricted case of an L-Processor) insofar as fabric accommodates a diversity of process types that are obscured in a tractable abstract representation. Ackoff (1973) makes the statement differently. Elements or "atoms" are systems; their interaction is implicit unless specifically "excluded"; inferences are multiple causal, rather than causal, Singer's producer product relation and its refinements.

In order to deal with analogy relations, it is convenient to develop the very terse definitions employed in the previous monograph (L^0 Procedures, $\underline{\text{Proc}}^0$; L^1 Procedures, $\underline{\text{Proc}}^1$; and so on). The notation introduced in the last section is used for this purpose. Since, in the last monograph, all procedures were qualified as undergoing execution in an L-Processor, no disparities exist. The main objects of the exercise are (a) to distinguish between a class of programs that compute the same *abstract* relation and a class of interpretations for whatever is computed. Due to the difference in interpretation, the results may be called different *topic relations*, even if they have an abstract relation in common; (b) to establish a correspondence between procedural representations and images depicting states of an L-Processor or an external processor (the modelling facility of the previous monograph).

Consider the notion of an L-Procedure (undergoing execution in *any* L-Processor, but some processor always at hand). Henceforward.

$$\text{Procedure} \triangleq \underline{\text{Proc}} \triangleq \langle \underline{\text{Prog}}, \underline{\text{Inter}} \rangle.$$

Thus, observing the artificial stratification of L, the L^0 procedure is

$$\underline{\text{Proc}}^0 i \triangleq \langle \underline{\text{Prog}}^0 a, \underline{\text{Inter}}^0 x \rangle$$

which computes, stabilises, or brings about a topic i.

Notice (as an important feature of the generalisation) that change in either right hand term may determine a fresh $\underline{\text{Proc}}^0$; thus, if $i \neq j$

$$\underline{\text{Proc}}^0 j \triangleq \langle \underline{\text{Prog}}^0 a, \underline{\text{Inter}}^0 y \rangle$$

which computes a topic $j \neq$ topic i, though since the $\underline{\text{Prog}}$ is identical ($\underline{\text{Prog}}$ a), topic i and topic j may share the same abstract topic relation, i.e., the relations in question (as in the first monograph, R_i and R_j) are isomorphic. This possibility is precluded by the other variation for $i \neq k$, namely,

$$\underline{\text{Proc}}^0 k \triangleq \langle \underline{\text{Prog}}^0 b, \underline{\text{Inter}}^0 x \rangle.$$

The term "concept" has the same meaning as it has before (a procedure under actual, or potential, execution) but is more con-

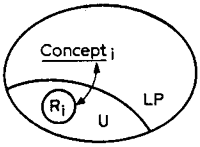


Fig. 4.4. Shorthand notation for action of concept i, compiled in an L-processor (LP) and bringing about relation, R_i in an outcome set interpreted in universe U.

veniently specified as follows:

Concept i $\triangleq$ Stable compilation of $\underline{\text{Proc}}^0$i.

We use the shorthand notation of Fig. 4.4 to indicate that concept i on execution brings about relation R_i in its interpretation set $\mathcal{U}$.

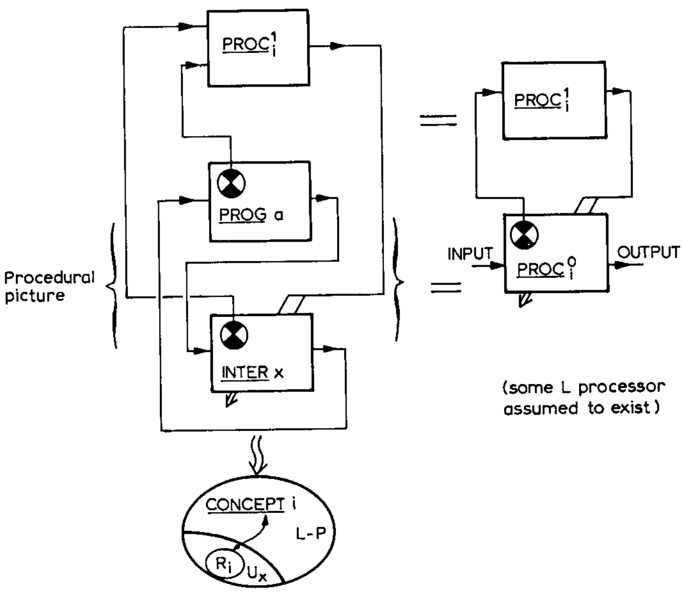


Fig. 4.5. Learning a concept. On left, procedural representation of $\underline{\text{Proc}}^0$i = ⟨Prog a, Inter x⟩. Below concept i is shown in notation of Fig. 4.3. LP denotes L-processor. Set of states U_x is reserved for interpretation of outcome domain in which concept i brings about R_i if it is executed. On right: notation as used in previous monograph where *some* L-processor is assumed to exist.

An L^1 procedure is

$\underline{\text{Proc}}^1 \triangleq \langle \underline{\text{Prog}}^1, \underline{\text{Inter}}^1 \rangle$

where Inter^1 need not be made explicit. The form of constraint is discussed in Chapter 5, Appendix 2.2; it consists in any workable structure existing in the L-Processor and possibly the repertoire of existing Proc^0. As in the previous monograph, the limiting case in which Proc^{1}i acts upon and reconstructs Proc^{0}i is a memory, the simplest form being merely a recompilation of Proc^{0}i. This is shown in Fig. 4.5, together with the stable compilation (in an L-Processor) to which it gives rise "℘" (that is the symbol "℘" links the procedural representation to the notation employed in Fig. 4.4.

More generally, Proc^1 carry out *constructive* as well as reconstructive operations and they must do so in the case of an *understanding* (the primary condition detected in a strict L Conversa-

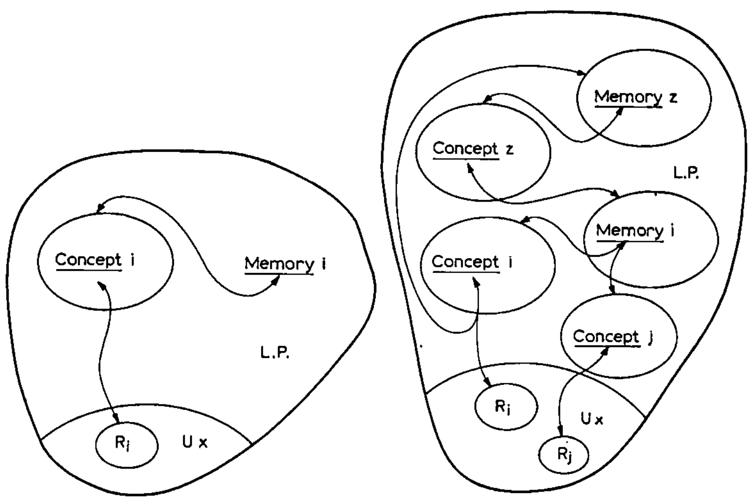


Fig. 4.6. On left, construction of memory comparable to construction for a concept; on right, bifurcating and looping constructions are permissible, insofar as stratification of $L = L^1, L^0$, (an artificial distinction, in any case) is preserved. Here, for example concept z is not allowed to bring about a relation R_z and neither concept i nor concept j can, as one of their immediate products, reconstruct memory i. This is a matter of edict, however, not of fact.

tion). The minimal form of an *understanding* is shown in Fig. 4.5.

Fig. 4.6 depicts the actual liberality of bifurcating and cyclic connections and reveals the frequently stressed fact that the stratification of L into L^1, L^0 is conventional, not factual. If imposed for convenience, the stratification disallows many cyclic organisations which could otherwise exist.

6. GENERALISATION OF ENTITIES IN THE CONVERSATIONAL DOMAIN

For notational clarity, the programs extensionally equivalent to (that do the same thing, by computing the same relation as) Prog^0i are represented as behaviour graphs (Chapters 1 and 2) denoted *BG*(i): meaning "descriptions of and precipitations for *model-building* behaviour". As noted before, all behaviour graphs are thus program graphs (for example, Chang and Lee 1973). The (many) programs exhibited in one behaviour program graph, *BG*(i), only represent $\underline{\text{Prog}}^0\text{i}$ (Section 4) since *BG*(i) is non Fuzzy. These representations are designated S $\underline{\text{Prog}}$ i (Section 4).

A modelling facility to accommodate non verbal explanations as compilations and interpretations of programs in *BG*(i) is a (restricted) universe of interpretation, or a set of a priori independent universes of compilation and interpretation. In other words, it is one or more processors, together with interpretation sets for the input and output domains of programs that may be compiled and executed. If there are several a priori independent processors, we use the neoglism "Lumped Modelling Facility" to denote the aggregate.

In either case, the modelling facility executes compiled programs as models to yield results in an interpreted input-output set (more usefully, in the product of *input-set* $\times$ *output-set* = *outcome-set*). Any correct model for *topic* i is such that the execution yields one or more outcomes (all of which belong to $R_i \subset$ *outcome-set.* Since a "Lumped Modelling Facility" is described by L-Predicates, the models that may be constructed in the facility form a model space. The facility is more restricted than an L-Processor due to the "clocking" restrictions (first monograph) upon the constituent processors. The graphical notation of Fig. 4.7 is used to represent a model. In this picture, which is intended to

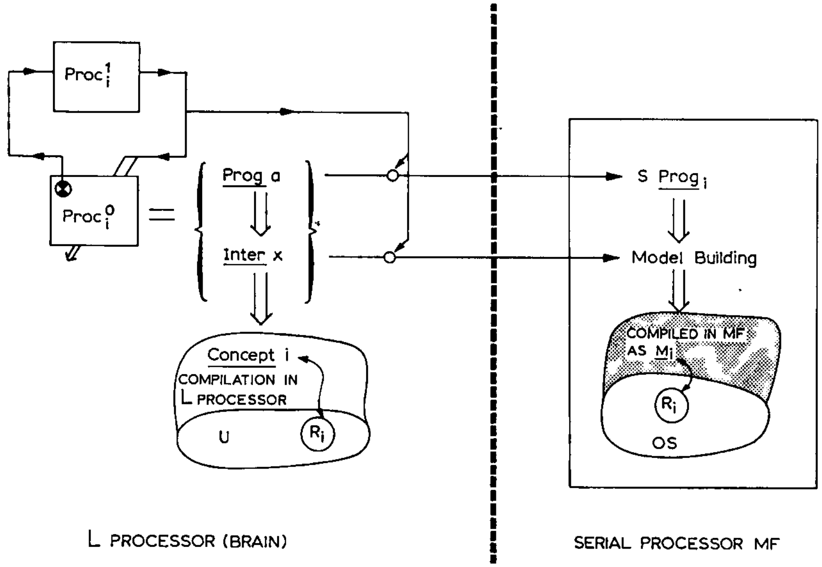


Fig. 4.7. Model building.

clarify the distinction between a model *external* to the L-Processor and a concept as a compilation *in* an L-Processor, the modelling facility *MF* is based on a serial or one clocked processor with an interpreted outcome set distinguished as *OS*. Some S Prog i, representative of Proc i, is compiled in the modelling facility as a model M_i. It is important to realise that whereas Prog a is a Fuzzy Program and Inter x is its Fuzzy Compilation (in an L-Processor, the student's brain in this case), as a concept the representative program S Prog i is serial and M_i is its compilation in the serial processor of *MF*. Model M_i is correct if the result of its execution is equivalent to the result of executing some program (S Prog i) in *BG*(i), and if it secures R_i in the interpreted outcome set *OS*. (Models for analogy relations requiring lumped modelling facilities with several independent processors are discussed in Section 9 and shown graphically in Fig. 4.8.)

One other feature of Fig. 4.7 is of importance. Just as the stable compilation of $\text{Proc}^0 i$ depends upon the operation of an L^1 process, $\text{Proc}^1 i$, so the selection of a representative S Prog i and its compilation in *MF* as M_i depends upon $\text{Proc}^1 i$.

The task structure *TS*(i) is an imperative form of the program graph *PG*(i). It represents all the demonstrations that can be given to a student using the modelling facility, and (as in the first monograph) is tantamount to a class of behavioural prescriptions for achieving the behavioural objective of satisfying R_i.

The entailment structure, *ES*, figures, as it did in the first monograph, in a dual capacity. On the one hand, it represents legal derivations of topics and thus *what may be known* (in the same way that *TS*(i) stipulates *what may be done* if the ith topic is selected). On the other hand, *ES* constitutes a modelling facility at the cognitive level in which the student exteriorises his actual derivation of a topic as a state marker distribution or learning strategy. In this capacity the entailment structure and the storage locations for marker placements (for the aim topic for the goal topic and so on), *ES* is an L^1 analogue for the L^0 modelling facility *MF* in which explanatory behaviour is exteriorised.

Finally, the *conversational domain* is the entire collection: entailment structure and the operator data base (first monograph) that back it up; for each *topic* i in the entailment structure either *BG*(i), or *TS*(i); the syntactic and semantic descriptions $D^1(R)$ of the derivations in *ES* and $D^0(R_i)$ of the compilations of each *BG*(i) in *MF*.

7. DIFFERENT TRUTH CRITERIA AND TRUTH VALUES

The following types of "truth" are generally recognised; correspondence truth, consensual truth and coherence truth.

Of these, correspondence truth is concerned with the result of testing that something has a mooted property, or that a given relation holds and is qualified by "in such and such (or *all*) worlds or universes of interpretation". If this qualifier is rescinded by supposing that any person or entity able to make a test is looking at the *same* world, then empirical evidence is obtained and a hypothesis based upon this evidence may be conditionally verified. (The contingency is present because things change, because the assumption of similarity is doubtful, because the relevance of data is never completely determinable, and because tests are fallible.)

Consensual truth is a form of gross accord between observers. In its naive form, consensus (over the admissibility of evidence, for example) is the outcome of a voting match between the observers.

But the refined versions of consensus admit for discussion in the course of reaching agreement, and in this case, consensual criteria are really being treated as the coherence criteria of the next paragraph.

The coherence truth of a proposition, p, is a question of the extent to which p forms part of a system of cogent inference with respect to some other corpus of propositions; for one example, those entailed by a prevailing thesis or a body of convictions, beliefs or (even) dispositions; for a further example, those propositions apposite to different possible worlds. Advocates of coherence truth include Bradley (1914); many of the notions are presaged in the writings of Leibnitz (especially in the sense of the "further example") and can be traced back as far as the ancient philosophers. The field is reviewed and an up to date coherence theory is developed by Rescher (1973). This recent theory is of peculiar interest insofar as one goal is to extract the maximum possible coherent content from a set Θ of generally inconsistent propositions, {p, q, ...}.†

Let Θ^* be Θ devoid of p. Now p is maximally coherent with Θ^* (thus, is a "strong" member of Θ), if p is a deductive consequence of the propositions in Θ^* (so that the negation of p is incompatible with Θ^*); p is coherent (to some extent) with Θ^*, if p is not incompatible with the deductive consequences of Θ^* and is thus a possible member of Θ. Now, given a set, Θ, it is possible to specify a family of non empty maximally consistent subsets of propositions (*mcs*) of Θ, such that any *mcs* is consistent, and such that the addition of any q in Θ to an *mcs* devoid of q renders that subset of propositions inconsistent. The coherently true content of the original collection might be specified as "that which is a deductive consequence of all the maximally consistent subsets" (Rescher's "I consequence"), or "that which is a deductive consequence of any of them" (Rescher's "W consequence"). In fact, Rescher recommends the use of intermediary criteria. A preference (an alethic or truth oriented preference) is employed to determine a set of eligible maximally consistent subsets of Θ, and the coherent content is whatever is a deductive consequence

† Our set "Θ" is Rescher's set S; as usual, the limitations of the alphabet make it impracticable to maintain a concordant notation, and we have used "S" for other purposes.

(Rescher's "P consequence") of any subset in this preferred set.

The theory is primarily concerned with working out the truth about a phenomenon based upon a set of observations and, perhaps, some existing observations. Hence, the propositions p, q, ... are data; they are candidates to be accepted or rejected according to whether they (and prior propositions) form a system, with the caveat that as much content as possible be extracted from the data. In order to count as a datum, however, the propositions (results of observation, for example) must have an extra logical claim to datahood and must also be sufficiently comprehensive to cover all possibilities relevant to the phenomenon under scrutiny. Similarly the preference criterion (in the original, an alethic preference unrelated to desire and attuned to objectivity) is also extra logical, and in the province of epistemology.

8. AGREEMENT AND COMMON MEANING

Our concern in this book is certainly not "logical" in the technical sense. It is psychological and epistemological. Consequently, our motives in mustering notions of coherence are distinct from Rescher's, and it is prudent to stress the differences at the outset. Except indirectly, the argument has little bearing upon rational assessment or even upon "necessary" or absolute truth. Nevertheless, the truth conditions of correspondence, coherence and consensus (as a form of coherence) hinge upon various kinds of agreement which implicate (at least) provisional and idiosyncratic truth.

Correspondence truth values (albeit local to a universe of interpretation) appear in adjudicating the "correctness" of a model; of whether or not a relation, R_i, is satisfied when the compiled model is executed, and whether or not the syntactic component (S Prog) of a model matches some other program or a class of programs, such as BG(i). In general, the logic is "non classical" both in the sense that it is a logic of action and in the sense that its truth sets are many valued (the valuations are of Fuzzy Predicates).

An external interpretation of $\underline{\text{Proc}}^0$i is the explanatory model preferred by a participant who is learning *topic* i. The correctness of this model (Section 6) depends upon whether or not its execution satisfies R_i. Correctness is thus, amongst other things, an index of correspondence truth, local to the universe of interpretation furnished by the modelling facility. Indirectly, correctness also implies that the representative program can be compiled and that its compilation as a model can be executed. Similar remarks hold good if the model itself is matched against the class of models *TS*(i) obtained by interpreting any of the programs in *BG*(i), all of which satisfy R_i. Moreover, both correctness and matching (against models in *TS*(i)) are special cases of a semantic or interpretative agreement; a participant agrees that the model (or the result of its execution) tallies with a canonical form.

The general case of semantic agreement involves two or more participants. That is, some other participant, often in a dominant and judiciary role (for example, a teacher), makes a demonstrative model in the same modelling facility. The result of executing the authority's demonstrative model is compared with the result of executing the submitted explanatory model, and the two participants agree that these results do, or do not, satisfy the *same* interpretation of a topic relation.

Such a semantic agreement is severely limited. It says nothing, of necessity, about general empirical "truth" or absolute rationality; nor does the related canon of correctness. For example, if the original thesis propounds a falsified theory, correctness means "correct with respect of some part of this false theory, or with respect of it all". Participants are obviously able to reach agreement upon irrational, or empirically refuted, propositions.

But, to do so is not pointless. Though the status of a semantic agreement is limited, it does mean more than a vague accord. The participants who agree have been able to interpret a relation (and a program which computes it) in some world, perhaps a very bizarre world, and they agree that these interpretations (of the relation) are the same, or are within tolerance. Moreover, in this universe, the compilations of their programs work to bring about the given result.

The companion notion of coherence is also essential to the idea of agreement. The main point is that coherence between the statements entertained by two or more participants implies a basically *syntactic* agreement, though depending upon the circumstances, more than syntactic agreement *may* be involved.

In the first monograph, we specified the mediator of cognition as a Psychological-Individual or P-Individual. Any P-Individual is the replication (or self-stabilisation) of a repertoire consisting in units $\langle \underline{\text{Proc}}^1, \underline{\text{Proc}}^0 \rangle$ (Section 5). The construct is essentially dynamic; the procedures making up the P-Individual are undergoing execution in some L-Processor. However, we do not insist that a P-Individual is localised, geographically, in a particular brain. Nor do we exclude the possibility that several P-Individuals cohabit the same brain, provided it is an L-Processor and thus is able to execute L-Procedures. As a matter of fact, both kinds of distribution of cognition are commonplace and are necessary features of a strict conversation, in which *understandings* are observable.

Having insisted that a P-Individual is a dynamic system, it is plausible to characterise it, alternatively, as some consistent and self-replicating system of hypotheses or beliefs, and thus to liken it to the sociological construct of a role. In this specification, "hypotheses" and "beliefs" are regarded as active cognitive processes "entertaining hypotheses" or "subscribing to beliefs", so that this picture of a P-Individual is quite similar to Kelly's (1955) picture of "man as an experimenter" or even, at a different and broader grain of theorising, Lewin's (1936) view in this matter.

Consider the artificial and imaginary expedient of freezing the P-Individual into momentary stasis. Under this imaginary assault, the "hypotheses" and "beliefs" make an appearance as "L Propositions". Call the set of L Propositions Propset. Manifestly, "any p, q, ... in Propset is coherent with Propset", i.e., the set of propositions representing the hypotheses or beliefs of the P-Individual at a particular instant are (L) coherent. If this were not the case, the P-Individual would not be (as asserted) self-replicating (though the converse of this contention to the effect that "if Propset is coherent then the system is self-replicating" is, clearly, not valid).

Coherence of Propset, in this sense, may have no greater status than a personal and private "truth"; the P-Individual's set of "beliefs" are amongst the deductive consequences of {p, q, ...} in Propset. To be more discriminating, we invoke instruments analogous, on the one hand, to Rescher's alethic preference ordering (so that only the deductive consequences of the preferred *mcs* are followed up) and, on the other hand, to the criteria of datahood (that the {p, q, ...} are truth candidates, both relevant and worth having). Lacking such an augmentation, a P-Individual may be nothing but a dreamer or a solipsist or a system that regurgitates the ultimately tautologous verb and adjective chains of an internal dictionary. In the extremity, a coherent Propset is a syntactic construction, and the further assertion that this Propset characterises a P-Individual leads only to the semantic inference that an L-Processor exists and is able to execute it. Perhaps the creature can do nothing except to say "I" repeatedly, like the bleating "point" of Flatland, in Abbot's (1884) geometrical fantasy.

Suppose there are two or more P-Individuals, A, B, in conversation, and their Propsets are constructed and symbolised as Propset A and Propset B. If the propositions in Propset A and Propset B are mutually coherent (so that Propset A, B is coherent), then the mutual coherence is an index of syntactic agreement between A and B. By the same token, there may be a syntactic agreement between factors of one P-Individual (A, for example) in respect of a conversational domain. This agreement is a statement of consensus (between A and B, or the factors of A) in terms of coherence. Consensus, in the sequel, is identified with such an agreement. But the statement, as it stands in its syntactic form, is minimal. Much more can be said if the conversation is *strict* and based upon *understanding* (in the technical sense of explanation conjoined with derivation) or the construction of $\underline{\text{Proc}}^1$i, $\underline{\text{Proc}}^0$i, as in Section 5.

Let a P-Individual engage in a strict conversation anchored upon a fixed conversational domain, taking place over occasions 0, 1, ... n ... N. Upon each occasion some topic in the domain is understood.

As in the first monograph, let π(n) stand for the repertoire of pairs $\langle\underline{\text{Proc}}^1\text{i}, \underline{\text{Proc}}^0\text{i}\rangle$ that are learned, reproduced, and stabilised at occasion n. Due to the construction of the conversational domain and the characterisation of any P-Individual, it is possible to order the repertoires π (n), as follows:

$$\pi(0) \subset \ldots \subset \pi(\text{n}) \subset \pi(\text{n} + 1) \subset \ldots \subset \pi(\text{N}).$$

With each π (n) associate a Propset (n); it will contain propositions asserting, or hypothesising, topics in the domain and relations between these topics. Each assertive proposition is rooted upon an *understanding*, and explanation and a derivation of some topic, held to evidence $\langle\underline{\text{Proc}}^1\text{i}, \underline{\text{Proc}}^0\text{i}\rangle$. The hypotheses concern topics up to and including the aim topic current at occasion n. The act of pointing out a topic to learn (issuing a command or asking a question, as the case may be) introduces at least one further candidate, and in general, the coherence of Propset is reduced by adding this candidate with hypothetical status. On the other hand, the act of *understanding* restores coherence and may increase it. That is, *Coherence* ($\underline{\text{Propset}}$ (n + 1)) $\geqslant$ *Coherence* ($\underline{\text{Propset}}$ (n)) and, in general, *Coherence* ($\underline{\text{Propset}}$ (n + 1)) > *Coherence* ($\underline{\text{Propset}}$ (n) (The Gestalt property, claimed for the conversational domain). *

Further, if the topic is correctly explained (as it must be for *understanding*), then the resulting proposition is credited with the weak correspondence truth, to which we previously alluded. If the topic is legally derived, then a similar credit is given to at least one proposition affirming a relation between topics. In brief, the coherent propositions are, by virtue of *understanding*, assigned a (weak) *semantic* truth value.

For a consensual externalisation, suppose two or more P-Individuals (A, B) or two or more factors of one P-Individual engaged in a strict conversation on a fixed domain. We recognise the following types of consensual agreement between one and the other.

(a) A syntactic agreement of degree depending upon the coherence of their $\underline{\text{Propset}}$.

(b) A semantic agreement regarding interpretations or models at level L^1 as well as at level L^0).

(c) If the participants have *both* syntactic and semantic agreement in respect of one or more topics, then these topics have the same *meaning* to the participants.

9. COMMON MEANING AND ANALOGY RELATIONS

Consider the conversations proper to learning about a given conversational domain. Such conversations may be of several types (reviewed in the next section), but the simplest kind amounts to a student engaging in "conversation with himself"; that is, a student represented as a pair of internal participants, one teacherlike and one learnerlike, who is "learning on his own".

* In practice, it is possible to determine the style of learning by examining the magnitudes of coherence values. For example, someone who recalls topics in a conversational domain by deriving them in many ways from other topics has a greater coherence, associated with his $\underline{\text{Propset}}$, than someone who learns and uses just one derivation.

Now ask, "Is there an inscription in a conversational domain which may be learned of a *common meaning* (Section 8(c) above) agreed between participants?" The reply to this question is affirmative, and the desired inscription is an Analogy Relation.

To see this clearly, distinguish between the syntactic and semantic components of a subject matter thesis represented in a conversational domain. The syntactic component is an expression of derivations of topic relations and the uninterpreted program graphs attached to each topic. The semantic component is made up of the modelling facility (the compilation/interpretation set) and the *description* of the entailment structure afforded by the descriptor values assigned in $D^1(R)$ (from which a description of the compilation/interpretation set is derived).

Specifically, an analogy relation, (Fig. 2.6) is distinguished from the derivations in a disjunctive or conjunctive substructure by the fact that a semantic component is essential to its *cyclicity* (first monograph) and is represented by the semantic predicate Dist which distinguished between universes that are related in the analogy by a *morphism;* in the limit by an *isomorphism.* At the risk of labouring this point, notice that conjunctive and disjunctive substructures are also cyclic, Fig. 2.3, but the cyclicity of the analogy alone depends upon Dist. In terms of the first monograph, this fact demarcates *isomorphism* (where there is one to one register between topic relations, but no identity) and the other relational operations able to preserve specificity all of which secure relational identity. More generally, the similarity part of an analogy relation is syntactic and the difference part is semantic, as minimally indicated by the predicate Dist.

This general statement is in complete accord with Hesse's (1963) elegant-analysis of the analogy relations of science. The similarity is expressed by a morphism (and ideally, an isomorphism) between rules or abstract systems or scientific laws; the difference is expressed by a possibly incomplete list of properties characterising universes of interpretation (for example, "optics" and "sound").

Hesse's argument is peculiarly germane to the current theory, since it stresses that material analogies (those which can be modelled in a modelling facility) are based upon similarities of a causal or functional sort; as a result, upon rules that can be expressed by means of programs executed in a serial processor (the compilations of which are finite state machines). Our terminology "syntactic" covers this case quite adequately but will, later on, allow access to less well structured analogy relations. The analogy between similar rules (programs) that are compiled and executed in different universes of interpretation may be expressed as an isomorphism between some of the properties of each of the universes (X, Y); namely, those properties which enter into a specification of the *outcome set* (Section 6). The list of properties pertinent to each universe of interpretation, for example, the lists:

Optics	*Sound*
Colour	Pitch
Intensity	Loudness
⋮	⋮

are isomorphically related, if the given rule relates those in "optics" and the same rule relates those in "sound" (for instance, a simple wave propogation equation involving these terms when it is interpreted). But the list may either be complete: each of the indefinite number of properties that might be cited can be given a "positive" (i.e., member of the list) or a "negative" value. Or it may be incomplete: some properties have undetermined relevance at the moment the analogy relation is stated; for example, the "medium" in which sound waves travel may or may nor correspond to a "medium" (the aether, historically) in which light waves travel.

If it happens that the syntactic rule corresponds to a Program $\underline{\text{Prog}}$ a, then (for colour and intensity relevant or positive optical properties, and pitch and loudness relevant sonic properties) the isomorphism in the analogy relation is:

Colour ⇔ Pitch
Intensity ⇔ Loudness

Given $\underline{\text{Prog}}$ a

Similarly, if x and y are outcome sets characterised by these properties, this is in accord with the formulation of Section 5, namely, for an analogy relation R_k between R_i and R_j

$$\underline{\text{Proc}}\ i = \langle \underline{\text{Prog}}\ a, \underline{\text{Inter}}\ x \rangle \Leftrightarrow \langle \underline{\text{Prog}}\ a, \underline{\text{Inter}}\ y \rangle = \underline{\text{Proc}}\ j$$

where Proc i, Proc j satisfy R_i, R_j.

The only significant difference is that Proc k is seen as a procedure which computes the value of a distinguishing predicate Dist x, y which determines in what respect the universes of interpretation differ. But it is surely the case that any student having Proc k in his repertoire is in a position to test any property that comes to mind, or is observed, as being relevant or irrelevant to the analogy relation.

All the analogy relations considered in the first monograph can be expressed in these terms; notably, those holding between the "real" and the "abstract" universes of interpretation in "probability theory". Several other specific examples, culled from our work, are discussed in connection with conversational domains.

The immediate point of emphasis is that R_i and R_j, whatever they represent, are presented to a student as distinct; they are the relations of different topics in the conversational domain. Insofar as the student regards them as distinct and consequently views them from a different perspective, he is, at any rate in a momentary way, represented as two distinct entities. Consensually, these may oscillate so that R_i is learned (or thought of, or attended to) at one moment and R_j at the next. Insofar as R_i and R_j both occupy his attention (or are learned about, still as distinct topics, simultaneously), the entities are two participants. Finally, if it happens that the student assimilates an analogy relation between *topic* i and *topic* j, then the (albeit transient) participants reach a *common meaning* which, if it tallies with the analogy relation inscribed in the conversational domain, is the specified analogy R_k between R_i and R_j. An understanding (explanation and derivation) of R_k is evidence to this effect.

Evidently, R_k is also a *common meaning* to comparable "participants" inside the subject matter expert who inscribed it as a topic in the conversational domain, and we return to the question of *constructing* analogy relations after detailing the act of *understanding* an analogy relation. Some groundwork is needed in order to update the exposition in the previous monograph so that the account adumbrates certain revised transactions in the operating systems (either CASTE or INTUITION as described in Appendix A) and the, now explicit, distinction between the Prog and Inter components of a procedure.

10. UNDERSTANDING OF AN ANALOGY RELATION: EXPLANATION

Consider first, the non verbal explanation or modelling of an analogy relation which is one necessary component of its understanding. Suppose a Lumped Modelling Facility consists of two a priori independent serial processors connected to one or moré outcome sets.)Generally, there are very many possible outcome sets, but they are invariably partitioned by the semantic descriptors into parts proper to each processor.) For example, in STATLAB (of the first monograph and the Appendix) there are two processors, one attached to the "real" universe of interpretation, and one attached to the "abstract" universe of interpretation. Their outcome sets consist in "simple events" or "composite events" or "measures" (in one case), and "simple results" or "composite results" or "frequency ratios" (in the other). Call one processor and its outcome sets X, and the other processor and its outcome sets Y.

In order to explain the analogy relation R_k between topic i and topic j, the student must ultimately do as follows:

(a) Build a correct model, M_i, of concept i which on execution in the modelling facility realises R_i in X.

(b) Build a correct model, M_j of concept j which on execution in the modelling facility realises R_j in Y.

(c) Couple X and Y so that the isomorphism between R_i and R_j is realised, and execute the models *simultaneously* to satisfy R_i and R_j. This coupling, a model M_k, satisfies the analogy relation R_k.

To summarise, clause (a) is evidence that $Proc^0 i$ exists; clause (b) that $Proc^0 j$ exists; and clause (c) that $Proc^0{}_k$ exists. If backed up by evidence for derivations, this provides evidence for a stable concept, and thus for an *understanding.*

Now clause (c) has greater content than it seems to have. In general, the simultaneous and successful execution of M_i and M_j implies more than a coupling between their outcome sets. It is possible if, and only if, the pair of a priori independent processors in the X and Y parts of the Lumped Modelling Facility are partially synchronised, either by interruption signals or by other methods. The crux of this requirement is not well illustrated by the examples in the first monograph (where, for the most part, the reality of concurrent execution of M_i and M_j was not encouraged and analogy relations were modelled formally). In fact, the subject matter employed permitted this glossing, though we noted persistent student demands to "compare models". As promised, the defects have now been remedied and concurrent execution is rendered mandatory. Its impact is easily imagined in the context of the "optics" and "sound" example, and the matter is pursued in Chapter 7 using the subject matter of energy conversion and simple thermodynamics.

With these comments in mind, Fig. 4.8 shows the structure built up in a modelling facility. M_i is the *compilation* in X of S Prog i (representing Proc i) with its outcome set (OSX) distinguished and having R_i as a subset. M_j is the compilation in Y of S Prog i (representing Proc j) with its outcome set (OSY) distinguished and having R_j as a subset. For correctness, M_i and M_j are matched against BG(i) and BG(j) and the satisfaction of R_i, R_j, is determined. This operation is not shown. The coupling and partial synchronisation appear in the picture as the connections between OSX and OSY, together with those between X and Y. If, and only if, the models can be jointly executed to satisfy R_i and R_j, the analogy relation R_k is correctly modelled.

The sense in which the entire model, M_k constitutes an index of

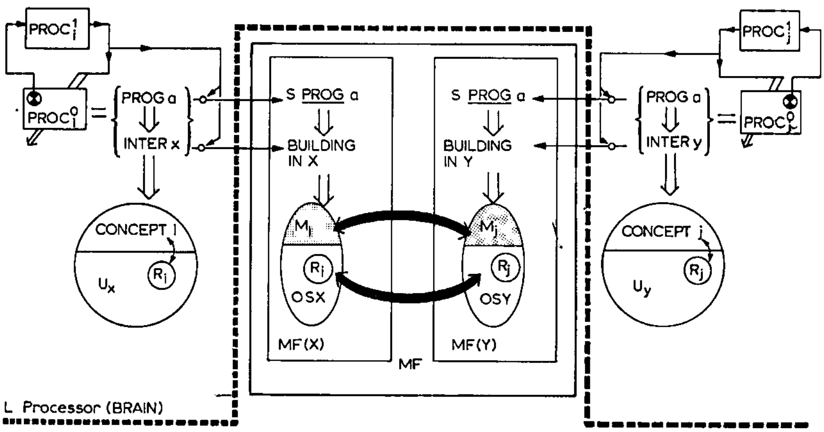


Fig. 4.8. The model for an analogy relation is a coupling (signified by ⇄ between models M_i and M_j) as a result of which the execution of M_i and M_j in *MF* is synchronised.

common meaning is shown in Fig. 4.8, which is no more than an outline sketch for the cognitive organisation we suppose to be responsible for the analogy relation model. As before, Proc i and Proc j are L^0 procedures in the cognitive repertoire. They consist in Prog and Inter components; namely, ⟨Prog a, Inter x⟩ and ⟨Prog a, Inter y⟩. Insofar as the student selects S Prog i (compiled as M_i in X of the external facility) and S Prog j (compiled as M_j in Y of the external facility), Inter x and Inter y are generating distinct compilations in the students brain (an L-Processor) for Prog a. These are sketched as Concepts i, j, conceived as internal representations on a par with the external models M_i in X and M_j in Y. Insofar as the student places x and y in register with the correct (in the sense of relevant) properties of X, Y, and to the extent that correctness is betokened by the successful (joint) execution of M_k he also has in his repertoire a further procedure Proc Dist = ⟨Prog Dist Inter x × y⟩ which is internally compiled in the product $U_x \times U_y$ and distinguishes between $U_x \times U_y$ appropriately. (And must do so, since under an operating system, Dist (x, y) is a semantic descriptor and is already marked as being understood.) That is, Proc Dist computes the distinguishing descriptor Dist (x, y).

The "internal" (or imaginary) participants said to reach "agreement" over a *common meaning* are centered upon Proc i and Proc j; they are held distinct by the action of Proc Dist; they have Prog a in common; their agreement amounts to a recognition of this communality, even though Proc Dist exists. The semantic (or correspondence) component of the agreement is the model for the analogy relation. Its syntactic (or coherence) component is the isomorphic register between Progs in Proc i, Proc j. We refer to the internal participants as "imaginary" because we are concerned with experiments or tutorials in a one aim at once facility, such as CASTE or INTUITION. Hence, although the foci of attention of the "participants" may be real enough to a student (and common experience suggests that they are), the transactions are not distinctly observable as exteriorised stretches of behaviour.

11. UNDERSTANDING OF AN ANALOGY RELATION: DERIVATION

Now, consider the other aspect of understanding an analogy relation: its derivation, which is exteriorised as a learning strategy traced out on an entailment structure. For a two term analogy * just four basic configurations are possible (though these give rise to innumerable variants). Assuming that the student's aim (his "focus of attention" or the "topic that he appreciates") is *at* or superordinate to the analogy relation R_k (Fig. 2.6), these configurations (Appendix A) are as follows:

(A) *Topic* i is understood, *topic* j is understood and the analogy relation is marked as goal which is a legal member of workset.

(B) *Topic* i is understood, the subordinates of *topic* j are understood and the analogy relation is marked as goal which is a legal member of workset.

(C) Vice versa, but *topic* j is understood instead of *topic* i.

(D) Neither *topic* i nor *topic* j is understood but the subordinates of at least one of them are understood. The analogy relation is marked as goal. Under the conditions discussed in the first monograph, this placement of markers does not admit to goal as a member of workset. However, in the revised operating systems that are currently in use, it does (and *may* do so because of the possibility of concurrent modelling).

Configuration A obtains if the student intends to learn the analogy relation as a relation between existing concepts for *topic* i and *topic* j. As a practical consequence, the student may (if he wishes) receive a demonstration of the isomorphism and of $\underline{\text{Dist}}$, and he must model the analogy, as in Fig. 4.8, if this *topic* R_k is to be marked as understood. Notice, however, that M_i and M_j both exist.

Configuration B obtains if the student intends to learn the analogy relation in terms of *topic* i and to derive an explanation of *topic* j in terms of R_k. As a practical consequence, the student may (if he wishes) receive a demonstration of R_k as a path to *topic* j, and for understanding of R_k he must model R_k which involves constructing M_j (since M_i already exists).

Configuration C is the reverse situation in which *topic* i is accessed through R_k. The student may (if he wishes) receive a

* That is, an analogy relation between two topics. Similar comments apply to analogies involving many terms or other analogy relations of the type exhibited in Chapter 2, but the configurations are much more complicated and are difficult to represent graphically.

demonstration of R_k as a path to *topic* j. He must model R_k if R_k is to be marked as understood; this entails the construction of M_i, but M_j exists.

Finally, the configuration D leads to a conditional transaction. The student may receive demonstrations of *topic* i and *topic* j (if he wishes). But in explanatory modelling, he can essay the construction of a coupling between unspecified and not-yet-understood topics. However, a model such as this is accepted conditionally. The analogy relation is marked as understood unconditionally if, and only if, M_i and M_j are produced (to be united by the coupling), as a result of which *topic* i and *topic* j or both of them will be marked as understood. In the process R_i or R_j or both of them will be marked as goals at the same moment as R_k. Since this implies that workset has more than one member, the manoeuvre is necessarily part of a holistic learning strategy and is, in fact, adopted by holistic students.

One psychological interpretation (which we favour as by far the most plausible) is that conditions A, B, C involve learning an analogy relation when one (condition B or C) or both (condition A) of the terms of the analogy are known already. In condition D, on the other hand, the analogy relation appears first of all and the terms (*topic* i or *topic* j or both) are understood because the analogy is known. For example, using the subject matter "energy conversion" of Chapter 7, the student in condition A discovers a relation ("heat conservation" cycle) between "heat engines" and "refrigerator" both of which are known to begin with; in condition B or C, he knows about "heat engines" or about a "refrigerator" and derives "heat conservation cycle" because of that. Of course, we may not exclude a global looking and comprehensive approach in these cases, since any student could fail to exteriorise his mental gambits. But in condition D, either "heat engines" or "refrigerator" or both are understood as a result of knowing about "heat conservation cycle" and in this case the student *must* be adopting a global method.

All of the conditions for learning an analogy relation are consonant with Fig. 4.8 and with the notion that the analogy relation is just the inscription of a common meaning (recall that each term is modelled, though it is only marked as understood if it was marked as a goal). On the other hand, the order of events and the type of interaction between <u>Procs</u> differ radically according to the condition selected. In particular, conditions A, B, and C involve the existence of Proc i or Proc j or both before there is an L^1 operation (a. $\underline{\text{Proc}}^1$) that places these concepts in register; whereas in condition D, this operation is performed over Dist (x, y) before Proc i and Proc j are constructed. This we can only construe (mechanically speaking) as implying the existence of a hybrid procedure, neither Proc i nor Proc j, which is differentiated to yield Proc i and Proc j.

12. THE ACT OF CONSTRUCTING AN ANALOGY RELATION

So much for *learning* an isomorphic analogy, as it is inscribed in the conversational domain. When it comes to *constructing* an analogy (during course assembly, or under the control of an evolutionary heuristic), the participants we dismissed as "imaginary" may be very real. These participants could be members of a team of subject matter experts, or equally they could be distinct cognitive organisations that are parts of one subject matter expert.

Since the course assembly heuristic EXTEND, considered in the first monograph, is (like CASTE) restricted to one aim at one, these interesting segments of cognition cannot be exteriorised in the system. But other heuristics to be described in Chapter 7 (as part of an operating system called THOUGHTSTICKER) allow for many aim topics.

ADDENDUM

Two recent papers by F. Varela ("A calculus for self reference", *Int. J. Gen. Syst.* 2, p. 5, 1975 and "The extended calculus of indications interpreted as 3 valued logic", *Notre Dame Journal Formal Logic*, 1976) provide a respectable means for talking of reflective and self reproducing systems (a fortiori, P Individuals) *within* the language L. The basic idea is to reserve a truth value for the condition of recursive or vicious circularity and the possibility of doing so stems from the calculus of distinctions and indicators (Spenser Brown, G. 1969, *Laws of Form*, George Allen and Unwin, London), to which the first monograph owes so much. The difficulty that different kinds of circularity are inadequately distinguished is resolved in a further paper, "The Arithmetic of Closure", which will be part of the proceedings of the 3rd European Conference on Cybernetics and Systems Research, Vienna, 1976; with this augmentation, it becomes possible to speak, similarly, of interactions between several distinct P Individuals.

Almost simultaneously J. Goguen ("Objects", *Int. J. Gen. Syst.* Vol. 1, p. 237, and "Complexity of Hierarchically Organised Systems and the Structure of Musical experience," *Technical Report* in Department of Computer Science, UCLA, 1976) has rooted general system theory in "objects" that depend (in a sense) upon observation and has shown how systems are amalgamated by dependency/independence, or synchronicity/asynchronicity, to create further systems.

The relevance of this work is evident; it is compatible with our informal argument, though more elegant. These significant innovations are currently being incorporated (under the notion of categories of "objects" that are P Individuals) and tangibly implemented, by Robert Newton, at this laboratory.

Chapter 5

Description Building and Procedure Building

Chapter 2 contained an informal discussion of categories of mental operation called "Procedure Building" (*PB*) and "Description Building" (*DB*), together with an allusion to a further, hazily specified category of operations called "Procedure Construction" (*PC*). Of these categories *DB* and *PB* (at least) were said to be global or local in form and we hypothesised that globality/localness is a substantially invariant propensity for a given student. Similarly, students may be characterised in terms of the efficacity of the *DB* and *PB* operations in their mental repertoire. "Efficacity" might be no more than preponderance, it might be a more subtle operational quality. At any rate, the characterisation is constant enough to transfer from one task to another and to demarcate sensible individual differences.

By conjoining the combinations *DB not PB, PB not DB, DB and PB, neither DB nor PB,* with the initial global/local distinction, we constructed a table with cells representing the learning performances of students with distinct "competence profiles" (that is, mental repertoires furnished with more or less efficacious *DB* and *PB* operations and particular dispositions to act as globally or as locally as circumstances permit). There is ample evidence, mustered and summarised in Chapter 2, in support of the empirical validity of these discriminations between competence profiles; the evidence is especially clearcut in the case of defects or pathologies of learning manifest repeatedly by people who have different mental equipment. The distinctions in question tally quite well with the predictions made in terms of the competence profiles. If the tutorial context is taken into account, it is possible to infer that the holist/serialist dichotomy (previous monograph) is a result of combining certain competence profiles with specific tutorial situations, especially those in which strict conversation is approximated and understanding is enforced.

Only one caution is required as a preliminary comment. The *DB* operations and the *PB* operations act upon $\underline{\text{Proc}^0\text{s}}$ in a mental repertoire. This statement should be taken literally; the operations act upon both $\underline{\text{Prog}}$ and $\underline{\text{Inter}}$ as the components (Chapter 4) of $\underline{\text{Proc}^0} = \langle \underline{\text{Prog}}, \underline{\text{Inter}} \rangle$.

It now makes sense to detail and enlarge upon the nature and significance of the *DB* and *PB* categories, and to some extent upon the *PC* category also. This endeavour entails translating *DB* operations and *PB* operations in terms of the L^1 procedures ($\underline{\text{Proc}^1}$) and the L^0 procedures ($\underline{\text{Proc}^0}$) which, according to the present theory, are the stock in hand of any mental repertoire whatsoever. The *PC* operations feature as essential ingredients of the mind, but they are ubiquitious, diverse and discussed in a much more cursory fashion.

1. THE GLOBAL AND LOCAL DISTINCTION

As in Table 13 of Chapter 3, we use the convention of *GDB*, *LDB*, to denote global and local *DB* operations, and by the same token, *GPB*, *LPB* to denote global and local *PB* operations. Both kinds of operation, when interpreted within the present theory as cognitive processes, are species of L^1 procedures ($\underline{\text{Proc}^1}$), and the *DB/PB* distinction is a means of partitioning the L^1 procedures into categories germane to the work in hand. This fact is not immediately obvious as the *DB* operate upon topic relations to produce new relations, and the *PB* operate upon L^0 procedures ($\underline{\text{Proc}^0}$), if a relation is given to produce new procedures. Thus:

$$DB(R_i, R_j) \Rightarrow R_k \; ; \qquad PB(\underline{\text{Proc}^0}\text{i}, \underline{\text{Proc}^0}\text{j}, R_k) \Rightarrow \underline{\text{Proc}^0}\text{k}$$

in which R_i, R_j and R_k may be regarded as descriptions of topic relations taken in extenso.

Calling the number of arguments to which the operation is applied the *scope* of the operation, any *GDB* or *GPB* has maximal scope (under the constraints imposed by a situation), which is represented as follows:

$$GDP(R_i \ldots R_j) \Rightarrow R_k \quad \text{and} \quad GPB(\underline{\text{Proc}}^0 i \ldots \underline{\text{Proc}}^0 j, R_k) \Rightarrow \underline{\text{Proc}}^0 k .$$

Similarly, any *LDB* or *LPB* has a minimal scope (the same caveat holding). Thus:

$$LDB(R_i, R_j) \Rightarrow R_k \quad \text{and} \quad LPB(\underline{\text{Proc}}^0 i, \underline{\text{Proc}}^0 j, R_k) \Rightarrow \underline{\text{Proc}}^0 k$$

The possible *scope* will often depend upon circumstances (the R_i, R_j, R_k involved, for example), hence the maximisation or minimisation caveat. But, it is difficult to imagine any situation in which either R_k or $\underline{\text{Proc}}^0 k$ might not be synthesised from a minimum number of constituents or from many. The bounds upon maximisation and minimisation can be formalised (at any rate in the case of R_i, R_j ...) either in terms of Ashby's (1964) Cylindrance (a measure of the minimal adicity of a redundantly specified relation), or more comprehensively in terms of Atkin's (1973) Connectivity Analysis of relational systems. The latter method has been elegantly applied by Aish (1974) to express the global and local propensities of designers, as a special but important case, their tendency to act in a holistic or a serialistic manner.

Noting that such a treatment is possible, the global/local distinction will be glossed over until the mechanism of mental computing is discussed (Section 8.2), in order to secure a lucid and unencumbered notation for expressing the sense in which the *DB/PB* distinction partitions the class of $\underline{\text{Proc}}^1$.

2. DESCRIPTION BUILDING

A description in an L-Processor is either the result of executing some $\underline{\text{Proc}}^0$ or the result of applying one or a finite series of $\underline{\text{Proc}}^1$ (imaging a derivation) to the result of executing some $\underline{\text{Proc}}^0$. Denote the result of execution $\underline{\text{Ex}}$ (to avoid confusion with the Execution Sequence (listing) $\underline{\text{Exec}}$ of the previous monograph). A topic relation, as an internal description, in extenso, is

$$R_i = \underline{\text{Ex}}\ \underline{\text{Proc}}^0 i; \quad \text{or} \quad R_i^* = \underline{\text{Proc}}_1^1 \ldots \underline{\text{Proc}}_\ell^1 (\underline{\text{Ex}}\ \underline{\text{Proc}}^0 i).$$

Thus $DB(R_i, R_j) \Rightarrow R_k$ is a shorthand expression for

$$R_k = \underline{\text{Proc}}_1^1 (\ldots(\underline{\text{Proc}}_\ell^1 (\langle \underline{\text{Proc}}_{\ell+1}^1 \ldots \underline{\text{Proc}}_m^1 (\underline{\text{Ex}}\ \underline{\text{Proc}}^0 i)\rangle, \langle \underline{\text{Proc}}_{\ell+1}^1 \ldots \underline{\text{Proc}}_n^1 (\underline{\text{Ex}}\ \underline{\text{Proc}}_j^0)\rangle))\ldots).$$

in which $m \geqslant 0, n \geqslant 0$ and $\ell + m \geqslant \ell + n, \ell > 0$. *

The trick in this definition is that ℓ, m, and n are finite. Descriptive chains, as derived through *DB* operations, are not endless compositions. The *DB* operator itself is to be conceived as a routine that is executed *until* its production (R_k) is *used* (by any of the *PB* operators) in order to build an L^0 Procedure which realises R_k. Failing that, the sequence terminates or is simply not a *DB* sequence. The *DB* are L^1 procedures, $\underline{\text{Proc}}^1$, the number in a chain is called its ℓ-distance.

In particular, an aim corresponds to some topic (the "most ℓ-distant" that can be appreciated or described), regarded psychologically, as a focus of attention. The aim is a description R_i^* at a maximum ℓ-distance from whatever $\underline{\text{Proc}}^0$i are undergoing execution. If the aim is referred to a conversational domain, then it means the displayed topic corresponding to a description at maximum ℓ-distance from whatever $\underline{\text{Proc}}^0$i are undergoing execution (R_i^* if this is a displayed topic relation, otherwise the topic nearest to R_i^* in the descriptor space).

3. PROCEDURE BUILDING

The *PB* operations are also a class of L^1 procedures, $\underline{\text{Proc}}^1$. The *PB* operators take an argument consisting in a description of a relation and the stable concepts in the repertoire from which the description is derived, and produce a further concept. The shorthand expression is given as

$$PB(\underline{\text{Proc}}^0\text{i}, \underline{\text{Proc}}^0\text{j}, R_k) \Rightarrow \underline{\text{Proc}}^0\text{k}$$

or (from Section 2)

$$PB(\underline{\text{Proc}}^0\text{i}, \underline{\text{Proc}}^0\text{j}, \underline{\text{Proc}}_1^1 \ldots \underline{\text{Proc}}_\ell^1(R_i, R_j)) \Rightarrow \underline{\text{Proc}}^0\text{k}.$$

In particular the $\underline{\text{Proc}}^1$ that merely stabilise or reproduce a concept as veridical memories are members of this class of $\underline{\text{Proc}}^1$. Hence,

$$PB(\underline{\text{Proc}}^0\text{k}, R_k) \Rightarrow \underline{\text{Proc}}^0\text{k} \quad \text{or} \quad PB(\underline{\text{Proc}}^0\text{k}, \underline{\text{Ex}}\ \underline{\text{Proc}}^0\text{k}) \Rightarrow \underline{\text{Proc}}^0\text{k}$$

* This is a convention. If $\underline{\text{Proc}}^0$ is stable, it will be stabilised by a memory; an L^1 procedure which may be written *PB*. If this is counted as one of the derivational procedures, then the inequalities become $m \geqslant 1$, $n \geqslant 1$, $\ell + m \geqslant \ell + n > 1$.

are general ways of stating that a concept Proc^0k is (as asserted) stable and compiled in an L-Processor by a memory, $\underline{\text{Proc}}^1\text{k}$. In a conversational domain (with cyclic and consistently related topics by definition), an understanding of Proc^0k (or *topic* k) consists in the set

$$DB(R_i, R_k) \Rightarrow R_j: \quad PB(\underline{\text{Proc}}^0\text{i}, \underline{\text{Proc}}^0\text{k}, R_j) \Rightarrow \underline{\text{Proc}}^0\text{j}: \ \underline{\text{Ex}}\ \underline{\text{Proc}}^0_j \Rightarrow R_j$$
$$DB(R_j, R_k) \Rightarrow R_i: \quad PB(\underline{\text{Proc}}^0\text{j}, \underline{\text{Proc}}^0\text{k}, R_i) \Rightarrow \underline{\text{Proc}}^0\text{i}: \ \underline{\text{Ex}}\ \underline{\text{Proc}}^0_i \Rightarrow R_i$$
$$DB(R_i, R_j) \Rightarrow R_k: \quad PB(\underline{\text{Proc}}^0\text{i}, \underline{\text{Proc}}^0\text{j}, R_k) \Rightarrow \underline{\text{Proc}}^0\text{k}: \ \underline{\text{Ex}}\ \underline{\text{Proc}}^0_k \Rightarrow R_k$$

for which Kallikourdis gives a general algorithm.

Since the $\underline{\text{Proc}}^0$ in a realisation of the formulae in Section 2 must be stable, it is clear that if there are *DB* in a repertoire, there must also be some *PB*, but the *PB* could conceivably be restricted to those $\underline{\text{Proc}}^1$s that reconstruct or reproduce $\underline{\text{Proc}}^0$s rather than those which construct them.

4. THE EXTERIORISATION OF AN *UNDERSTANDING*

An *understanding*, the pivotal condition for a strict conversation and, according to this theory, the prerequisite for any perma-

TABLE 5.1

$PB(\underline{\text{Proc}}^0\text{i}, R_i) \Rightarrow \underline{\text{Proc}}^0\text{i}$ (Concept for topic i is stabilised)	(1)
$PB(\underline{\text{Proc}}^0\text{j}, R_j) \Rightarrow \underline{\text{Proc}}^0\text{j}$ (Concept for topic j is stabilised)	(2)
$\underline{\text{Ex}}\ \underline{\text{Proc}}^0\text{i}$ in L-Processor $\Rightarrow R_i$ (Students concept of topic i)	(3)
$\underline{\text{Ex}}\ \underline{\text{Proc}}^0\text{j}$ in L-Processor $\Rightarrow R_j$ (Students concept of topic j)	(4)
$\underline{\text{Ex}}\ M_i$ (Based on S $\underline{\text{Prog}}$ i) in modelling facility $\Rightarrow R_i$ (Evidence for (1))	(5)
$\underline{\text{Ex}}\ M_j$ (Based on S $\underline{\text{Prog}}$ j) In modelling facility $\Rightarrow R_j$ (Evidence for (2))	(6)
$DB\ (R_i, R_j) \Rightarrow R_k$ (Description of R_k from topic i, topic j)	(7)
$PB(\underline{\text{Proc}}^0\text{i}, \underline{\text{Proc}}^0\text{j}, R_k) \Rightarrow \underline{\text{Proc}}^0\text{k}$ (Construction of $\underline{\text{Proc}}^0\text{k}$ given (1), (2), and (7)	(8)
$PB\ (\underline{\text{Proc}}^0\text{k}, R_k) \Rightarrow \underline{\text{Proc}}^0\text{k}$ (Concept for topic k is stabilised)	(9)
$\underline{\text{Ex}}\ \underline{\text{Proc}}^0\text{k}$ in L-Processor $\Rightarrow R_k$ (Students concept of Proc^0k)	(10)
$\underline{\text{Ex}}\ M_k$ (Based on S $\underline{\text{Prog}}$ k) in modelling facility $\Rightarrow R_k$ (Evidence for (10))	(11)
Evidence of (5) and (6) and (11) is Evidence that concept k is understood provided R_i, R_j and R_k form part of a cyclic and consistent mesh so that R_i, R_j are part of R_k	(12)

nent retention of a concept (Section 2), is the conjoint activity of *DB* and *PB* operations. Evidence for the *understanding* of a topic relation, R_k (the acquisition of a concept $\underline{\text{Proc}}^0$k and a memory Proc^1k, to stabilise it), is stated in Table 5.1. Prior *understanding* of topic relations R_i and R_j is assumed.

5. COMBINING OPERATIONS

Apart from the *DB* and *PB* operation categories, it is proposed that further L^1 procedures exist in any mental repertoire, and they are given the general title "Procedure Combining" (*PC*) operations. These are characterised by the formula

$$PC(\underline{\text{Proc}}^0\text{p}, \underline{\text{Proc}}^0\text{q}) \Rightarrow \underline{\text{Proc}}^0\text{r}.$$

The salient difference between *PB* and *PC* is that the latter (*PC*) does not take a description as one of its arguments whereas the former does so.

The result of applying a *PC* is a program which may, in principle, be compiled and executed (in that sense the "combination" is not arbitrary or haphazard). For example, we might set p = i, q = j, and r = k to obtain the product of Section 4. On the other hand, there is, in general, no guarantee that the product (though realisable) will either be useful or viable in the sense that it is stabilised in the existing repertoire.

There is no objection to postulating a "description combining" operation also. However, its form is identical with the "description building" operation (*DB*) so that the postulate is redundant; that is, *DB* operations could be renamed as combinatory rather than constructive. The issue at stake is really the existence or non existence of a coupling between what may be described and what may be done (computed, brought about, stabilised) as follows.

Consider a repertoire consisting only of *PC* operations and *DC* (alias *DB*) operations, devoid of *PB* operations. Within such an organisation descriptions are computed from the result of executing some *PC* engendered $\underline{\text{Proc}}^0$; but there is no guarantee that this procedure is either useful or viable (in fact, in the absence of *PB* operations "viable" is ambiguous). Similarly, the *PC* operations generate procedures. Such chains of computation could, and possibly do, go on endlessly. They are reminiscent, at the descriptive level, of the arbitrary reprogramming which Evans (1967) regards as a constituent of dreaming; at the operational level, of trial and error. Without further embellishment, there is no coupling condition of the type "memory" or "understanding". Moreover, I have deliberately refrained from equating the levels of activity to the strata L^1, L^0 of the conversational language, L, for just this reason.

Within L, the L^1 descriptors are of things which can be computed or done or that survive as cyclic structures; either that, or the descriptors are evanescent. L^0 procedures, in turn, do things and may also be described. True, the descriptions may be many stages removed from whatever is described, but they are not just arbitrary burgeonings. In a strict L Conversation, it is only possible to observe (as *understandings* and the transactions that exteriorise *understandings*) mental events of this type.

The flux of activity thus discernible, addressable, and manipulable as part of a P-Individual, is the construct which I have elsewhere called a "language oriented system" (Pask 1970) in sharp contrast to a "taciturn system", developed and amplified in Von Foerster (1971) and Von Foerster and Weston (1974). The distinction still seems apposite; a coupled *DB*, *PB* system is "language oriented" or, to qualify it specifically, "L oriented". The *PC* system is "taciturn" or, to qualify it specifically, "L taciturn".

6. COMMENTS ON THE *PC* OPERATIONS

PC operations are surely required to account for the ubiquitous phenomena of adaptation and probably play an essential part in maintaining cognitive fixity. We conjecture that the *PC* procedures are intimately related to the brain, qua L-Processor rather than the integral cognitive organisations (P-Individuals, for example) which inhabit and are executed in the brain. In its role as an L-Processor, the brain is a matrix (a modular computer) made up from ongoing *PC* operations. A simple model of such an equipment is discussed in Appendix B.

PC operations may be held responsible for all manner of conditioning, chaining, and a certain kind of evolutionary learning; as later, selective evolution based upon weak interaction, generation and recombination rules. Essentially, this is trial and error learning, moderated by constraints prohibiting fatuous constructions that cannot be executed.

According to this view of a brain as a taciturn system (an L-Processor), it makes sense to say "we condition a brain" or that "the brain is observed to adapt". It is also likely that brains engage in more or less continual "trial and error" learning, though we prefer to reserve the word "learning" for phenomena that are deductively based and characteristic of language oriented systems; notably, P-Individuals which inhabit brains and appear in this analysis as collections of *DB* and *PB* operations. From the present perspective, we do not "condition" P-Individuals, but talk to them as L oriented systems, and teach them. Conversely, we do not "teach" a brain.

The taciturn and language oriented varieties of systems obviously interact. But, in an educational context, it does not seem too difficult to distinguish between them. *DB/PB* learning and the understandings to which it gives rise is more efficient, by many orders of magnitude, then *PC* "trial and error learning" (which we do not refer to as "learning" at all). This difference is highlighted by numerous studies. Landa's (1971) data on method learning in language comprehension bears impressive testimony to the distinction. Landa's discussion of what it means to learn a logical principle (that any principle is *interpreted*, for example in language usage) makes the same point, though a different terminology is employed. Again, in Scandura's (1973) work, there is ample evidence of a clearcut demarcation, and (with similar reservations over the difference in terminology) his categories of "rule" and "higher order" rule learning are identifiable as *DB*, *PB* mediated *understandings.*

7. INTERPRETATION IN TERMS OF MACHINE COGNITION AND ABSTRACT SYSTEMS

One advantage of partitioning the L^1 procedure into *DB*, *PB* and *PC* is that the learning predicted by conversation theory can be placed in register with well-known processes in the field of cognitive science.

Various algorithms exist for constructing fresh algorithms as compiled programs. Chang and Lee (1973) present their own algorithms and review the field.

It is probably fair to say that all efficient constructive algorithms rely upon a distinction between two aspects of program construction. On the one hand, a relation is described. On the other hand, a program is constructed from existing routines (perhaps as basic as machine code instructions) that if subsequently executed, will satisfy the relation.

For example, consider the "Monkey, Box and Banana problem" (MBBP), so often quoted in the literature of Artificial Intelligence. The relation described is a relation between the elements or sub-relations of the "Monkey, Box and Banana" situation (box position, monkey moves and so on), such that MBBP is solved.

In the context of computers it is legitimate to assume certain prerequisites and invariances which cannot be taken for granted in the field of mental activity; for example, that compiled programs remain as stable entities in machine storage and that a fixed set of primitive operations and order relations is known at the outset. If these assumptions are made explicit, they stand in place of dynamic activities which we, from a psychological stance, introduce as part of the process in order to secure equisignificant invariances. Under this transposition, an efficient constructive algorithm, typified by Chang and Lee (1973), has an outline (Table 5.2) identical with the skeleton of *understanding* given in Table 5.1.

Other (fundamentally different) kinds of program construction are far less efficient if a relation can be described. (They are not simply "less efficient" without qualification; under certain conditions they come into their own.)

Evolutionary construction of the sort predictable in a repertoire filled with *PC* operations has been examined and extensively simulated by Fogel, Owens and Walsh (1966). The compiled programs produced as the result of this construction are finite state machines and their input/output sets are interpreted in an (internal) universe of number sequences under a criterion that is satisfied if the next output states of a machine predict the next number in an arbitrary sequence. This criterion is a synonym for a relation which is satisfied (if the criterion is satisfied), and successful machines are those that yield satisfactory predictions.

Initially, finite state machines are produced by random "mutation". The successful variants in a 1st generation are preserved and

TABLE 5.2

Entry in Table 5.1	Process and Inference	
(1), (2)	Basic routines (in the limit, machine instructions and indexed storage locations) exist	Assumption (A)
(3), (4)	The basic routines can be executed	Assumption (B)
(5), (6)	Operation of the basic routines can be deciphered externally and placed in register with variables describing parts of the problem	Assumption (C)
(7)	*DB* (Functions specifying basic routines) ⇒ MBB (Description of MBB Problem computed or externally specified)	Process (I)
(8)	*PB* (Basic routines, MBB) ⇒ Compiled MBB Program	Process (II)
(9)	MBB Program is compiled or stable	Assumption (D)
(10)	MBB Program can be executed	Assumption (E)
(11)	Operation of MBB Program can be deciphered and placed in register with MBB problem variables	Assumption (F)

mutated to form a 2nd generation (others being discarded), and so the process continues. However, as soon as a population of machines is in existence, the random "mutation" is replaced by recombination rules for forming fresh machines, and these rather than the mutants are the variants tested against the criterion and recycled. At this stage, the process is open to representation in terms of *PC* operations, if i_1, i_2 ... j_1, j_2 ..., index the machines (alias procedures) in the current generation.

$$PC(\underline{Proc}^0 i_1, \underline{Proc}^0 j_1) \Rightarrow \underline{Proc}^0 k_1$$
$$PC(\underline{Proc}^0 i_2, \underline{Proc}^0 j_2) \Rightarrow \underline{Proc}^0 k_2$$

The most successful of the $\underline{Proc}^0 k_1$, $\underline{Proc}^0 k_2$ ... are selected (together with some $\underline{Proc}^0 k$ and $\underline{Proc}^0 j$, if they have equal merit) and are recycled.

The evolutionary paradigm is relatively inefficient (though it gains in flexibility as it loses in efficiency). There are, of course, many heuristically-governed, evolutionary-style, artificial intelligence systems intermediary between the *PC* type and the *DB, PB* type, of which the earliest and one of the most elegant is Selfridge's (1959) Pandemonium.

Such intermediaries are believed to characterise mental as well as machine organisation. However, the crucial *understanding* condition is wholly concerned with *DB, PB,* learning. Similarly, insofar as the stable re-entrant organisation of a P-Individual is a collection of *understandings*, any P-Individual is formulated in terms of *DB/PB* operations (in that sense, it is processor-independent).

8. EXPERIMENTAL POSSIBILITIES DUE TO THE *DB/PB* DISTINCTION

Our original motive for classifying cognitive operations as *DB* and *PB* was to explain the empirical competence profiles of Table 3.13 and recapitulated in Table 5.3. The explication is not entirely straightforward because of an indeterminacy in the object of observation which is said to *be* competent (in particular, to have one or other competence profiles). Similar indeterminacies are believed to hamper most types of educational testing, and the easy way out, consisting of glossing over the mixed characterisation either of competence or properties such as "intelligence quotient" or "specific aptitude scores," seems to produce a good deal of harmful and unnecessary obfuscation. Within reason, the parochial discussion of the competence profiles in Table 5.3 can be generalised to cover the wider field of examination, mental testing, assessment procedures and the like.

8.1. Dual Aspect of Competence or Dual Referants of this Property

According to our theory, at least two subjects of observation can be credited with a competence profile.

(a) Competence is a property of a repertoire of *DB, PB* operations which form a P-Individual in some conversational domain(s). In this case, the competence determines the extent to which this repertoire forms a P-Individual in this particular conversational domain.

(b) Competence is a property of a brain, or more generally, an L-Processor. In this case, the competence determines how certain

TABLE 5.3

A Cluster of Mechanisms Sufficient to Account for the Competence Profiles

	Long *DB* Chain Length High *PB* Efficiency	Long *DB* Chain Length Low *PB* Efficiency	Short *DB* Chain Length High *PB* Efficiency	Short *DB* Chain Length Low *PB* Efficiency
TLC like	*GDB* and *GPB* bias	*GDB* bias	*GPB* bias	Neither bias (*PC*)
GPS like	*LDB* and *LPB* bias	*LDB* bias	*LPB* bias	Neither bias (*PC*)

TLC like = Resembles Quillian's "Teachable Language Comprehender" or de Faivre's "Fuzzy" ("global" paradigm). GPS like = Resembles Ernst, Newell, Shaw and Simon's "General Problem Solver" ("Local" paradigm).

DB and *PB* operations will be executed (supposing they are presented for execution) and even whether or not they can be executed in any way. By hypothesis, the competence of a brain reflects the composition of *PC* operations which are executed in order for the brain (or L-Processor) to act as a computing medium that accommodates *DB* and *PB* procedures.

8.2. Tentative Stipulation of Competence Profiles

The profiles of Chapter 3 (Tables 13, 14, 15) can be reconstructed (Table 5.3) using two parameters of *DB*, *PB* operations; the mean length (ℓ, m, n) of the *DB* chains and the efficiency (speed, numerosity) of *PB* operations. The global/local (row of Table 5.3) distinction is identified with a tendency, on the part of a processor (brain) to execute whatever *DB* or *PB* are presented in a particular fashion. Recalling that *DB*, *PB* (or Procs in general) are, by postulate, compiled Fuzzy Programs, it is clearly not absurd to say that they can, and generally will, be executed differently by different processors. Choosing a plausible distinction, a high adicity processor accepts a Fuzzy Program and computes in parallel, i.e., it runs the program without resolution, each stage in computation resulting in a set of data which is input to the next step. A low adicity processor serialises the computation, so far as possible, by tricks equivalent to the expedient of numerical resolution (for example, sélecting a maximum value as representative of an extremum such as the Fuzzy Output from a previous stage in the computation). For biological processors, literal *numerical* resolution is improbable; hence, "tricks equivalent to".

8.3. Interaction

The qualities of competence labelled in Table 5.3 are to some extent separable; the column labels refer chiefly to properties of a program suite, and the row labels refer chiefly to a processor type. But the separation is unrealistic for two reasons; first, insofar as any manifestation of competence involves programs and a processor in which they are undergoing execution, and second, because the processor characterisation is believed to represent a dynamic process (the execution of *PC* operations that maintain the brain as a computing medium able to accept and execute *DB*, *PB* procedures).

From the first, our distinction between 8.1(a) and 8.1(b) is (in any actual experiment) a distinction between ways of looking at the same system; in 8.1(a) as a *language oriented* system, in 8.1(b) as a *taciturn system.* From the second, any actual execution of *DB*, *PB* procedures is likely to influence the *PC* operations which sustain the processor. Moreover, from Section 7, it is believed that intermediary types of operation exist.

8.4. Experimental Situations and Basic Indeterminacy

The conditions that favour observation of the language oriented (or 8.1(a)) aspect of competence reduce the information available about the taciturn system responsible for the 8.1(b) aspect of competence. The converse also applies to conditions which tap the (b) aspect of competence and reduce the information available about the (a) aspect. Between them, these trends introduce a measure of indeterminacy; not so much about the value of competence as an operational and predictive quantity, but in respect of the object manifesting competence. That is, an index of competence is contextually bound.

To see this, notice that the (a) aspect calls for information about *understandings* and that *understandings* are only determinable in a conversation; a Piaget like or Vygotsky like or a Landa like interview; a paired experiment; a peer group discussing a project; or (the case to be examined since it is relatively simple though no more effective) a strict conversation anchored upon a conversational domain and maintained by an operating system. In all such situations, the class of procedures which engender *understanding* is liable to be distributed; it is almost nonsensical to say "who is responsible for that *understanding*? who has it?" In our theory, the class of procedures is a P-Individual (so by definition is the conversation itself), but the problem of distribution besets any theory whatsoever. Due to distribution, the process under scrutiny may not be exclusively accommodated in one brain, and the measurements and observations refer to the entire situation.

Conversely, observations of the (b) aspect of competence (of the brain as a taciturn system) are favoured by approaching the stimulus/response or behaviouristic paradigm as closely as possible. For example, stimulus/response, small item tests, are quite effective instruments. The price paid in the limit is that no *understandings* are observable.

8.5. The Function of Complete and Attenuated Operating Systems

In the microcosm of a strict conversational operating system, these peculiarities are open to analysis, though the operating system itself (CASTE or INTUITION) does no more than an interviewer or the experimenter engaged in teachback (previous monograph). The operating system:

(1) Guarantees that if a student learns in any way about the conversational domain, then his learning amounts to a series of *DB/PB understandings*, so that he may be characterised as a P-Individual in this domain.

(2) It furnishes assistance, by augmenting the student's repertoire and the computing facilities of his brain, qua L-Processor, so that within limits a student can act as a P-Individual in this domain.

Function (1) is sufficiently explained by Table 5.1. To make a convincing case for (2), it is necessary to retrieve the detailed transaction types of the previous monograph, and this is done in Table 1.2.

If the student is versatile (the *DB* and *PB* competence profile of Table 5.3), no assistance is needed, even though it is at hand. If his competence profile is *PB* not *DB* (Table 5.3), then the operating system guided by the entailment structure carries out external *DB* operations that are surrogates for those which could otherwise be executed by the student. If his competence profile is *DB* not *PB* (Table 5.3), then it externally furnishes surrogate *PB* operations. Finally, if the student is neither *DB* nor *PB* (Table 5.3), the operating system literally tells the student what to do (there is some rather shaky evidence for a positive transfer effect).

Now, as external observers, we can quantify the student's competence in a taciturn (8.1(b)) sense insofar as the student does function as a P-Individual only if he receives a (measurable) amount of help from the operating system, that is, to this extent only is it possible to make a firm demarcation between the row categories of Table 5.3 (low and high adicity, *PC*: local and global).

Little can be said of the student column categorised as neither *DB* nor *PB*, since he may or may not act as a P-Individual in the conversational domain.

Students having the *DB* not *PB* competence profile fall quite definitely into holist *GDB* behaviour if they are *PC* characterised as global (high adicity) learners and into serialist *GPB* behaviour if they are *PC* characterised as local (low adicity) learners. The *PB* not *DB* competence profile is similarly dichotomised (*GPB* and *LPB*) in terms of the demonstrative assistance they need in order to satisfy *understanding* (condition 12 of Table 5.1).

Finally, having the competence profile *DB* and *PB*, versatile students are not unambiguously distinguished in terms of *PC* competence since they do not need assistance. These students do exhibit a learning strategy which is either holist or serialist in form, and this suggests that their *PC* competence favours global or local processing. The trouble is that cognitive fixity, which is a predictable consequence of *DB/PB* organisation, would lead on its own account to a clearcut demarcation or distinction in learning strategies, so that the observed dichotomisation of behaviours may be (and probably is) due to this effect rather than a processor bias that renders students only able to learn in one way or the other. These arguments are summarised in Table 5.3 and gain support from the studies of Chapter 2, where the operating system is abraded either by relacing the *understanding* condition, or by withdrawing the potentially available assistance.

To summarise the matter: If the demand for explanation is replaced by a correct response criterion (multiple choice questions), then some students (*DB* not *PB*, on this or other grounds) are liable to "globetrotting" defects which are characteristically either discursive (*GDB*) or normally channeled (*LPB*) while other students are unaffected (*PB* not *DB* or versatile). If the entailment structure is abraded and descriptive data is withdrawn, some students evidence the defect of "improvidence" (*PB* not *DB*, on this or other grounds) but are not as seriously affected. Finally, there are some students (all of those acting like "neither *DB* nor *PB*" in an operating system, perhaps others also) who seem able to learn very little unless given a specific and phased sequence of instructions; in fact, unless they are conditioned by one of the less exciting kinds of behaviour shaping.

9. PARADOXICAL FEATURES OF THE COMPETENCE RESULTS

If the objects of observation in 8.1(a) and 8.1(b) are lumped together, many common observations appear paradoxical. For example, it is queer to remark that a student (the lumped entity) deliberately adopts a mismatched learning strategy, i.e., his disposition does not tally with his competence. But the existence of this divergence is a strong result.

It is equally difficult to comprehend the Jekyl and Hyde demeanour of many students which leads them to learn and think in one way of academic subjects and in another way of the rest (manifest as the curiously strong serialistic disposition induced apparently by institutional training and often running contrary to competence, either in test or practice). The data referenced in Chapter 3 give only a mild mannered expression to the facts which, once aired, turn out to be part of conventional wisdom. These students have not only different styles, dispositions and learning strategies, but different personalities; they live in different realities; they deploy the external data storage in their environment (files, book arrangements, recall cues) quite differently with one persona and the other. Only if they are versatile do they function in each role with comparable efficiency.

Both of these phenomena are marked enough and important enough to take in earnest, and both are paradoxical, unless the convenience of viewing the student as a lumped entity is discarded. Any trenchant explanation must make some distinction akin to 8.1(a) and 8.1(b), and this particular way of carving the cake does at least dispel the air of mystery.

For the student, qua P-Individual is formulated to place connation on a par with cognition; viewed thus, as a language oriented system, he may have a will or disposition to do what he cannot do effectively; further, it is not unreasonable to suppose that more than one P-Individual inhabits the same brain. Regarding the processor which is said to have a certain *PC* competence, it could be just one brain or it could, more realistically, be considered as the total environment encountered in each area of activity, institutional and extra curricular. To a large and significant extent, this environment is structured individually (for example, by arrangements of external data and recall from storage). The processor which is *PC* competent includes all of these structures, as well as the more obvious augmentation provided by masters and peers.

10. ANALOGICAL TRANSFORMATIONS

The *DB* and *PB* and *PC* distinction permits the prediction of mental transformations, involving analogy learning. Recalling the discussion of analogy in Chapter 4, $\underline{\text{Proc}}^0\text{i}$ is a compiled program:

$$\underline{\text{Proc}}^0\text{i} \triangleq \langle \underline{\text{Prog}}\ \text{p}, \underline{\text{Inter}}\ \text{x} \rangle.$$

PB acts upon both components, $\underline{\text{Prog}}$ and $\underline{\text{Inter}}$, of $\underline{\text{Proc}}^0$.

DB acts upon interpreted relations (sets in some internal universe X, Y, U). Since the distinguishing predicate of an analogy is itself a relation, $\underline{\text{Dist}}(\text{x}, \text{y})$ which is *given externally*, DB may act upon it as one argument and perform a transformation

$$DB(\text{R}_\text{i}, \underline{\text{Dist}}(\text{x}, \text{y})) \Rightarrow \text{R}_\text{j} = \underline{\text{Ex}}(\underline{\text{Proc}}^0\text{j})$$

where, in the simplest case, *DB* realises isomorphism so that $\text{R}_\text{i} \Leftrightarrow \text{R}_\text{j}$; that is, if R_i is interpreted in X, its form is copied into Y. At this stage, $\underline{\text{Prog}}$ p in $\underline{\text{Proc}}^0\text{i}$ may be given a different interpretation, that is:

$$PB(\underline{\text{Proc}}^0\text{i}, \text{R}_\text{j}) \Rightarrow \underline{\text{Proc}}^0\text{j} = \langle \underline{\text{Prog}}\ \text{p}, \underline{\text{Inter}}\ \text{y} \rangle.$$

Moreover, if $\underline{\text{Dist}}(\text{x}, \text{y})$ is given externally, R_i and R_j need not be isomorphic, providing the type of morphism is properly spelled out.

Conversely, if R_i and R_j are given externally and the analogy relation is said to be an isomorphism by some external agent or specification, it is possible to write a transformation like

$DB(\mathrm{R_i}, \mathrm{R_j}) \Rightarrow \underline{\mathrm{Dist}}(\mathrm{x}, \mathrm{y})$
$PB(\underline{\mathrm{Proc}}^0\mathrm{i}, \mathrm{R_j}, \underline{\mathrm{Dist}}(\mathrm{x}, \mathrm{y})) \Rightarrow \underline{\mathrm{Proc}}^0\mathrm{j}.$

These transformations may be countenanced within the compass of one P-Individual (generally in any "One-aim-at-once" or "one focus of attention" experiment) because external information is furnished which stipulates that X and Y are distinct universes united by the analogy relation. As a result, distinct compilation and interpretation sets may be reserved in the brain (generally in L-Processor storage), and the computation may go on uniformly, apart from the distinctions thus stipulated. X and Y, so united, have comparable internal representations. In fact, one meaning of P-Individual is a set of processes that are not independent and are able to interact because a dependency exists (equisignificantly stated as "uniform computation" and "synchronicity").

The one-aim-at-once condition is fairly innocuous in the context of learning where what may be known and done is spelled out (in a conversational domain, for example). However, the one-aim-at-once restriction imposes very serious constraints upon the unguided generation of analogy relations (for example, generalisation based analogies) and upon the production of interesting novelty.

11. INNOVATION AND GENERALISATION BASED ANALOGY RELATIONS

Let us focus the discussion upon analogy relations that straddle two or more a-priori-independent universes of compilation and interpretation.

The act of learning as analogy, when the distinguishing predicate Dist(x, y) is externally delineated by information from an entailment structure or any other source, differs fundamentally from the act of creating the analogy relation *de novo*, when the distinguishing predicate is invented. In the previous monograph, without special emphasis upon analogies, this act figured as "predication". Our key point is that learning an analogy can go on in a cognitive system is informationally closed, apart from the specificity injected by way of guidance. The process need involve no more than an application of *DB* and *PB* operations already in the repertoire, and much the same comment applies to other than analogical syntactic derivations; for example, forming new rules or concepts by iterating or combining those that exist.

In contrast, creating an analogy relation between two or more universes calls for the construction of a semantic predicate, Dist(x, y). Any cognitive system able to perform this feat must be informationally open, and the sort of openness considered amounts to the juxtaposition and (partial) coalescence of two (or more) systems which have distinct and a-priori-independent "internal representations" — one a "representation" of X, and one of Y.

This state of affairs is captured in Gergely and Nemeti's argument, as it is sketched in Chapter 4, and this slant upon their argument is developed in Chapter 6. From a psychological point of view, the events in question may be characterised as the juxtaposition and partial coalescence of two or more a priori asynchronous and independent P-Individuals; or as the coexistence and subsequent integration of two or more aim topics; or as a division of attention between two or more topics (whether or not the two foci of attention are externalised and objectified as aim selections).

Before depicting this important process, it will be prudent to press home an already made distinction between "many goal" situations (of the sort encountered in quite ordinary holistic learning) and the very different class of "many aim" situations pertinent to the immediate issue. Subsection 11.1 is a digression intended to serve this purpose; the main line of argument is resumed in Section 11.2.

11.1. Diversity Under One-Aim-at-Once

One characterisation of a serialist student in a strict conversation is that he has an aim topic (his maximal focus of attention, the most distant topic he appreciates) and only *one* goal that he chooses to learn about, the one member of his legitimate workset. In contrast, a holist student appreciates a topic well in advance, he aims for it, and his workset includes several subordinate topics which he has chosen as goals to learn about.

We often cannot (and need not) discriminate the possibilities that a holist student deals with the goal topics simultaneously "in parallel" and the possibility that he scans them in an order of his own choice, usually leaving one topic before it is fully learned, dealing with another, and returning later on to the original. The crucial feature is that in either kind of holism, the topics in work-set are considered in the context of the aim topic and that the exteriorised behaviours are synchronised with respect to aim behaviour and each other. Scanning is just as good a synchronisation as a parallel approach, and there are grounds for believing that apparent simultaneity (even in the case when an analogy R_k is explained by the simultaneous execution of M_i and M_j) is really a complex and probably variable topic scan.

Unequivocally, the serialist's exteriorised behaviour is also synchronised with respect to the aim topic. The behaviour in this case is literally sequential.

Under one aim circumstances, observations are made of one, and only one, P-Individual; for example, using the expedients described in the previous monograph. It will be recalled that a strict conversation (amongst other things, a means for securing one-aim-at-once) is defined as a P-Individual in its own right. Although this P-Individual may have factors that are also P-Individuals, they are synchronised under execution and, in that sense, are dependent. The conversation manifest at an interface is *the* P-Individual actually observed. As before, the locus of *this* P-Individual in the conversational domain is the current aim topic; this is a more precise way of stating the commonplace dictum that a student, qua sentient cognitive system, is located at his focus of attention and is thereby identified.

11.2. Many Aim Systems

It has been argued that nothing essentially novel (or, at any rate, no predicative or semantic novelty) can arise until there are two or more aims (alias two or more a priori asynchronous and independent P-Individuals). In the sequel, it is assumed that the two P-Individuals (which may be executed in one brain or several) address their attention to, and formally aim for, two topics with relations R_i and R_j respectively, which are interpreted in a-priori-independent universes (X and Y, respectively). However, the two P-Individuals are in a position to interact and may wholly or partially coalesce, losing some or all of their independence. A creative act, such as the production of an analogy relation, comes about due to their interaction, and this interaction may be of two different-sounding but essentially similar kinds: (a) By a linguistic exchange, as in Chapter 4, or (b) As the concurrent and interactive execution of procedures in each P-Individual. Of these, (a) is a perspective proper to "language oriented" systems, as distinguished in Section 5, and (b) is a perspective proper to "taciturn" systems.

For conformity with the rest of this chapter, it is desirable to express the joint analogical transformation in the form

$$\mathit{DB}(\mathrm{R_i}, \mathrm{R_j}) \Rightarrow \mathrm{R_k}$$
$$\mathit{PB}(\underline{\mathrm{Proc}}^0\mathrm{i}, \underline{\mathrm{Proc}}^0\mathrm{j}, \mathrm{R_k}) \Rightarrow \underline{\mathrm{Proc}}^0\mathrm{k}$$

As it stands, the form is unacceptable, because the *DB* and *PB* operations are defined as acting within one mental repertoire. By edict, $\mathrm{R_i}$ and $\mathrm{R_j}$ do not, at the instant concerned, have a uniform internal representation. ($\mathrm{R_i}$ is interpreted in X and $\mathrm{R_j}$ in Y; $\mathrm{Proc}^0\mathrm{i}$ and $\underline{\mathrm{Proc}}^0\mathrm{j}$ operate within repertoires that are, at this stage, still independent.) On the other hand, if interaction can take place (clearly it *can* if the P-Individuals are executed in the same brain, and interaction has been posited anyhow), then the expression is not nonsensical, simply non standard. In order to indicate that transformations of this type do not have the same meaning as the standard *DB* and *PB* transformations, they are distinguished by adjoining an asterisk: thus

$$\mathit{DB}^*(\mathrm{R_i}, \mathrm{R_j}) \Rightarrow \mathrm{R_k}$$
$$\mathit{PB}^*(\underline{\mathrm{Proc}}^0\mathrm{i}, \underline{\mathrm{Proc}}^0\mathrm{j}, \mathrm{R_k}) \Rightarrow \underline{\mathrm{Proc}}^0\mathrm{k}$$

Regarded from the language oriented perspective, these expressions represent linguistic transactions whereby one P-Individual is able to describe and manipulate the descriptions and operations used by the other P-Individual, and of course vice versa. The conversation can be realised either by providing a metalanguage to accommodate these transactions or by enriching L so that it can express interpersonal hypotheses as well as hypotheses which refer directly to topics.

From the taciturn perspective, the asterisk marked expressions mean that all operations (*DB*, *PB*, or Proc) are executed, perhaps concurrently, in a distributed L-Processor.

Chapter 6

Conversations with Many Aim Topics

The discussion in this chapter develops the conversational paradigms, represented by Icons in the previous monograph, and sets the stage for an essay into situations characterised by more than one aim selection at once. There are several objects in view.

(a) More than one user can learn a subject matter represented in a conversational domain; the most interesting situations involve group or team activity (as distinct from "multiple access" to a large CAI system).

(b) Although some work has been done with groups (the verbal communication between members is extremely informative), the data have not been fully analysed and are not reported. Instead, we take the opportunity to introduce multiple user versions of CASTE and INTUITION in which the verbal communication between the users is replaced by a series of quasi mechanical and exteriorised transactions. The crucial feature of these transactions is that they exteriorise not only hypotheses (on the part of one participant or the other) about topics in the conversational domain but also mutual or personalised hypotheses on the part of one participant about the other.

(c) It is quite possible for more than one aim topic to exist in a suitably liberalised operating system, even if there is only one user. Formally, this state of affairs represents the coexistence of more than one P-Individual (externalised at the interface with the conversational domain) in the same brain or L-Processor. Intuitively, the same state of affairs images one person having more than one focus of attention or more than one concurrently entertained perspective and roles.

In order to make sense of this statement, we digress in Part B, Sections 9, 10, 11, into some distinctions between the notion of an aim topic and the similar but only superficially identical notion of a focus of attention. Salient aspects of the literature are reviewed in order to bring these ideas into register.

The effort is eminently worthwhile, for during the earlier part of the discussion, it is possible to show that analogy construction is dependent upon a (usually transient) many aim condition and that nearly all analogy construction is loaded with innovation. Loosely, one student with many foci of attention is organisationally equivalent to many students with one focus each, and both organisations are capable of innovation.

Part A. Representation of Many Aim Operation

1. GENERALISATION TO CONVERSATION WITH MANY PARTICIPANTS OR MANY AIM TOPICS

In order to obtain a facile representation of many participant and/or many aim, conversations within a uniform framework, it is necessary to simplify the Iconic schemes of the first monograph. Of course, the simplified schemes must accommodate all of the one aim constructions, of which the fundamental construction is the neutral and minimally biassed "cognitive reflector", of Icons 3 or 4 (previous monograph), repeated as the first part of Fig. 6.1.

An initial step in this direction is taken by drawing the transcription in Fig. 6.1 which also depicts a "cognitive reflector". The regulatory heuristic, B, which maintains a strict conversation on a fixed conversational domain $D^1(R)$, $D^0(R)$ — or, under concrete interpretation, *ES*(R), *TS*(R) — is accommodated in a separate processor (not usually an L-Processor) corresponding to β in Icon 3 or 4. Due to the action of this heuristic and the norm accepted with the tutorial or experimental contract, the participant A (usually a student) is divisible into a learnerlike component a_L and a teacherlike component a_T. These components are also "participants" but they are restricted by the constraints just mentioned, so that for any occasion, n, there is one and only one common aim topic which is psychologically one focus of joint attention. However, the composite participant $A = a_T, a_L$ may learn about, and

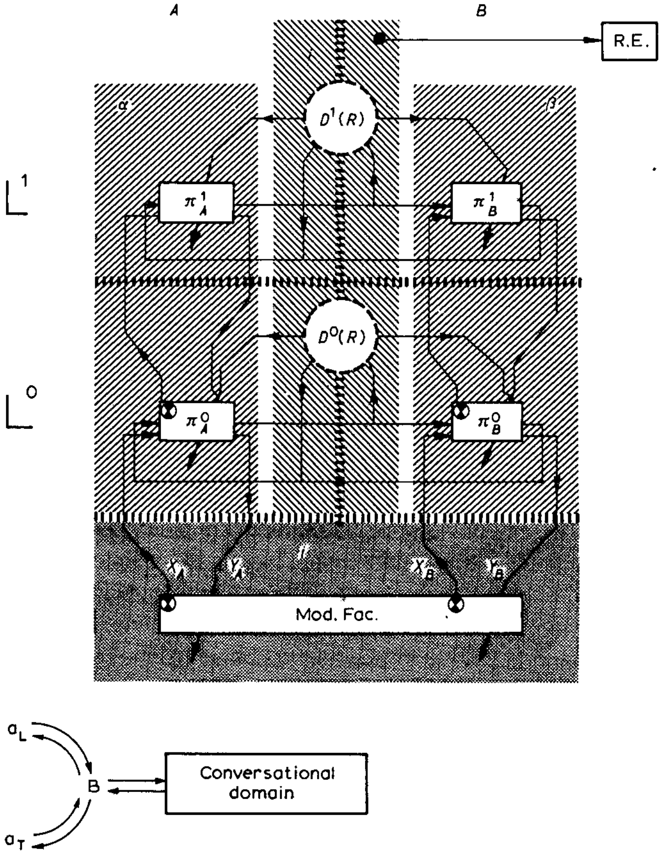


Fig. 6.1. "Cognitive Reflector" icon (from *Conversation, Cognition and Learning,* first monograph) and crude outline of "Cognitive Reflector" as pair of cognitive organisations a_L, a_T and a regulating heuristic B which controls access to conversational domain.

come to understand, one or several topics selected as goals which are members of his workset.

By the expedient employed in Fig. 6.1, we have thus represented learning as a conversation between the component participants of A; namely, a_L and a_T, regulated by the heuristic procedure B (rather than representing it as we did in the original Icon 4 as a conversation between A and B, with B occupying a neutral role as the "cognitive reflector"). So, if topic i is the aim and if topic j (or a topic class j) is the goal it is permissible to speak of a_L, a_T agreement with respect to an explanation of R_j in the context of B and of a_L, a_T agreement with respect to a derivation of R_j, under R_i, in the context of B. Together, these agreements correspond to the sprout or growing point of a strict conversation as defined in the previous monograph.

From a mechanistic or operational (or dynamic) point of view, the essential constraint imposed by the one-aim-at-once condition is a "local" or "partial" synchronicity with respect to the aim topic and all transactions that refer to it.

Since a_L and a_T are both executed in an L-Processor (and generally the same L-Processor, one brain) their constituent procedures (both $\underline{\text{Proc}^1}$ and $\underline{\text{Proc}^0}$) may be executed asynchronously. But, insofar as a_L and a_T coalesce to form an unspecific P-Individual A, the pertinent procedures must be locally synchronised. If the P-Individual A is unspecific, the synchronising events are not directly observable, though we have conjectured that A's awareness arises from (indeed *is*) the local synchronicity (alias, "information transfer" alias "program sharing") of an internal and generally unobservable "conversation". The peculiarity of the constructions in Fig. 6.1 and (later) in Fig. 6.2 is that the synchronising events are mediated through B and, given the experimental contract, synchronicity is enforced by B with respect to an aim topic in the conversational domain. That is, when both a_L and a_T attend to one aim topic, the procedures executed by these participants are coupled with respect of that particular aim. Hence, "local" synchronicity gains meaning as an observable; it is "synchronisation of $\underline{\text{Proc}^1}$ (aim) in the L^1 repertoires of a_L and a_T," which is manifest as A's learning strategy (i.e., a marker distribution model executed in the entailment structure display *ES*). By the same token, there is a local synchronisation of $\underline{\text{Proc}^0}$ (goal) where the goal is legitimate under the chosen aim and is a member of A's workset. This synchronisation is the construction of a model representing the (agreed) $\underline{\text{Proc}^0\text{(goal)}}$ in the Lumped Modelling Facility shown as *MF*. If there are several goal topics (R_j is a class of topic relations), then either the models are built and executed (under the control of a modelling facility processor clock) in sequence as subgoal models, or else these models are constructed in the a-priori-independent parts of a Lumped Modelling Facility, one to each part. Moreover, since each part of the modelling facility has a distinct processor clock, the models are executed in a facility-wise independent manner. But all of the models for goal topics refer to the aim topic, as a result of which their construction is coupled through the L-Processor which executes A.

In the special case when the goal topic is an analogy relation (as discussed in the last chapter), several models are built and executed in different parts of the Lumped Modelling Facility (the models representing the terms or relata of the analogy), and these a-priori-independent models are *executed* (not simply *constructed*) in a locally synchronous manner. The introduction of the couplings that secure this degree of synchronisation represents the analogy relation itself; this, in other words, is the model for the analogy relation between the terms.

The functional coordination of the composite participants a_L and a_T is shown in Fig. 6.2 where the "interface" of the original Icon is made explicit. At level L^0 (of $L = L^1, L^0$) there is a modelling facility (in general, a Lumped Modelling Facility containing several a-priori-independent processors), which is the vehicle for demonstrations given by a_T to a_L and explanatory models produced by a_L for agreement by a_T. The L^1 box, *ES*, is also a modelling facility, in practice the entailment structure display in which derivations of topics are modelled as learning strategies or state marker distributions.

Moreover, the aim topic is selected by choosing values of the semantic descriptors (L^1 predicates) of a conversational domain and the aim is validated, perhaps after a sequence of explore transactions, as noted in Chapter 1. (Recall that aim validation has been introduced into CASTE fairly recently; the validating transactions are not mentioned in the previous monograph, though they correspond to estimation of d_0, which was discussed in theoretical terms.) The conversational domain ($D^1(R)$, $D^0(R)$ or *ES*, *TS* under interpretation) is elided in Fig. 6.2 and its remnant is the Box D. That is, we assume that topics and their entailment relations are described and that for each topic i there is a pointer to some *PG*(i). Both kinds of data are available to $A = a_L, a_T$ (the unidirectional connections from D to a_L and from D to a_T), under the restrictions imposed by B. Moreover, B regulates all interactions at the interface (explanatory or demonstrative modelling in *MF* and the determination of learning or teaching strategies in *ES*) as indicated in Fig. 6.3 by the (dotted) bidirectional connections. In particular, B

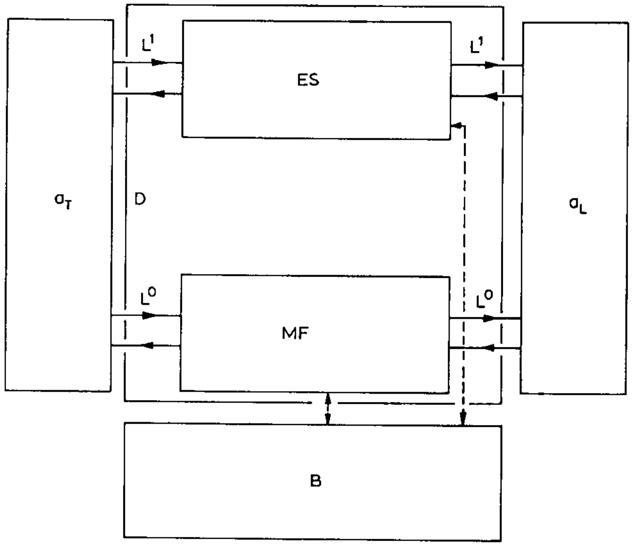


Fig. 6.2. "Cognitive Reflector" in enough detail to show *understanding.* As before, a_L and a_T are cognitive organisations, usually embodied in the same brain and B is the regulating heuristic securing *understanding* for each topic picked out for learning. B exercises overriding control upon access to entailment structure and modelling facility. *ES* = entailment structure for accommodating L^1 (derivation) models as overt learning strategies. *MF* = lumped modelling facility for L^0 explanation and for L^0 demonstration.

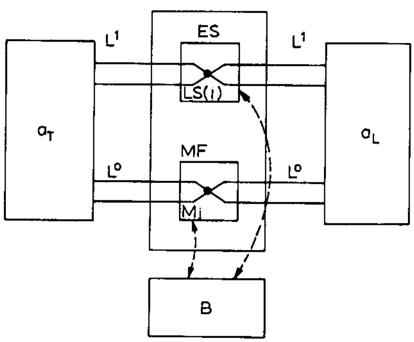


Fig. 6.3. Insertion of aim i and goal (or set of component goals) j. Any learning strategy delineated in the ES display acts as a model. LS(i), under aim i (of how topic i becomes known). The model M_j for any goal j under aim i is constructed in the modelling facility, MF.

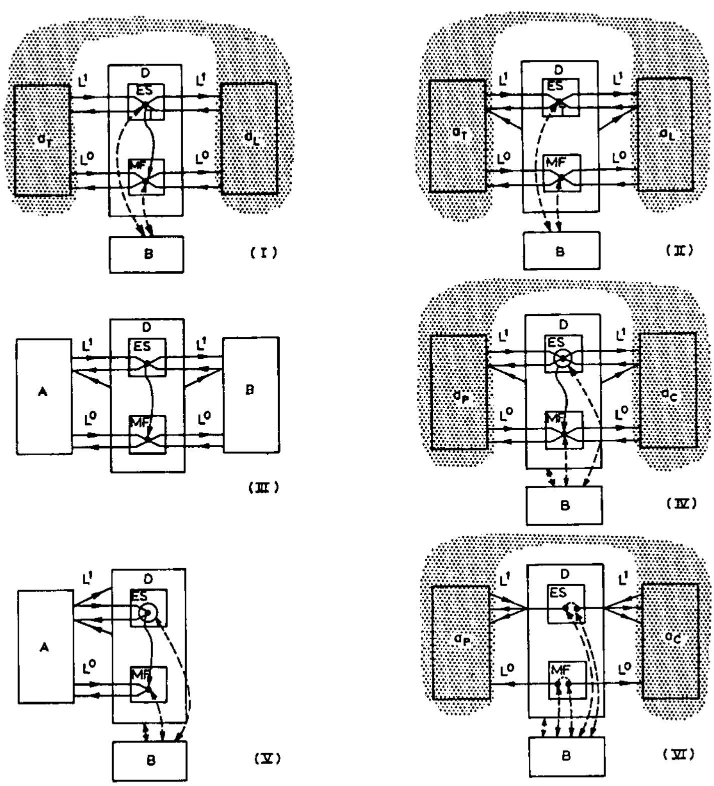
regulation ensures (as in the first monograph) that a strict conversation is reducible to ordered occasions, n, n + 1, ... upon each of which there is an *understanding* of some topic relation.

The construction is completed in Fig. 6.3 by inserting the aim and goal current at the n^{th} occasion.

Equipped with these conventions, it is possible to represent in outline all of the conversation types developed in the Icons of the first monograph, and to encompass without changing the conventions many participant and many aim conversations which have not previously been represented.

The conversation types due for discussion in this book are shown in Fig. 6.4(I) to (XII).

Of these pictures, (I) and (II) show the cognitive reflector construction, with (I) and without (II) the possibility of selecting



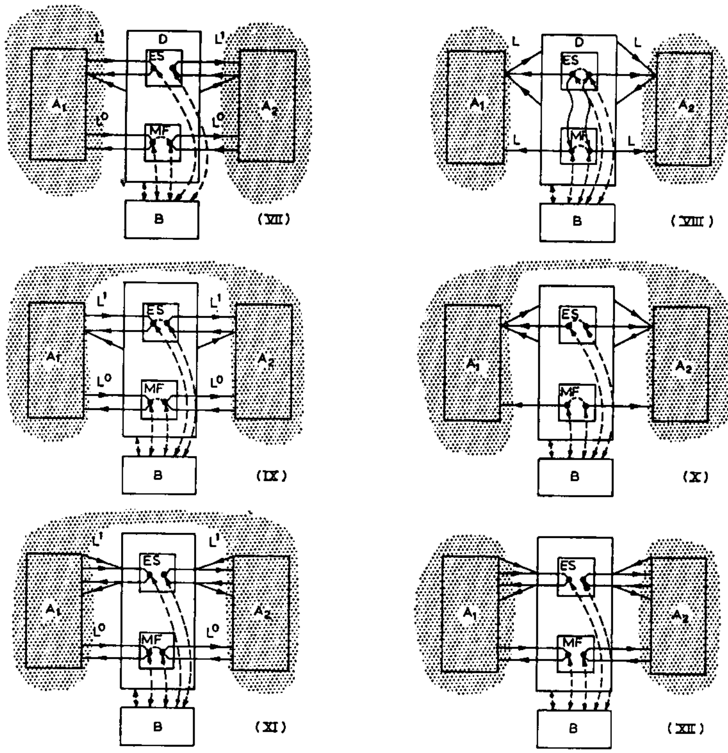


Fig. 6.4. Paradigms for one aim and many aim conversations discussed and detailed in the text. Of these 4(VII) to 4(XI) count as many aim conversations of various types, and Fig. 4(I) to 4(VI) as one aim conversations only. Shading distinguishes one or several brains (L Processors) $\alpha\beta$.

amongst several families of descriptors of the conversational domain. T_i in *ES* (at level L^1) is the aim topic and is connected by a data link to the program graphs (task structures) of one or more goal topics in workset, which are being modelled in *MF* at level L^0.

Picture (III) shows a conversation between a pair of distinct participants which *happens to be* a strict conversation because one of the participants (B) is not only a sentient individual, but *also acts* as a regulating heuristic. This circumstance, which was introduced initially in the first monograph (Icon 4), is exemplified — supposing the transactions are an approximate to those of a strict conversation — by a Piagetian interview or a paired experiment (B the interviewer), by an implementation of the teachback technique (B the participant experimenter), or by a real life tutorial (B the teacher).

In picture (IV), B is a heuristic pure and simple, as in (I) or (II). However, it is an evolutionary heuristic, encouraging development of the conversational domain, such as the EXTEND program in the first monograph. A is a source or subject matter expert (possibly a student who has opted into this role). The circle surrounding the aim topic indicates that the source or subject matter expert is free to originate a topic which is not part of the conversational domain. Insofar as he is able to satisfy the constraints upon learnability and memorability imposed by B, the topic will *become* part of an *enlarged* conversational domain. It is still the case that one and only one aim topic exists at once, namely, the novel topic currently undergoing incorporation.

The gross representation of (IV) is refined in (V) and (VI), by depicting two internal participants which make up A. Since A is a subject matter expert, these components are more aptly called "proposer" and "critic" (Minsky's locution), and they are labelled a_p and a_c (rather than a_T and a_L) for this reason.

The two distinct refinements, (V) and (VI), appear because it is both propitious and operationally mandatory to distinguish between the syntactic and the semantic components of a thesis which is under exposition (at this stage in the exposition just topic T is being added to the thesis).

On the one hand, Picture (V), the description of the conversational domain is held constant and a fresh syntactic derivation is established; this is the basic operation governed by EXTEND. On the other hand, Picture (VI), the form of the thesis is held constant whilst this form is given a fresh semantic interpretation by way of a new description. This is the "choice and the evaluation of descriptors" phase of EXTEND, using the repertory grid technique (Chapter 1, Chapter 3, and Icons 15, 16, and 17, in the previous monograph).

Before turning to the many aim conversations shown in (VII) and (VIII), notice that all of these one-aim-at-once conversations, either on a fixed or an evolving conversational domain, can be accommodated as special cases of the scheme outlined in Chapter 4, Section 1. The specialisation is introduced by setting $L = \mathcal{L}$ (just one language), or in case there are analogy relations, by setting $L = \langle S, \underline{\text{Inter}}\ i, \underline{\text{Univ}}\ i \rangle$ or $\langle S, \underline{\text{Inter}}\ j, \underline{\text{Univ}}\ j \rangle$ so that any analogy is depicted as a morphism (usually an isomorphism) between different models for some identical or similar syntactic expressions. This expedient is satisfactory provided that analogies are learned (from their descriptions in the conversational domain) and are not constructed de novo. The expedient remains satisfactory for the limited, and far from innovative, analogy constructions encompassed by EXTEND; that is, the analogy relation is treated as a fresh topic on a par with others, since it relates topics which already exist in the conversational domain without recourse to the analogy relation. To go further than that, and to accommodate forms of conversation in which the analogy relation is invented first of all and the terms of the analogy (its relator) appear as a result of this invention, it is necessary to introduce the two (or more) aim-at-once constructions shown in (VII), (VIII), (IX) and (X).

We use the notation A_1, A_2 to represent two coexisting P-Individuals, each of which might be factored independently to yield restricted participants: $A_1 = a_{T1}, a_{L1}$ and $A_2 = a_{T2}, a_{L2}$ (or $A_1 = a_{P1}, a_{C1}$ and $A_2 = a_{P2}, a_{C2}$). These P-Individuals are not locally synchronised by the heuristic B and may act independently as indicated by the simultaneous presence of two aim topics. Psychologically, A_1 attends to one topic and A_2 to another; A_1 models a topic in one universe of interpretation, A_2 models a topic (perhaps the same topic) in a distinct universe of interpretation. From the perspective of Section 1, A_1 and A_2 have different languages (so that L is a set of languages $\mathcal{L}_1$, $\mathcal{L}_2$ ...), though certain A_1 statements in $\mathcal{L}_1$ of L may be agreed, at the syntactic level of consensus to have the same formal consequences as certain A_2 statements in $\mathcal{L}_2$ of L.

If it happens, as in (VII) and (VIII), that A_1 is executed in a processor α and A_2 in a distinct processor β, then the syntactic agreement is a consensus between people or cohesive groups A_1, α and A_2, β which may later be strengthened by semantic agreement into a common meaning (accord, cooperative interaction, mutualism).

If it happens, as in (IX) and (X), that A_1 and A_2 are executed in the same L-Processor, a brain, then this agreement sets the stage for an innovation which will occur if the syntactically common statements (call them set E) can be given a compatible interpretation by A_1 and A_2; that is, E gains a common meaning for A_1 and

A_2. If so A_1 and A_2 fuse into one P-Individual $A = A_1, A_2$ with respect to the innovation which is the meaning of E.

Such a fusion is also the "analogy relation first" construction of an analogical topic. By parallel with (V), Picture (IX) represents the syntactic component of an innovation, where distinct universes of interpretation are held constant as a framework. By parallel with (VI), Picture (X) represents the generation of further universes of interpretation as means for realising distinct compilations of the same program.

The artificial calibre of the convenient demarcation between syntax and semantics is conceded immediately. In the sequel, particular significance is credited to the case in which (IX) and (X) coalesce as a hybrid form, approximated by Picture (XI), in which changes of program structure and changes of interpretation are inseparable. In the fields of social anthropology and sociology, similar interest may be attached to the hybrid of Picture (XII). Though it is beyond the scope of our empirical enquiry, we conjecture that (XII) represents a peculiarly stable social group, a persistent cult, an urban civilisation, or a cohesive society.

2. IDENTIFICATION WITHIN THE GENERALISED THEORY OF LANGUAGE

One of the chief results of the work on the theoretical scheme outlined in Chapter 4, Section 1 is an account of the conditions under which entities with different sublanguages, $\mathcal{L}_1$ in L and $\mathcal{L}_2$ in L, may communicate. These theoretical results have been applied (by Gergely and Nemeti) to the interaction between scientific disciplines having disparate languages, or calculi, or models, and to the interaction between social systems.

An indication of the process, as they envisage it, is given in Fig. 6.5, and may be regarded as a cooperative or mutualistic interaction between persons or societies C_1 and C_2. Using the notation of Chapter 4, Section 1, C_1 and C_2 are characterised (given calculi 1 and 2) as a pair of systems $\mathcal{L}_1 = \langle S_1, \underline{\text{Inter}}_1, \underline{\text{Univ}}_1 \rangle$ and $\mathcal{L}_2 = \langle S_2, \underline{\text{Inter}}_2, \underline{\text{Univ}}_2 \rangle$, where S_1 and S_2 are the true statements (or productions under the given calculi) of $\mathcal{L}_1$ and $\mathcal{L}_2$ ... That is, there are models M_1 in $\underline{\text{Univ}}$ 1 (for S_1) and M_2 in $\underline{\text{Univ}}$ 2 (for S_2), which are interpretations of these statements. The truth criterion, in this

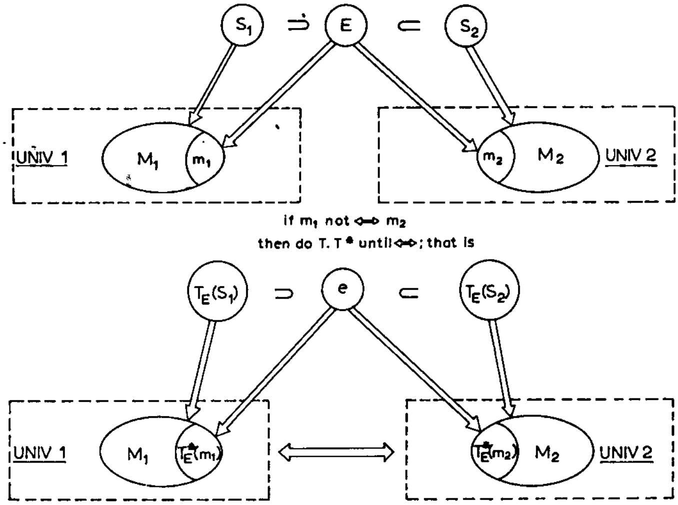


Fig. 6.5. Outline of the transformation required for "common meaning" agreement between participants. Broad unidirectional arrows stand for relation between a class of statements and its model; the bidirection arrow ⇔ stands, as usual, for isomorphism.

case, is veridicial, (for example, the result of empirical testing carried out by C_1 and C_2 independently) and the truth in question is a correspondence truth.

Suppose that certain statements $E \subset S_1$, $E \subset S_2$ are held in common as (syntactically) agreed by C_1, and C_2; that is, the statements of E form a coherent set. Agreement hinges upon a *consensual* agreement; that is, upon a coherence ordained syntactic agreement (Ch. 4 Sect. 7). We are anxious to investigate the circumstances under which C_1 and C_2 attach the same meaning to statements in E, given the existence (as parts of M_1 and M_2 of models m_1, m_2; for E in Univ 1 and Univ 2 that are held, by C_1 and C_2 to represent the correspondence truth of statements in the set E. The required equisignificance obtains if there is an isomorphism from m_1 to m_2 (written, $m_1 \Leftrightarrow m_2$).

Usually, this condition is not satisfied; at most, there is homomorphism preserving only some of the relations in the models and losing specificity. However, it is possible to construct transformations, which we shall here designate T and T*, that are coupled and operate upon S_1, S_2 and m_1, m_2, respectively, * with E as a parameter such that $T_E(s_1)$ and $T_E(s_2)$ generate a usually more complex set, e, of agreed statements, and $T^*_E(m_1)$ is C_1's model of e. $T^*_E(m_2)$ is C_2's model of e, and $T^*_E(m_1) \Leftrightarrow T^*_E(m_2)$ is the common meaning of the (usually more complex) set of statements, e, that are shared by C_1 and C_2 (obtained as a closure of the model space under the originally agreed set of statements, E). The crucial feature of this construction is the fundamental coupling between T and T^*; in order to obtain common meaning, it is generally necessary to modify the statement set and the interpretations. Moreover, although these processes might be isolated under special conditions, they are as a rule inseparable.

To obtain an immediately apposite identification, notice that T represents the act of reaching a syntactic (coherence based) agreement and that T^* represents the act of reaching a semantic (correspondence based) agreement, together an act of establishing a *common meaning*. Now call $C_1 = \langle A_1, \emptyset \rangle$ and $C_2 = \langle A_2, \emptyset \rangle$ (where $\emptyset$ is a variable with values α, β, ...). The legitimacy of this identification is evident in the case when $\emptyset$ assumes distinct values (corresponding to α and β in Fig. 6.4), since the L-Processors are specified at the outset as distinct universes of interpretation. The legitimacy of this expedient when $\emptyset$ assumes the same value (the P-Individuals are compiled and executed in the same brain, or L-Processor) depends upon the assumption that procedures contain a compiler and that they construct distinct "possible worlds" upon compilation. We took this as a plausible hypothesis in Chapter 4, Section 1 and certainly consider it to be experientially (though not empirically) justified. Later on it will be possible to buttress the hypothesis and support it on logical grounds.

Now the argument just put forward, that T and T^* are *in general* coupled, has as a consequence that the most general constructions of Fig. 6.4 are the hybrid organisations in 6.4(XI) and 6.4(XII),

* As in the previous monograph, the normally Fuzzy reproductive processes can be represented or simulated (Loefgren 1972) as a productive/reproductive Turing Machine which produces and reproduces Turing Machines (representing Progs in the repertoires π_1, π_2, of A_1, A_2). S_1, S_2 are sets of their code numbers and productions. The interpretation functions may be given as fixed (the form I/F of Section 1) or, since α and β are discriminated, in the generative form (Inter of Section 1) calculus 1 and calculus 2 are production systems for these (abstract) machines.

where the act of reaching syntactic or coherence based $\langle A_1, \alpha\rangle$; $\langle A_2, \beta\rangle$ agreement (reflecting T in these pictures) is inseparable from the act of reaching semantic or correspondence based $\langle A_1, \alpha\rangle$ $\langle A_2, \beta\rangle$ agreement, reflecting T^*. Reintroducing the postulate of Section 2, 6.4(XII) is identified with a *natural language* dialogue; L^1 and L^0 coalesce into a natural language, L. The modelling facilities *ES*, *MF* likewise coalesce and become the universes of interpretation of a natural language, namely, as postulated in Chapter 4, Section 2, a set of Fuzzy Sets. Under this identification e is a social metaphor, and it designates, as its *common meaning*, an interpreted analogy relation.

On the other hand, 6.4(XI) represents a slightly different situation insofar as the P-Individuals are compiled and executed in the same L-Processor, and agreements are reached within this medium (between $\langle A_1, \alpha\rangle$ and $\langle A_2, \alpha\rangle$). Once again, L^1 and L^0 coalesce and so do the modelling facilities, *MF* and *ES*. The only kind of modelling facility which satisfies this requirement as a physical entity is an L-Processor, and if this is identified with a brain, then the common meaning encompassed by e and its interpretation is *thought* — constructive or *innovative* thought, if e is, as usual, greater than E.

The remaining, more tractable, pictures in Fig. 6.4 represent special cases of these general paradigms.

All of the "many aim" (more than one coexisting P-Individual) pictures 6.4(VII), (VIII), (IX) and (X) represent an act of agreement about *common meaning*, and as a corollary of the present argument, such situations are likely to foster creativity or innovation which can be observably exteriorised under particular constraints proper to the interpretations (of course assembly and so on) furnished in Section 1.

In contrast, the one-aim-at-once constructions (namely Fig. 6.4(I), (II), (III), (IV), (V), (VI)) do not have this property. The inference is not that a human being *cannot be creative* under these circumstances. The constructions simply assert realisable experimental, tutorial or expository situations in which creative or inventive acts cannot be sensibly *exteriorised for observation*; so that, even if they occurred, such acts, insight apart, would be confused with mistakes or haphazard events.

Moreover, within the experimental framework of the many aim conversations (reified as a many user version of CASTE or its surrogate INTUITION, and a course assembly system called THOUGHTSTICKER), it is possible to suggest mental mechanisms for the creativity and invention which *is* observed and to provide evidence that these mechanisms are in human beings responsible for the transformations T, T*.

3. GENERAL DISCUSSION

The innovative mechanism to be postulated is readily conceived in terms of the thoroughly tangible analogy modelling operations which were discussed in detail in Chapter 4, Sections 10 and 11. Any model for an analogy relation R_k between topic relations R_i and R_j is a coupling M_k between a pair of distinct models M_i, M_j realised in a-priori-independent parts of a Lumped Modelling Facility. Usually, this does involve a partial synchronisation between the a priori asynchronous processors X, Y in the Lumped Modelling Facility, and at a theoretical level the partial synchronisation is always mandatory.

However, M_i and M_j are compilations of serial representatives S Prog i, S Prog j, of Proc i and Proc j, so that synchronisation is achieved by expedients such as "interruption" and "hold" signals. Hence, M_k is really the compilation of a further serial program (of a kind often called an executive program).

A more general proposal for a mechanism realising the coupled transformations T, T* depends upon the apparatus discussed and developed in both Chapter 4 and Chapter 5. The procedures under consideration are Fuzzy (Chapter 4, Section 5; Chapter 5, Section 11), and their interaction, coupling and local synchronisation in an L-Processor is imaged in Chapter 4 as the interplay of memories or concepts or both. Chapter 5, Section 10 and 11 presented a more specific mechanism using the Proc^1 categories of *DB*, *PB* and *PC* operations.

Moreover, at that juncture, we posited a boundary condition upon the interaction (here identified with the outcome of T, T*) to the effect that the Fuzzy Procedure resulting from local synchronisation or coupling is usually larger than the original procedures. Isomorphism between a pair of original concepts is the limiting case, the exception rather than the rule. Generally, the syntactic component (Prog) of a concept must be modified and enlarged before it is possible to secure isomorphism between compilations of models. Thus, in the context of Chapter 5, speaking of analogy construction, most analogies are founded on generalisations, only a few on isomorphism. Within the overall picture of agreement between P-Individuals executed in the same brain or in several, the analogy construction is a special but important case of

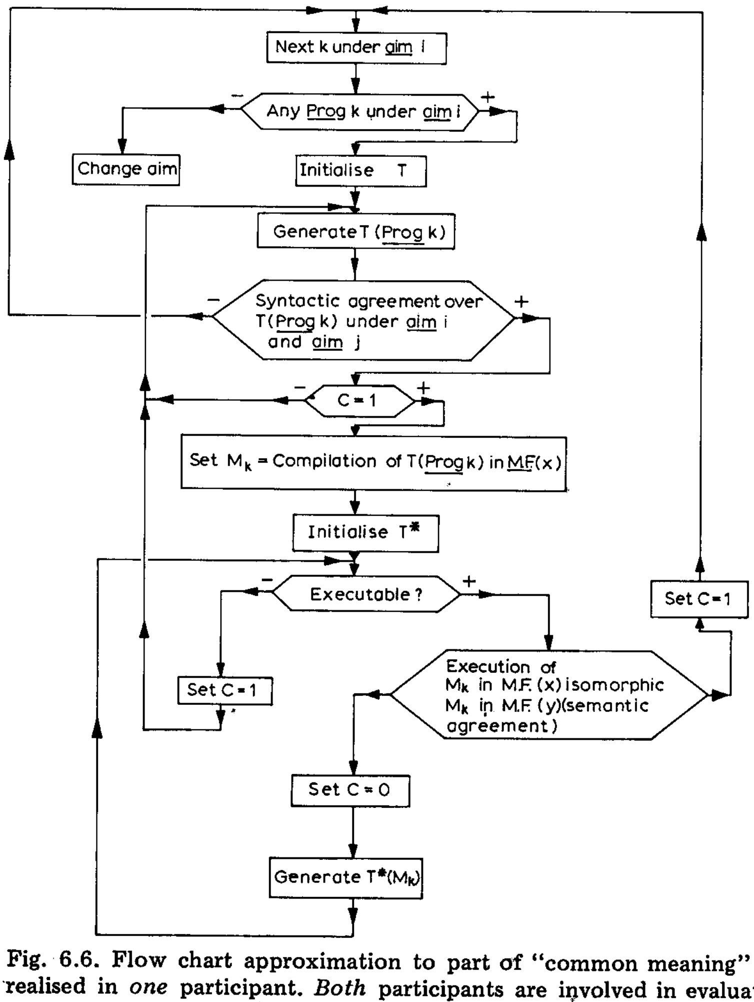


Fig. 6.6. Flow chart approximation to part of "common meaning" process realised in *one* participant. *Both* participants are involved in evaluating the tests in "syntactic agreement" and "semantic agreement" and the process is interrupted at these points. Parameter C is artificial expedient used to represent process serially.

achieving agreement that furnishes a common meaning. For interpersonal dialogue $\langle A_1, \alpha \rangle$ with $\langle A_2, \beta \rangle$, the analogy exists at the syntactic level between the productions of A_1 and A_2; at the semantic level, it induces an isomorphism between compilations/interpretations in the distinct L-Processors α and β. For analogy construction, where only one L-Processor (α, say) is involved, the analogy exists between distinct internal compilations (Inter x/Inter y) or between models M_i, M_j in distinct modelling facilities MF(x), MF(y).

The argument is summarised as follows: a mechanism is believed to exist in mental activity and to have an intimate relation to awareness (since, in conversation theory, consciousness depends upon local synchronisation of a priori asynchronous processors). To reach steady states, this mechanism must be augmented by a boundary condition, and this was introduced as a postulate in Chapter 5, begging the question of what the boundary condition *is*. Starting from the argument in Chapter 4, we imported a set of results (Andreka, Gergely and Nemeti) on model matching and interpreted the transformations T, T^* as the genesis of common meaning, but without stating a mental mechanism which would secure this result. Finally, it is proposed that common meaning is the boundary condition required to govern the process in Chapter 5, and this process is the mechanism required to realise T, T^* and achieve a common meaning.

Fig. 6.6 is a crudely flow-charted approximation to the entire process. It is assumed that distinct P-Individuals exist, that their universes of interpretation and compilation (α, β or X, Y) are held distinct, that each P-Individual has the *isomorphism* operator in his repertoire, and that there is an internal or external channel of communication sufficient to establish local synchronicity.

4. TWO AIMS, ONE TO EACH OF TWO USERS

Suppose there are two users (people, respondents) indulging in dialogue. How should an external observer of their conversation detect the existence of two aims (in a non trivial sense), and what evidence should he accept for the coexistence of two P-Individuals. Since I am anxious to maintain the possibility of experimentation, the conditions to be listed are almost obsessively mechanical.

First of all, the conversational paradigm must be modified to allow for the existence of many aims at once, and this involves replicating all of the apparatus underlying the entailment structure display, the modelling facility, and most of the other parts of an operating system — either CASTE or INTUITION (Fig. 6.7). There are two distinct entailment structures (two replicas) on which separate marker distributions are displayed as the two separate learning strategies of the participants; two records are kept of their explanatory models.

Finally, there are two aims, one to each user. Though the aims may point to the same topic (that is, the node picked out in one

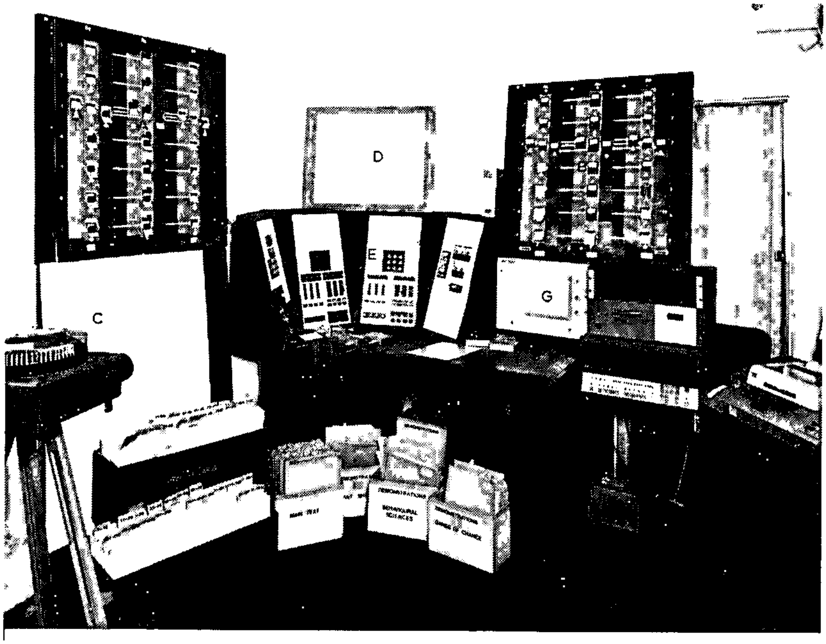


Fig. 6.7. Group learning on INTUITION system for a pair of participants (1 and 2). A = Entailment structure (as in Fig. 1 for participant). B = Entailment structure (a duplicate of participant 1 structure). C = Random access slide projector for descriptive materials. D = Screen visible to participant 1 and to participant 2 jointly. E = STATLAB modelling facility used by participant 1 and participant 2. F = Conditional probability "boxes" and "delay" boxes for modelling stochastic processes. G = Mini BOSS equipment. H = Control and recording equipment for regulating interaction.

user's entailment structure may be in register with the node picked out in the other user's entailment structure), the two aims are separately validated. This means (as in Chapter 1) that each user separately has a substantial zero value of doubt, d_0, regarding the topic description occupying his attention, and even if the aims are in register, the users may have reduced their attentional doubt, d_0, by entirely different explore transactions. Of course, the users *need* not have aims in register and (before they interact) are very unlikely to do so.

In such an arrangement, associated with a fixed conversational domain, it is possible for two participants to learn independently and for the operating system to gather information about the independent learning strategies and the independent explanatory models they produce. Similarly, the heuristic can react to them independently.

If the two participants, human beings, $\langle A_1, \alpha \rangle$ and $\langle A_2, \beta \rangle$ are to engage in collusion, then they must be furnished with a communication channel. Surely, this may be verbal and graphical; for example, $\langle A_1, \alpha \rangle$ may talk to $\langle A_2, \beta \rangle$ and they could look at each other's learning strategies exhibited on their entailment structure displays. They could also cooperate by demonstrating topic relations to each other and by joint model-building. Unfortunately, some aspects of the interchange are not readily interpretable by the heuristic B, and in particular B is unable to sense the fact that $\langle A_1, \alpha \rangle$ does (or does not) entertain hypotheses about $\langle A_2, \beta \rangle$ (in contrast to hypotheses about the topics being learned); and vice versa, of course, $\langle A_2, \beta \rangle$ may or may not entertain hypotheses about $\langle A_1, \alpha \rangle$ of which B is necessarily ignorant. This defect is damaging because if B takes $\langle A_1, \alpha \rangle$ and $\langle A_2, \beta \rangle$ as a-priori-independent (on the grounds that A_1 and A_2 are housed in different brains α, β, and need not interact through the operating system), then B must sense the extent to which A_1 and A_2 do interact with each other (not simply with the operating system) in terms of their mutual and person directed hypotheses. Similar remarks apply to the external observer if he remains utterly dispassionate and refrains, for example, from interpreting spoken dialogue.

The minimal sampling arrangement for mutual (I/You, not I/it) hypotheses is an IPM interchange between $\langle A_1, \alpha \rangle$ and $\langle A_2, \beta \rangle$, mediated by the FRIM device described by Icon 24 and Fig. 9.10 in the first monograph. (Recall the change in notation: to tackle many aim systems the participants are now called A_1 and A_2, while in the first monograph they feature as A and B.) With the changed notation, an IPM response to a PQuest (multiple choice, list, or assessment question) is a double hierarchy of replies; for example, regarding the evaluation of some property of *topic* i, presented jointly to $\langle A_1, \alpha \rangle$ and $\langle A_2, \beta \rangle$, we have:

1 (i) What A_1 thinks of *topic* i,
1,2 (i) What A_1 thinks A_2 thinks of *topic* i,
1,2,1 (i) What A_1 thinks A_2 thinks A_1 thinks of *topic* i.

On repeating the hierarchical construction for the other participant, independently, the following responses are obtained from the perspective of $\langle A_2, \beta \rangle$:

2 (i) What A_2 thinks of *topic* i,
2,1 (i) What A_2 thinks A_1 thinks of *topic* i,
2,1,2 (i) What A_2 thinks A_1 thinks A_2 thinks of *topic* i.

In the simple IPM test, the scores are collected independently as lists and compared for later reference. Using FRIM, the participants, having stated their (independent) hypotheses, receive an immediate stage by stage feedback (first monograph) which allows them to resolve differences and reach agreement (if they wish to do so) on the spot; not necessarily agreement over topic i, more often agreement to differ and agreement about why they differ (Fig. 6.8).

We intend to use the existence of feedback manipulable mutual hypotheses as the evidence for cogent interaction between the participants $\langle A_1, \alpha \rangle$, $\langle A_2, \beta \rangle$ and to say, in general, that two P-Individuals exist if there are aims i, j such that appropriate matching scores or comparisons are obtainable with respect to the values of the descriptors of the aim topics, and similar matches are obtained in respect of PQuests (as in the first monograph, multiple choice or list questions) spanning topics k that are goals, under the distinct aims, common to both aim topics.

The argument depends quite critically upon the fact (given, in an operating system) that the aims chosen by the participants are both validated. As a result, both participants have a near zero attentional doubt, d_0, in respect of their own aim, or differently phrased, both participants have some description of the aim topic which is compatible with the (possibly redundant) descriptor

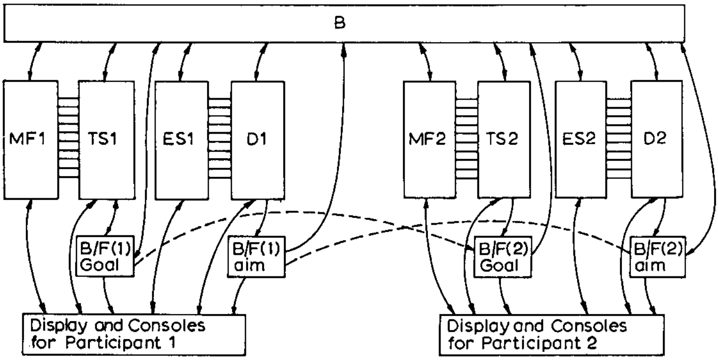


Fig. 6.8. A many participant (many user) operating system in which interpersonal communication takes place through the boxes B/F(1) aim, B/F(1) goal and B/F(2) aim, B/F(2) goal. The label "B/F" denotes "BOSS/FRIM equipment"; the B/F aim boxes are aim validation devices and the B/F goal boxes are confidence estimation devices.

values assigned on the conversational domain, by the subject matter expert.

Since the point is important, it is worth looking at the matter from a viewpoint which some readers may find more explicit. Consider the descriptors as semantic differential indices (Osgood et al. 1957). If *topic* i is validated as one participant's aim, and *topic* j is validated as the other participant's aim, then both participants have located the topics they appreciate as points (relative to their own perspective in the matter) in an Osgood-like semantic-space. Quite possibly, *topic* i and *topic* j are distinct. Whether or not this is so, the possible set of (semantic differential) attributes is available to both of them. They both have unlimited explore transactions. It makes sense to compare their attitudes, noting that participant A_1's perception of *topic* i may (or may not) differ from A_2's perception of *topic* i; that A_1's perception of *topic* j may differ from A_2's perception of *topic* j; and that A_1 and A_2 may or may not see *topic* i and *topic* j as similar.

Use δ to denote a descriptor having real values (+, − not the null value*) on a *topic* i and index it (δ_i). If $\langle A_1, \alpha\rangle$ and $\langle A_2, \alpha\rangle$ are anxious to interact, then they must satisfy the conditions given below. (Note the inversion of indices, i is still A_1's aim topic, and j is still A_2's aim topic.)

$1(\delta_i)$ may or may not match $2(\delta_i)$
$1(\delta_j)$ may or may not match $2(\delta_j)$

But, if not, then

$1,2(\delta_i)$ must match $2(\delta_i)$
$1,2(\delta_j)$ must match $2(\delta_j)$

and

$1(\delta_i)$ must match $2,1(\delta_i)$
$1(\delta_j)$ must match $2,1(\delta_j)$.

If this condition is satisfied for all the descriptors with (+, −) values on topic i and topic j (as a matter of practice, all those used by the pair of participants for gaining access to the aim topics), then: Either $\langle A_1, \alpha\rangle$, $\langle A_2, \beta\rangle$ agree about the description of their (possibly distinct) aim nodes, or even though the aims have a different meaning, the participants are alive to the differences and have accurate hypotheses in this respect. This is a *semantic agreement* index and an approximation to Fig. 6.4(VIII).

If $\langle A_1, \alpha\rangle$, $\langle A_2, \beta\rangle$ enter into these mutual hypothetical transactions and also provide the required matching scores, then one participant's entailment structure display (its configuration of markers is this participant's learning strategy *LS*) is made available to the other participant, and vice versa. Moreover, if this combination is satisfied, $\langle A_1, \alpha\rangle$ and $\langle A_2, \beta\rangle$ share the results of explore transactions, and in addition to this, $\langle A_1, \alpha\rangle$ and $\langle A_2, \beta\rangle$ may adopt a joint learning strategy, worked out on the entailment structure display. The participants are now in a position to cooperate in learning. As a rule (though various heuristics have been used and are being tested out experimentally), the potentially possible modes of cooperation are as follows.

(a) $\langle A_1, \alpha\rangle$ models a topic as a demonstration to $\langle A_2, \beta\rangle$ (thus, $\langle A_1, \alpha\rangle$ is acting as a genuine teacher), and vice versa.
(b) Within restrictions (noted in Chapter 4) upon complete overall explanation, $\langle A_1, \alpha\rangle$ and $\langle A_2, \beta\rangle$ build and submit a joint explanatory model.

Either (a) or (b) or both are permitted for any topic k, such that k is both in the EntSet (i) and in the EntSet(j) (hence, it is a possible goal topic), and such that the following conditions are satisfied for R_k. For any R_k it is possible to construct a list of spanning PQuest k of alternative sets (AltSets, previous monograph) in which only one alternative (Alter$^+$) is correct. In essence, the Alter in AltSet k figure as plausible solutions to problems posed in respect to realising R_k, and the groupings into AltSets are designed to set up a one-and-only-one-correct situation.

Using BOSS (Belief and Opinion Sampling System, previous monograph, Chapters 4 and 6), each participant can provide an index of veridical certainty, a confidence estimate that peaks for Alter$^+$ in the AltSet k of PQuest k. These indices disignated θ (the Shuford Scores of the previous monograph) are written θ_1 for $\langle A_1, \alpha \rangle$ and θ_2 for $\langle A_2, \beta \rangle$, and θ values are elicited in respect of any R_k for which the participants wish to cooperate. Let θ_0 be a criterion value (about 0.8 is usual), then the condition that $\theta_1 > \theta_0$ *or* $\theta_2 > \theta_0$ *or both* indicates that one or other participant or both of them are able to set about solving problems under the topic relation R_k.

It is also possible to obtain an unconstrained confidence estimate indicating the participants' doubts about problem solving under R_k, whether or not the participants favour a *correct* solution (that is, an estimate of each participant's prospective doubt, d_2 of Chapter 11 in the previous monograph). Moreover, the d_2 estimate makes sense since attentional doubt, d_0, is nearly zero (assured by aim validation). As a slightly different exercise, it is easy to match BOSS responses to PQuest k, obtained in the unconstrained mode, in an IPM or FRIM hierarchy.

Use $1(R_k)$ for $\langle A_1, \alpha \rangle$'s confidence estimate; $2(R_k)$ for $\langle A_2, \beta \rangle$'s. Use $1,2(R_k)$ and $2,1(R_k)$ for the confidence estimate obtained to express A_1's belief about the confidence estimate that A_2 will produce, and A_2's belief about the confidence estimate that A_1 will produce (both of them given the same question, namely PQuest k).

Notice, as an operationally important point, that both matching scores based on the form of prospective doubt and correct belief scores, θ, are obtained from the same response, for θ is derived by a mechanical comparison between the confidence estimate and Alter$^+$ (which is unknown to the participants).

If the participants desire to cooperate at *topic* k (either by method (a) or method (b)), and if the EntSet condition is satisfied, then they may do so provided that

(1) $1(R_k)$ matches $2(R_k)$ or, if not, then $1,2(R_k)$ matches $2,1\ (R_k)$

(from which, since d_0 is nearly zero for both participants, these participants either have the same form of prospective doubt, d_1, or if not, they recognise the difference that exists between them). And

(2) Either $\theta_1 > \theta_0$ or $\theta_2 > \theta'_0$ or both

(an optional, but salutary, condition; at least one of the participants has a chance of solving problems correctly with respect of R_k).

If so (and if cooperation takes place), the participants are reducing their individual prospective doubts, d_1, by information about the form of their mutual prospective doubt. This realises the syntactic agreement of Fig. 6.4(VII).

5. IMPROVED OPERATING SYSTEMS FOR TWO USERS

Both the *semantic agreement* index and the *syntactic agreement* index can be refined, using the following techniques:

To refine the semantic agreement, the FRIM responses to a PQuest are replaced by FRIM responses to Thomas's "Exchange Grids" where the participants are allowed to construct and compare their own descriptors as well as the values of fixed descriptors. The technique is an elegant and basic extrapolation of the repertory grid technique for eliciting "personal constructs" (alias descriptors), mentioned in Chapter 1 and crystallised in Icons 15, 16 and 17 of the first monograph. Although only recently introduced into our operating systems, Thomas has employed the "exchange grid" method extensively, both manually (Thomas 1971) and using computer administration (Thomas 1970). The results from these studies are extremely coherent and informative.

The previous notation $1(\delta_i)$; $1,2(\delta_i)$, and so on, is generalised to accommodate exchange grids by writing δ_i for the constructs or descriptors at *topic* i and Δ for a vector $\delta_{i1}\delta_{i2} \ldots \delta_{j1}\delta_{j2}$ so that an exchange grid comparison has the form:

$1,2(\Delta), 1(\Delta)$ For $\langle A_1, \alpha \rangle$

$2,1(\Delta'), 2(\Delta)$ For $\langle A_2, \beta \rangle$.

This process of reaching semantic agreement is a more informative realisation of Fig. 6.4(VII), in which the descriptors are regarded as personal constructs (Kelly 1955), rather than the attributes in a semantic space; i.e., the system is an open system, rather than a closed system.

Regarding the *syntactic agreement*, the refinement is obtained (A) by adjoining a "dummy" (L^0) modelling facility to each working position, so that one participant can deliver an IPM response by "making the model he thinks the other participant will make", and (B) by adding further markers, so that one participant can model on the entailment structure display "the learning strategy he thinks the other participant will adopt". This L^1 or learning strategy hypothesis may contain a redundant semantic component (picked up already by the exchange grid system) insofar as the entailment structure display represents some (but not all) of the L^1 semantic descriptors.

Insofar as the participants reach agreement at the syntactic level, they not only reduce their individual prospective doubt, but also their retrospective doubt (d_1 of Chapter 11 in the previous monograph) and do so by exchanging information about the form of each other's doubt (both prospective, d_2, and retrospective, d_1).

The "dummy" models and the hypothetical learning strategies enter into FRIM comparison and feedback, as before. They are shown in Fig. 6.9, using the following notation.

$1(LS_i)$ is $\langle A_1, \alpha \rangle$'s learning strategy under aim *topic* i.
$1{,}2(LS_j)$ is $\langle A_1, \alpha \rangle$'s hypothesis about $\langle A_2, \beta \rangle$'s learning strategy under aim *topic* j
(and, vice versa, $2(LS_j)$ and $2{,}1(LS_i)$, for $\langle A_2, \beta \rangle$.

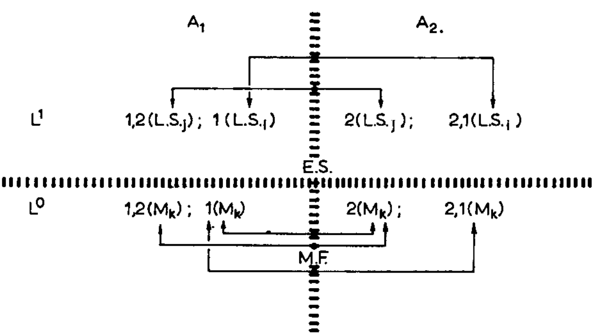


Fig. 6.9. Generalised system. Comparisons of models and learning strategies are indicated by connecting links.

$1(M_k)$ is $\langle A_1, \alpha\rangle$'s explanatory model, complete or incomplete, for any *topic* k in EntSet i and in EntSet j;

$1,2(M_k)$ is $\langle A_1, \alpha\rangle$'s hypothesis about the explanatory model which could be, or is, constructed by participant $\langle A_2, \beta\rangle$ at the same *topic* k (and, vice versa, $2(M_k)$ and $2,1(M_k)$, for $\langle A_2, \beta\rangle$.
(and, vice versa, $2(M_k)$ and $2,1(M_k)$, for $\langle A_2, \beta\rangle$.

Thus, Fig. 6.9 depicts a realisation of Fig. 6.4(VII).

The modified operating system is a realisation of Fig. 6.4(VII) and Fig. 6.4(VIII), in which these constructions are alternated in reaching syntactic and semantic agreement. The kinds of agreement are, however, phased distinctly, and the system should not be confused with the hybrid form of Fig. 6.4(XII).

6. OPERATION

Experiments have been carried out with the system described in Section 5 and a simplified version (common modelling facility) of the refined system. The chief importance is to provide a standard condition for group learning on a par with CASTE or INTUITION as a standard condition for individual learning. The systems are quite practicable, but the experimental work must be regarded as a pilot study.

(a) Some (but not all) pairs $\langle A_1, \alpha\rangle$, $\langle A_2, \beta\rangle$ interact to form groups. Once formed, a group of participants appears to have stability due to a fixity effect. Not surprisingly, stable groups learn successfully and benefit from cooperative interaction.

(b) As might be anticipated, the personality (chiefly manifest in the participant's choice and use of descriptors), as well as the learning style and competence, influences the formation of groups which act as P-Individuals in the conversational domain.

It looks as though matched combinations (serialist/serialist, or holist/holist) are more effective and thus are predicted to have a greater chance of being stabilised by cognitive fixity. However, a serialist participant and a holist participant can also coalesce, and a few instances have been observed. The aims of the participants remain distinct, and there is a division of labour in respect of model building and demonstration. Though $\langle A_1, \alpha\rangle$ has accurate hypotheses about $\langle A_2, \beta\rangle$, and vice versa, they do not agree to adopt the same learning strategy, even though each participant knows why the other learns as he does.

7. THE MEANING OF STABLE CONFIGURATIONS WITH MUTUAL HYPOTHESES

Let π_1^1i, π_0^0i denote (as in the first monograph) the cognitive repertoire of a given P-Individual A_1; similarly, π_2^1i and π_2^0i stand for the cognitive repertoire of P-Individual A_2; in each case, the π^1 component is the L^1 component and π^0 is the L^0 component (of Proc^1s and Proc^0s). This notation is extended to cover the mutual hypotheses entertained by the P-Individuals A_1, A_2 (or the participants $\langle A_1, \alpha\rangle$ and $\langle A_2, \beta\rangle$ by the following expedient.

$1(\pi^1) = \pi_1^1 = \{\underline{\text{Proc}}^1\text{i}\}$ in A_1: $2(\pi^1) = \pi_2^1 = \{\underline{\text{Proc}}^1\text{i}\}$ in A_2.

$1(\pi^0) = \pi_1^0 = \{\underline{\text{Proc}}^0\text{i}\}$ in A_1: $2(\pi^0) = \pi_2^0 = \{\underline{\text{Proc}}^0\text{i}\}$ in A_2.

Iterating the notation

$1{,}2(\pi^1)$ = A_1's hypotheses about A_2's L^1 repertoire.

$1{,}2(\pi^0)$ = A_1's hypotheses about A_2's L^0 repertoire.

And, vice versa, for the P-Individual A_2, as

$2{,}1(\pi^1)$ = A_2's hypotheses about A_1's L^1 repertoire.

$2{,}1(\pi^0)$ = A_2's hypotheses about A_1's L^0 repertoire.

The repertoires π^0, π^1, $1(\pi^0)$, and so on are specified "relative to the <u>EntSets</u> of the <u>aim</u> of $\langle A_1, \alpha\rangle$ and the <u>aim</u> of $\langle A_2, \beta\rangle$ insofar as these <u>EntSets</u> have members in common". But, if the participants agree with respect of their semantic interpretations (that is), $1(\Delta) = 2(\Delta)$, as well as the mandatory condition, that $1{,}2(\Delta) = 2(\Delta)$ and $2{,}1(\Delta) = 1(\Delta)$), then if both participants aim for the head topic under the agreed descriptors, all members of their <u>EntSet</u> are held in common. So the disclaimer is not, in practice, as drastic as it seems to be.

Suppose there is a joint semantic agreement and syntactic agreement between participants $\langle A_1, \alpha\rangle$ and $\langle A_2, \beta\rangle$ (with constituent P-Individuals A_1 and A_2). This joint agreement implies the existence of a further P-Individual A constructed in Fig. 6.10. Further, the

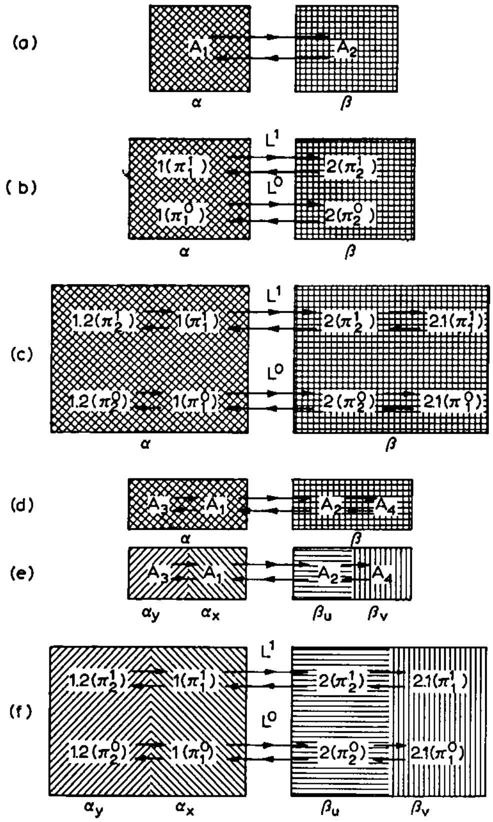


Fig. 6.10. "Conversation breeding". Common meaning agreement may give rise to the construction of further, viable, P individuals insofar as 1's hypotheses about 2 and/or 2's hypotheses about 1 are self replicating. If so, the compilations in α and/or β are partitioned (the notation "α_x, α_y; β_u, β_v"). Key: (a) $\langle A_1, \alpha\rangle$ reaches common meaning with $\langle A_2, \beta\rangle$. (b) Expansion of (a) prior to common meaning agreement representing hypotheses about the agreed topic. (c) Expansion of (a). The hypotheses entertained by 1 (alias $\langle A_1, \alpha\rangle$) about 2 (alias $\langle A_2, \beta\rangle$) and vice versa. (d) Condensed form of (c). (e) Condensed form showing segregation of independent compilations in previously homogeneous L processors (in distinct brains). (f) Expansion of (e).

matching of representative models and hypotheses (Fig. 6.9) is evidence (so far as an external observer is concerned, *the* evidence) for the existence of such a configuration.

The really important point is that $1{,}2(\pi^1)$, $1{,}2(\pi^0)$, and $2{,}1(\pi^1)$,

$2,1(\pi^0)$, respectively, *may also* be self-replicating and, consequently, count as P-Individuals *in their own* right (albeit, compiled and undergoing execution in the same brain α or β, as $\langle \pi_1^1, \pi_1^0 \rangle$ and $\langle \pi_2^1, \pi_2^0 \rangle$, respectively). As a result, a larger P-Individual containing the concepts and memories that are common to $\langle A_1, \alpha \rangle$ and $\langle A_2, \beta \rangle$ is generated by a common meaning agreement between these participants. But there is a converse and equally important result.

If the conversation between $\langle A_1, \alpha \rangle$ and $\langle A_2, \beta \rangle$ is halted, for whatever reason, then an *internal* to α or *internal* to β conversation may take place between the fresh P-Individuals induced by mutual hypothesis-making, and it will be recalled, *some* conversation *must* take place. Finally, conversations of the external or the internal type *must* take place whilst consciousness is maintained (previous monograph, "man is designed to learn"). One reason for truncating a particular conversation (say $\langle A_1, \alpha \rangle$ with $\langle A_2, \beta \rangle$) is that A_1 and A_2 reach common meaning. Or, phrasing it differently, transactions addressed by A_1 to A_2 or by A_2 to A_1 feature as the *provocative* transactions (i.e., such transactions involve mutual hypothesising). From the previous monograph the learning condition can be alternatively stated as, "there must be some (any, in fact) provocative transactions".

Thus, conversations breed conversations provided only that the personally hypothetical structures are self-replicating. The mechanism is sketched in Fig. 6.10 and is dubbed a "conversation breeder" for later reference.

Amongst the other prerequisites for conversation breeding (for example, that personal hypothetical structures are syntactically self-replicating), there is one of special interest; namely, that $\langle 1,2(\pi^1), 1,2(\pi^0) \rangle$ and $\langle 1(\pi_1^1, 1(\pi_1^0)) \rangle$ must have an independent compilation and interpretation in α (the brain or L-Processor), similarly for $\langle 2,1(\pi^1), 2,1(\pi^0) \rangle$ and $\langle 2(\pi_2^1), 2(\pi_2^0) \rangle$ in β. It is thus, perhaps, that distinctions are generated; at least this is one view to adopt about the otherwise slightly arcane notion of "predication" (previous monograph). In Fig. 6.10 the independent portions of the brains or L-Processors are symbolised α_x, α_y, and β_u, β_v. The P-Individuals "bred" by the process are concisely designated by $A_3 = \langle 1,2(\pi^1), 1,2(\pi^0) \rangle$ and $A_4 = \langle 2,1(\pi^1), 2,1(\pi^0) \rangle$. Certainly the process may be iterated within a brain or L-Processor and is limited only by the fact that not all the conditions for self-replication of the "offspring" (A_3, A_4) are satisfied. As a further point, the process has a base definition, "There is a conversation". But this may be an internal conversation, in α for example, obtained by setting $\alpha = \alpha_x$, $\beta = \alpha_y$, $A_2 = A_3$ in the first stage of the process.

One unsatisfactory aspect of the notion "conversation breeding" is lack of any cogent reason why distinct P-Individuals operating as unities in distinct interpretations (α_u, α_v, or β_x, β_y) should come into existence. The question is not absurd; without importing further constraints, there is nothing to prohibit undifferentiated growth, rather than the development of discrete entities. Very similar difficulties beset generative theories in biology and are typified by asking why organisms should be distinct rather than aggregated into splodges like the polyps in a coral reef.

Sometimes it is possible to answer the question on energetic grounds; sometimes this mode of argument is less convincing, even though energetic and spatial considerations surely contribute to the observed segregation of organisms (critical mass/volume ratio, critical efficiency/communication balance, and so on). In all cases, there is recourse also to immunological or genetic incompatibility, both as a special discriminating agent, and as a means of maintaining the biological individuality of an organism during its life span.

By the same token the present difficulty, "Why are there distinct perspectives rather than one gigantic splodge of attention?", calls for similar treatment. One answer is furnished in Chapter 7, Section 4.

8. COMMON MEANING AGREEMENT IN A HYBRID SYSTEM

Since the internal conversations do not penetrate an interface, they are not open to direct external observation. But conversation breeding is not a strange phenomenon. Really, it rephrases the contention of phenomenological and transactional psychology that a "self" exists insofar as there are "others" and that if there is a "self", there must be "others" and that in a slightly obscure sense (though here some clarity is gained), the "self" is "made up from many others".

A more pedestrian, but no less important, interpretation is as follows:

Suppose that L is a *natural language* (Fig. 6.4(XII)). If so, the joint P-Individual A may be *realised*, rather than evidenced, to an external observer. Alternatively, suppose the construction is performed when $\alpha = \beta$, so that there is a *uniform L-Processor* and that participants $\langle A_1, \alpha \rangle$ and $\langle A_2, \alpha \rangle$ inhabit it (the position indicated in Fig. 6.4(XI)). If so, the joint P-Individual A of Fig. 6.10 may also be *realised*, rather than *evidenced*.

Succinctly, the barriers of an interface and a stratified conversational language $L = L^1, L^0$ no longer block certain transactions. Under these circumstances, not only can $1(\pi^1)$ construct $1(\pi^0)$ and $1,2(\pi^0)$, but also $2(\pi^0)$. Vice versa, not only can $2(\pi^1)$ construct $2(\pi^0)$ and $2,1(\pi^0)$, but also $1(\pi^0)$; not only can $\langle 1(\pi^1), 1(\pi^0) \rangle$ construct $1,2(\pi^1)$, but also $2(\pi^1)$; not only can $\langle 2(\pi^1), 2(\pi^0) \rangle$ construct $2,1(\pi^1)$, but also $1(\pi^0)$. The system is self-replicating in its proper conversational domain.

Fig. 6.4(XII) represents a depth interview using natural language (and is the last elaborate construction that captures the essence of such a conversation). Fig. 6.4(XI) is (as maintained in Section 2) the minimal construction for thought. In this case, however, the empirical enquiry can penetrate further into the inscrutable mental activity called innovation; moreover, the enquiry can be conducted without relinquishing the convenience of operating systems that are at any rate partially mechanised.

Part B. Attention

9. P-INDIVIDUALS, THE FOCUS OF ATTENTION AND ONE OR MORE AIM TOPICS

The term *attention* is used ambiguously in some of the psychological literature. The different shades of meaning are probably most obtrusive to psychiatrists with information theoretic training who are anxious to apply measures of signal rate, redundancy, etc., in comparing normal and abnormal behaviour (Thomas 1970), and to educational psychologists eager to employ information processing schemes in the context of full blooded learning and teaching situations (Entwistle 1975). Naturally, we experience similar problems with the present approach, and at this point it becomes necessary to deal with the matter.

Our discussion closely parallels Thomas' (1970) analysis and is not likely to cause much dispute. Psychologists such as James

(1890) or Bartlett (1932) or Kelly (1955) use "attention" for a locus of awareness; the field of attention is the scope of awareness; its content determines the nature of awareness, roughly the usage employed in this book. Thomas has a slightly narrower interpretation in mind (maximising information feedback with respect to satisfying a task criterion in the current environment). Interestingly enough, a similar idea is implicit in Bryan and Harter's (1899) classic paper on the telegraphic coding skill, though measures of selective information were not available at that date.

Two other meanings (at least) are given to "attention". For circumstances under which the respondent receives and processes an input of sensory data (auditory, visual or whatever), it is customary to speak of "selective attention" (the extent to which "relevant" signals are processed and "irrelevant" signals excluded). This meaning is employed by Broadbent (1957) and Treismann (1966) in connection with "missed signal" and "perceptual filtering" experiments, Welford (1968) in the context of single channel operation, and by Tanner and Swets (1954) when discussing receiver operating curves and signal detection theory in general. As an alternative, when there are several modalities, criteria of relevance, or signal sources, the "division of attention" is of primary interest; for example, in studies of vigilance and perception (Broadbent 1971) or in the multiple channel and scanning experiments performed by the authors already mentioned and by Conrad (1954), Poulton (1953, 1960), Mackworth (1959), or Yntema and Mueser (1960, 1962). Under these circumstances "attention" unqualified is sometimes used as an index of the receiver's capacity and flexibility, the number or variety of information channels he is able to deal with successfully. The two meanings "selective attention" and "division of attention" are obviously compatible, and under special circumstances, come into register with attention as a "scope of awareness". Hence, our usage often conveys the flavour of attention as an omnibus term for the overall properties of an information processor, for which Atkinson and Shiffrin's (1965, 1967) scheme (sketched in the Introduction) is an appropriate paradigm.

Formerly, "attention" and "span of attention" were sometimes taken as synonyms for "size of sensory buffer", or "span of apprehension" (digit span or Miller's 1956 "Magic Number 7 ± 1" of "chunks held in immediate memory"), thus making attention a property of the register, or the short-term store, rather than a property of the entire system. This usage is nowadays substantially abandoned. So far as this book is concerned, at any rate, no such connotation is intended.

What are the differences between "attention" as scope of awareness (SAA) and "information processing attention" (IPA)? The outstanding distinction between them is that SAA refers to an awareness or perhaps to a consciousness (with *someone* of *something*), whereas IPA is uncommitted in this respect. In contrast, IPA has a very strong commitment to the input and output operations of the processor, including the function it/he is designed/instructed to perform, whether it/he is aware of the performance or not. Similarly, unless SAA is constrained by the requirement that something (a relation to be computed) exists in consciousness, the respondent's awareness might refer to any inputs/outputs, or to none at all. There are thus a number of plausible situations in which SAA and IPA may be used independently, and under these conditions, the indices attached to SAA and IPA should not be expected to covary.

Surely, most conditions are *not* of this kind; most conditions of immediate concern are *not*. Even so, SAA and IPA still have a modicum of independence. Nobody overlooks this fact. For example, Treissmann points out that there must be a leakage of information around sensory filters (the leakage being part of SAA, though the filtered messages are formulated in terms of IPA), and Sutherland (1964, considering "sensory analysers" rather than "filters") makes a similar observation.

10. ATTENTION AND "PARALLEL ACTIVITY" AS A "PSEUDO-PROBLEM"

In the present theory of conversations we are, however, treading over perilous ground. The <u>aim</u> of the participants was introduced as a surrogate for their attentional focus (in one-<u>aim</u>-at-once conversations) for several reasons; one of them, to avoid confusions which might easily arise if "attention", a more usual term, had not been continually qualified as "SAA" or "IPA" or "so much of one and so much of the other". No great difficulties crop up in loosely equating <u>aim</u> and "focus of attention" (or awareness of goals under <u>aim</u>), provided that only one-<u>aim</u>-at-once conversations are under discussion. The only problem which does appear in this context was considered in Chapter 5, Section 11, but is illuminating enough to bear recapitulation.

For *either* a serialist *or* holist participant (A) the aim topic in the conversational domain is a locus of awareness in one of the following senses.

(a) It is a topic (the maximally distant topic) which A is able to appreciate and describe.

(b) If A is on his own (interacting with the cognitive reflector heuristic B) then the aim topic is a point at which normally asynchronous processes are locally synchronised (the region of synchronicity includes goals in workset under the aim, intermediary topics, and the aim topic itself). If the processes in question are exteriorised by B's action, then "A's awareness" becomes "A's consciousness" (apparently, *with* B *of* aim) and the statement is empiricised.

(c) If several participants (A_1, A_2) are learning, then statement (b) stands, given the further condition that some of the processes which become locally synchronised under a common aim topic belong to A_1 and other to A_2.

Of these clauses, (a) is normative and it appeals to a notion of consciousness (the appreciation of the aim topic).

Even so, the scope of consciousness is operationally determinable to the extent that it is exteriorised in any strict conversation. At the outset, when topic i is the aim, A's awareness is the description of topic i which is given as the basis for the aim validation (to secure $d_0 = 0$). Later on, if aim becomes understood, the scope of A's consciousness is the series of L transactions or L statements that are exchanged with B and lead towards the achievement of an *understanding.*

In contrast, Clause (b) or Clause (c) or both form the basis for a partial mechanistic explanation of consciousness, insofar as (b) or (c) delineate the conditions prevailing at any point in the conversational domain where SAA exists and (by hypothesis) prevailing for any conscious event, observable or not.

In the case of a serialist, for whom goal = aim, it seems easy to equate SAA with aim and to place SAA in register with IPA, since the participant is working on/learning about the (one) goal topic which (usually) is the aim topic. For a serialist having one goal in his workset and one (but a distinct) aim, it becomes necessary to recognise that the content of SAA is broader than that. The participant entertains hypotheses, images, and thoughts other than those proper to the one goal topic, and as a result, it is provident to revise the seemingly easy equation between SAA and IPA for all occasions in a serialist learning strategy whether goal = aim or not. To be conscious of a topic in a learning situation means more than simply behaving sensibly in respect of that topic. We may equate SAA (goal) with IPA (goal) but not SAA (aim) with IPA (aim). When using aim in place of the participant's *focus of attention*, we refer to SAA (aim). There is no need to comment further unless it is pointed out that we have thus contrived a plausible but unusual meaning for "having one thing in mind at once" or "attending to one thing at once".

The behaviour of a holist, however, is more difficult to square up with ready identification between aim and a *focus of attention.* For, in this case, there are several goals simultaneously in the workset. These may be learned about in any order or in parallel, though the learning processes are invariably referred to the current aim topic and in this manner are coupled together and synchronised.

Now, on sound evidence, both from experience and from experimental studies, the most significant aspects of cognition are serial and take place literally one-at-once. There is only one focus of attention (SAA) at once, and (apart, perhaps, from the parallel loading and unloading of sensory buffers) there is one dominant operating channel at once (one IPA). Arguments of this kind are used by Simon (1973), for example, in the context of problem solving, learning and other highly intellectual skills.

The fact that only one event can be reported at once in a protocol is incidental (after all, metaphors, especially poetic metaphors, stand for many events). The curious singularity of mental activity is no artifact of reporting method; it is a deeply investigated phenomenon, the meaning of which is captured best by inspecting tailor made information processing programs such as EPAM (Simon and Feigenbaum 1964, Fiegenbaum 1964), although the same organisation is embodied in most of the larger scale artificial intelligence programs.

It is undetermined (Chapter 5, Section 11) whether the holistic participant, for whom the aim topic synchronises learning over several goals, really learns in parallel or addresses the goal topics in some idiosyncratic sequence. Hence, the holist behaviour in no way denies the general statement of singular mental activity. Nor, of course, does it affirm the statement, but (as a conjecture in the matter) most holists address goals by idiosyncratic scanning sequences, replete with interruptions. If anything can be said on this score, holist behaviour furnishes evidence in *favour* of Simon's view; indeed, the view generally espoused by cognitive psychologists.

The position is summarised in Fig. 6.11 where the goals are associated with specific loci of IPA and so is aim itself; the plain lines stand for an arbitrary (but typical) series of activity initiations; the dotted lines stand for couplings, control interactions, or synchronising operations and may be much more complex (for example, extending from goal to goal). In such an arragement, there is one locus of IPA attention at once with the possible exception of autonomous processes which may overlap if they have determined stopping criteria. There is also one SAA locus of attention at once; namely, SAA (aim) carries an awareness of the process bearing the name of the aim topic. SAA (goal) is not defined, nor, so we believe, may it be defined (it is approximated only, even in the case when the aim topic is the one goal topic).

So far, in other words, conversation theory is in accord with the consensus of informed opinion and the vast majority of observations. At first sight, this conclusion seems to be at odds with the previous insistence that L-Processors, and brains in particular, are concurrent and a priori asynchronous systems. On closer scrutiny, however, the impression of disparity is seen to be spurious. For an aim topic corresponds to the control centre of a stable organisation (a P-Individual), and although an L-Processor is made up from a priori asynchronous parts, the P-Individual is a synchronous system, executable just insofar as these parts are brought into local synchronicity.

Our contention, spelled out in greater detail, is that one P-Individual has one aim and one locus of SAA attention at once; it may or may not have several IPA loci of attention; if so, then one is active at once (with the generally conceded exceptions noted during the description of Fig. 6.11). For a one-aim-at-once conversation, this contention tallies with a statement like "each person has one locus of attention at once" which, with due precautions to

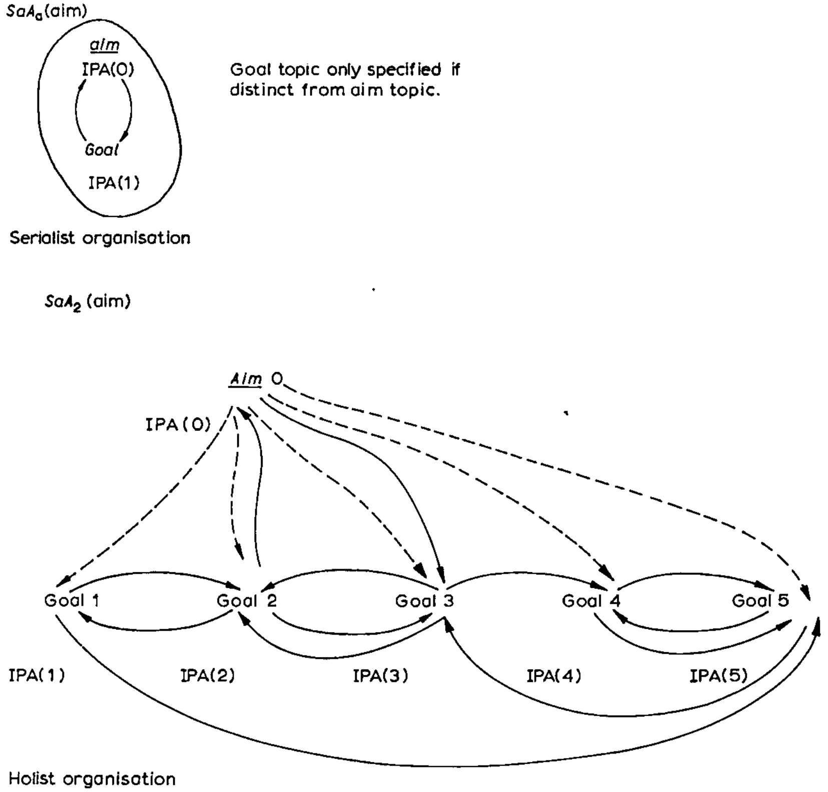


Fig. 6.11. Synchronised execution of mental operations concerned with one aim (the holist organisation and the serialist organisation; of which, in this respect, the latter is trivial). It is essential to distinguish this paradigm from the many aim paradigm, as only the many aim paradigm involves the synchronisation (perhaps partial and local) of previously asynchronously executed P-individuals.

avoid confusion between SAA and IPA, applies for "either kind of attention". For more than one-aim-at-once (two P-Individuals), conversation theory leads to some novel, though not counter-intuitive, predictions, especially in the perplexing case when the two P-Individuals are accommodated in the same brain.

This circumstance might be dismissed as merely imaginary. If you are asked what you are attending to, there is a school of thought (*not* the one-focus-at-once school of cognitive psychology) which maintains you will always reply "*topic* 1" or "*topic* 2" or else "nothing". Without denying the fact that you can be and often are so single minded (the experimental contract of a strict one aim conversation demands this attitude, for example), it is counterfactual and even nonsensical to assert that your reply is always single minded. Could you really attend to "nothing" for instance. Perhaps all you mean by "nothing" (supposing the response is uttered) is that you cannot think of an apposite phrase. Here, the reporting method *does* produce artifacts, "nothing" and various "absurd" topics, just as surely as it *does not* produce artifacts in the earlier mentioned studies.

Again, from a factual point of view, is it possible to have an attentive organism that cannot change its attention? Presumably not, though the argument is complicated by the different usages of "attention". For example, *most* of the "leaks" around Treissmann's filters could be ascribed to regarding "attention" as IPA, and the change from one IPA to another as taking place under the governance of an unspecified SAA. However, if *all* the leaks were of that kind, the SAA mechanism would become a switching homunculus, distinct from or outside the organisation.

More parochially, the formulation of conversation theory holds that the minimal observable event is a conversation (albeit, a conversation taking place in the one brain), and any conversation can be factored into more than one P-Individual (A_1 and A_2 in our pictures). Of these, only one need have an aim topic in a conversational domain, and using the present equipment, only one is fully observable. The operating systems of the next chapter permit greater freedom in this respect.

11. ATTENTIONAL UNCERTAINTY

It is possible to overcome some of the constraints imposed by a reporting language by recourse to the expedient discussed in the previous monograph. There we considered the estimation of degrees of doubt, d_0, d_1, d_2. Of these, d_1 and d_2 specify doubt about *how* to solve a problem and doubt in regard to a set of specified *outcomes* or solutions, given that d_0 is substantially zero; d_0 is an index of doubt about which topic occupies the attention (in the sense of doubt over the current aim topic), and a problem is specified if, and only if, d_0 is zero valued, otherwise d_1 and d_2 remain undetermined.

When the possible topics are displayed (for example, in the entailment structure of a conversational domain), d_0 is fixed most of the time at a vanishing value. But, in between occasions in a conversation, d_0 may (with individual differences) assume a transient high value and typically does so each time the aim is reselected. When the conversational domain is open ended (as it is in the systems of the next chapter), the values of d_0 are more regular (though still individually distinct); quite appreciable intervals are occupied by a state of uncertainty when the aim is undecided.

It is fairly easy to obtain a more telling measure (call it d^*) by calculating an uncertainty index from a confidence estimate over any finite set of topics and permitting bimodal or multimodal (subjective) probability distributions. If $d_0 = 0$, then $d^* = 1$ (for one topic is selected with certainty). Otherwise $1 > d^* \geqslant 0$.

Under these circumstances, participants give the following types of introspective reports upon the occasions when d_0 decreases (and $d^* > 0$), "I saw it" or "I had a flash of insight". On these grounds (taken in conjunction with the theoretical argument already presented), it seems reasonable to suppose that the moments of insight are in register with the coalescing of two P-Individuals to form a (usually larger) P-Individual with a freshly constructed aim topic of which the larger organisation is conscious and is able to describe insofar as the fresh aim is validated.

Some linguistically competent people are also able to report the *process* of coalescence, which in theory should image the construction of an analogy relation. The reports, when they are elicited, turn out to be verbal metaphors and thus *do* designate analogies. For the case in which two P-Individuals co-exist, the most that can be done is to obtain reports (preferably through the sampling arrangements described in this chapter) of one individual's hypotheses *about the other*, in addition to an hypothesis of his own about the *current aim topic.* Apart from this mechanism of describing a dual situation in terms of oneself and another (real or imaginary) participant, there is a phase prior to a coalescence in which the participant is unconscious of the duality and is conscious only of thought.

Such moments, followed by insight (we hypothesise by coalescence), are not much studied and are often believed to be uncommon. The evidence of uncontrolled introspection/retrospection does not support this belief apart from situations where there is a definite task (for example, learning in a fixed conversational domain). Preliminary observations of behaviour in an open conversational domain also suggest that the frequency of insightful incidents is fairly high, and the relatively regular variation of d_0 under these open ended circumstances lends credence to this point of view.

Chapter 7

Innovation and the Operation of THOUGHTSTICKER

1. INTRODUCTION

"Innovation" is used to denote a *process* without commitment to its originality or creative value. Innovation is distinct from learning insofar as it involves the existence of two or more P-Individuals (recognised by the existence of two or more simultaneous aims, or *foci of attention*) that are subsequently coalesced, at the moment of innovation, into one. If the P-Individuals are cognitive organisations in separate human brains, their distinction is in general guaranteed, and their coalescing is signified by an agreement over the common meaning of the topics under discussion. If the P-Individuals are compiled and executed in the same human brain, there is (under propitiously chosen circumstances) an alternation between many aim and one aim behaviours.

1.1. Although the theoretical notions are quite generally applicable, the investigations have so far been confined to the process of course assembly. This limitation is a mixed blessing. On the credit side of the balance, it is possible to recognise configurations in a conversational domain (representing the thesis which is evolving throughout course assembly). On the debit side, there is no means of telling in terms of *content* whether a topic is innovated or recalled. In either case, there is a memory-computation which refers to concepts existing in a repertoire. These concepts are recomputed or reconstructed (like repetitions of the "Indian Ghost" story in Bartlett's (1932) classic study. If the reconstructions are accurate enough, they may constitute recall (of the "Indian

Ghost" story, or equally well of previously known facts).

In terms of content alone, the reconstruction is seldom entirely veridical. It contains fresh elements or fresh combinations of existing elements. On a broader front, consider the "recall" of historical facts (assuming only they are not merely rote learned for repetition, parrot-fashion). Is this recall an innovation or a reconstruction? Does the respondent delve into his repertoire *for the facts* or does he use his repertoire for computing the solution to a historical problem (a gap where some event "must" have occurred, for example, a mode of transport that "must" have existed).

Perhaps the respondent invents a leader because he is told about a movement. Perhaps he recalls "Napolean". Perhaps the respondent "invents" the use of carts and carriages (counterfactually) because the Aztecs were a highly organised civilisation. Perhaps he recalls the passages from Von Hagen (1962) arguing that wheels, though used on children's toys, were never recognised by the Aztecs as mechanically useful. Given that, he may either invent or look up the litter (like a sedan chair) as the conveyance these people must have used. It seems likely that both activities accompany the mental operation, though one or other may be dominant at a particular instant. The whole business of scoring tests and examinations for divergent and convergent thinking is plagued by such ambiguities, which remain so long as content is emphasised.

1.2. From the present point of view, all the mental operations of course assembly are many faceted, and no attempt will be made to distinguish the "recall" of a topic and the "invention" of a topic. Indifference on this score is legitimate until the originality and value of invention come under discussion. Until that juncture the essential distinction is wrought in terms of process alone; whether one focus of attention is involved (which is learning), or whether several foci of attention are involved but coalesced in the process (which is innovation). So far as content is concerned, both learning and innovation have components of recall and invention, often in roughly equal measure.

2. INFORMAL DISCUSSION

If two subject matter experts are engaged in natural language dialogue, expounding a thesis to an interrogator or analyst (as they do during the informal course assembly process described in the first monograph), it is frequently possible to observe incidents that look like innovation and are by hypothesis indicative of innovation.

To illustrate the argument, suppose the thesis bears on the subject matter of energy conversion. Prior to the incident the following configuration exists. One expert is concerned with a subthesis; for example, the notion of heat engines, such as steam engines, that use a temperature difference to harness energy for doing mechanical work. The other expert is concerned with some other subthesis; for example, the "obviously" (to the already knowledgeable) converse case of a refrigerator or a heat pump in which mechanical work is done to maintain a temperature difference. The experts' subtheses generally range over wide and quite different interpretations. For instance, the steam engine subthesis ranges over historical technology, Newcommen and Cawley pumps, Watts mining pumps, marine engines, piston engines in tramp steamers, piston driven railway engines, and Parson's Turbine. The refrigerator subthesis ranges over domestic refrigerators, ice cream carts as improvident users of Freon, ice boxes, and heat exchangers in ecologically desirable dwellings.

It should be evident from these examples that an interpretation means, in this context, a "natural language interpretation". Although it is true that most of the examplars do correspond to an existing or historical actuality, it is certainly not always true that they have the generality they are credited with in the subthesis. For example, though an early Watts steam engine (using atmospheric pressure to drive the piston beam down upon condensing steam) is an instance of steam engines in general, it does not, unless explicated at some depth, illustrate the principles of expansion (piston) engines or the need to employ many stage expansion. Quite possibly, the machine is only mentioned (as a historically existent example of a steam engine) rather than described in sufficient detail to explain what steam engines (this one included) really are. This fact is not at odds with the regulation carried out by the interrogator/analyst to ensure learnability and memorability if the exemplars in question do have a limited explanatory power and are, within the limts of a part of the subthesis, legitimately derived.

2.1. *Observable Mechanisms*

From time to time, the experts, who ultimately are both anxious to delineate a thesis upon energy conversion, feel impelled to explain one subthesis in terms of the other. This is an empirical fact. The innovation originates in the ensuing interlocution which typically includes the following kinds of transaction between the participants (henceforward called Expert 1 and Expert 2, for Subthesis 1 and Subthesis 2 respectively), all of whom are monitored by and interact with the interrogator/analyst as he makes certain that the learnability/memorability conditions are satisfied.

2.1.1. Expert 1 makes an hypothesis about the explanations and derivations given by Expert 2 of all or some of the topics in Subthesis 2; vice versa, Expert 2 makes a personal hypothesis regarding the explanations and derivations of Expert 1, in respect of Subthesis 1.

2.1.2. On the basis of these hypotheses, Expert 1 builds up the explanations and derivations, he believes Expert 2 would have built up for Subthesis 1, and Expert 2 builds up a similar set of postulated explanations and derivations which he believes Expert 1 would have used in delineating Subthesis 2.

2.1.3. If possible, Expert 1 and Expert 2 reach mutual agreement in respect of their interpretations of each others subtheses: a process involving variations to be discussed in Sections 2.2., 2.3. and 2.5.

Insofar as their endeavour is successful, the experts establish a common meaning (in the sense of Chapter 4, Section 9) which is inscribed as an analogy relation in the thesis; the analogy holding between some or all the topics which make up Subthesis 1 and Subthesis 2.

2.1.3.1. The hypothesis building which is performed in Section 2.1.1. may be, to a greater or lesser extent, accomplished before the interlocution. (This in no way means it does not occur; merely, that our linear account of the matter is oversimplified; taken as conceded throughout.) It *is* performed before the current interlocution whenever, as is mandatory in systematic course assembly, the thesis (and thus its subtheses as parts) is displayed in a developing entailment mesh.

2.1.3.2. The hypothesis building which goes on (Section 2.1.2.) above may also be accomplished to some extent before the current interlocution. It *is* accomplished beforehand insofar as there are mutually agreed parts of the entire thesis. These, if they exist, are inscriptions of a common meaning and are analogy relations strictly between subtheses previously constructed by Expert 1 and by Expert 2, respectively. It is sometimes maintained that previously agreed parts of a thesis (as produced by a course team of experts, for example) constitute areas of consensus. This contention is accepted only if consensus is given the coherence based connotation discussed in Chapter 4, Section 7. If consensus is supposed to mean that Expert 1 and Expert 2 (or the body of experts in the course team) solemnly vote upon the nature and inclusion of topics, we deny that any learnable and memorable thesis can be output in this manner. Even if voting or the like is introduced as a procedure, it is quite artificial (it may serve an administrative purpose, but it does not bear directly upon the process under scrutiny).

2.1.4. The matching operation of Section 2.1.3. is precisely the operation described in Chapter 4, Section 8; namely, a coherence agreement is reached regarding a syntactic topic or set of topics such that all interpretations of the topic (those of Expert 1 and Expert 2 in this case) are isomorphic (semantic agreement between the experts). Generally we also require that the interpretations are represented at this stage in the process as models in a common (though lumped) modelling facility. Either this requirement must be introduced or some other means employed for matching verbal interpretations as isomorphic or not.

2.1.5. Assume, as before, that the experts have subtheses headed by "Heat Engine" (HE) and "Refrigerator or heat pump" (RP) and that matching starts in respect to this head topic. To some numinous person, it is obvious that Expert 1 (heat engines) can see a refrigerator as a kind of heat engine, and vice versa, that Expert

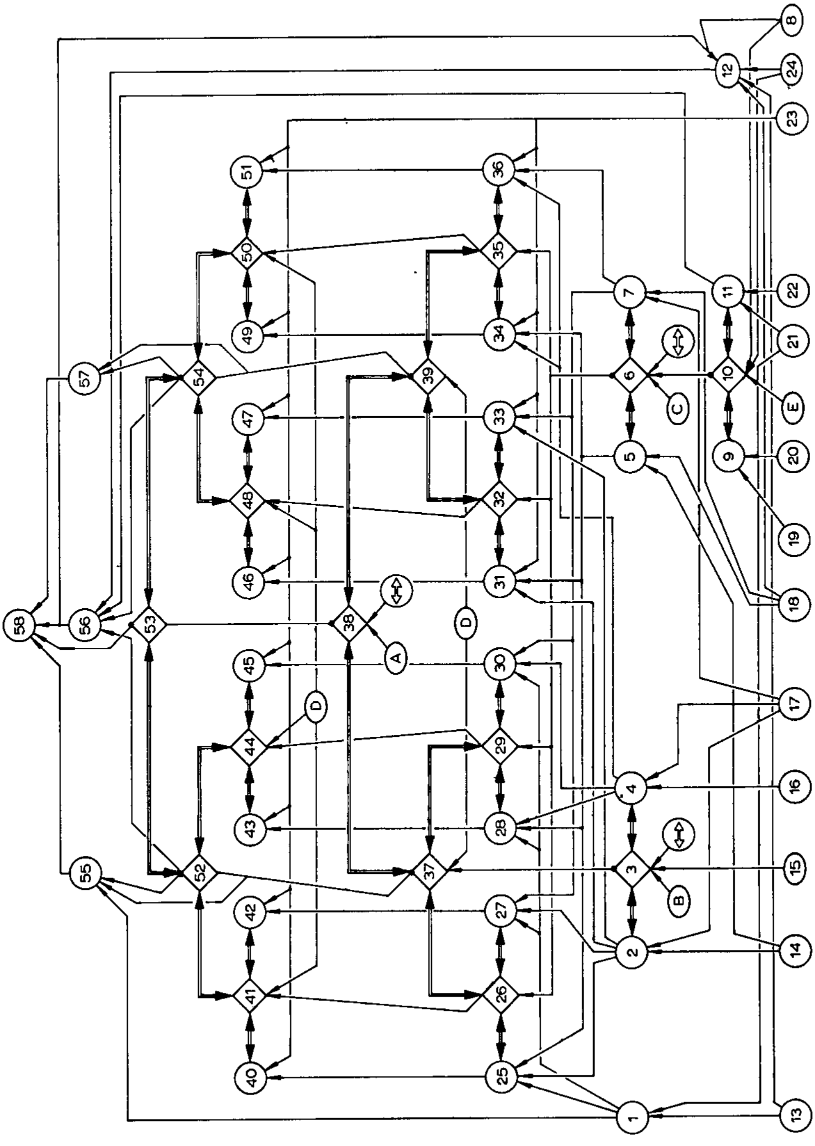


Fig. 7.1. Fragment (between $\frac{1}{2}$ and $\frac{1}{4}$) of a thesis on "Heat Engines". Topics directly concerned with the inversion "temperature difference into mechanical work output" and "mechanical work done to maintain temperature difference" enter thesis chiefly at the analogical relation of node 38 and are not shown. Other topics are listed on the adjacent page with the analogy relation first. The remaining topics are listed in detail apart from the central core (25—36 and 40—51) where they are tabulated under visually clear descriptor values.

Analogies
53, 58: "Temp. Diff. in"/"work out" (left): "Work in"/"Temp. Diff. out" (right)
52, 57, 37, 39, 3; Piston/volume operated cycles (left)
Impeller/turbine operated cycles (right).
41, 44, 48, 50, 28, 29, 32, 35, 6; systems losing working fluid though retaining heat content stored by fluid (left) and systems preserving working fluid (by condensing device or the like) on right.
10: storage on heat in working fluid (left) and storage of mechanical energy (right).
In grouped analogy relations the similarity is identical; the difference depends upon the contexts that are related.

Other Topics
1. Temperature difference/conversion/work done
2. Piston heat exchanger
3. Turbine heat exchanger
5. Working fluid discarded if heat extracted
7. Working fluid in a closed system
8. Degree of organisation
9. Heat storage medium (state change also explained)
11. Storage by inertial medium
12. Temperature as mean kinetic energy and/or level of organisation (potential)
13. Pressure/volume
14. Change of volume/force
15. Cyclic operation
16. Change of pressure/velocity
17. Thermally insulated enclosure
18. Loss
19. Specific/latent/heat
20. Fluid as storage medium (in one state/in liquid/gaseous states)
21. Velocity/force
22. Momentum/mass/inertia
23. Repeated application of energetic transformation
24. Heat (thermal energy)
55. Temperature difference/work output of heat engine
56. Composition of heat pump (refrigerator) with heat engine (possible if energy supplied, impossibility of perpetual motion in mechanical system)
57. Work input/temperature difference output of heat pump
58. Efficiency of a heat engine

Descriptors (determined by an analogical relation): A. "Systems that convert temperature difference into mechanical work" (+) or "work to produce and maintain a temperature difference" (—). B. "Piston Impulsion" (+) "Turbine Impulsion" (—). C. "Lose fluid" (+) "Retain Fluid" (—). D. "Iterated System" (Double or Multiple Expansion) (—) "Simple System" (+). E. "Storage heat" (+) "Storage of Mechanical Energy" (—).

25: A, +; B, +; C, +; D, + (For example, simple steam engine, losing steam)
27: A, +; B, +; C, —; D, + (For example, simple condenser engine)
28: A, +; B, —; C, +; D, + (For example, simple outlet turbine)
30: A, +; B, —; C, —; D, + (For example, simple condensing turbine)
31: A, —; B, +; C, +; D, + (For example, refrigerator losing fluid, e.g., "Freon")
33: A, —; B, +; C, —; D, + (For example, refrigerator with "absorber" fitted)
34: A, —; B, —; C, +; D, + (For example, impelled refrigeration plant)
36: A, —; B, —; C, —; D, + (For example, impelled refrigeration plant recondensing "Freon")
40: A, +; B, +; C, +; D, —
42: A, +; B, +; C, —; D, —
43: A, +; B, —; C, +; D, —
45: A, +; B, —; C, —; D, —
48: A, —; B, +; C, +; D, —
47: A, —; B, +; C, —; D, —
49: A, —; B, —; C, +; D, —
51: A, —; B, —; C, —; D, —

All the examples as given above, except that engine or heat pump operation is iterated to provide a multiple expansion or multiple compression machine which extracts more heat at given temperature difference for doing work or (within limits) vice versa.

The "syntactic" descriptor, "depth," is vertical displacement from the head topic 58 (efficiency of a heat engine). Almost any semantic descriptors may be added; for example, "Marine engines" or "Properties of matter" or "Making up for lost heat".

2 can see a heat engine as a kind of refrigerator; the topics are surely not identical, but there is a very substantial isomorphism between their interpretations. However, the joint requirement (imposed by the course assembly system) that a topic is an explanation and not simply a mentioning or classification of named entities means that the analogy relation (referenced as "Heat Exchange Work Cycle" or HWC) has a syntactic or formal component, which represents the similarity between topic HE and topic RP, and a semantic component, representing the difference by virtue of which HE (heat engine) and RP (refrigerator, heat pump) are definitely not *identical.*

2.1.6. The difference component of an analogy relation either is, or is based upon, one or more semantic descriptors which are stipulated and agreed by the experts. The agreement in this respect may be given many different names as an indefinite number of descriptors could be mustered to establish the required distinction. One distinction made by real experts working upon this subject matter was tag named "*converse*" meaning that the following discrimination can be made. "Heat engines use thermal energy available because of a temperature difference to do mechanical work; *conversely* refrigerators or heat pumps use mechanical work in order to maintain a temperature difference between the energy of two different regions (for example, the ice compartment and the room)." This distinction is shown in Fig. 7.1, alongside several others: the distinction "piston/turbine" proper to an analogy "conversion mechanisms" (CM), and "impeller/volume change" proper to an analogy "circulation of the working fluid" (CWF).

Now, although the agreed semantic distinction, or the descriptor on which it is founded, can be chosen from an indefinite number of possibilities, the choise is *not* unrestricted. The chosen descriptor must serve to discriminate the cases HE and RP under whatever the experts have agreed to be the similarity which is shared by HE and RP. In respect of this syntactic agreement the observed instances are divisible into two quite different categories. These categories amount to the *limiting common meaning agreement* of Chapters 4 and 6 (models are placed in register without the need to modify their formal structure) and the *general common meaning agreement* in which models are placed in register as a result of a formal restructuring.

2.1.7. Agreements of the first kind are rare. One of them is shown in Fig. 7.2. The analogy relation (HWC) is supported by a strict isomorphism; in Rapaport's (1972) terms, this is a "mathematical isomorphism". It is modelled by concurrently executing models for HE and RP, each in its own universe of compilation and interpretation, with the proper couplings or correspondes established. It might also be modelled in a distinct (mathematical) universe, but the isomorphism itself (represented in Fig. 7.2 by "⇔") belongs to none of these universes; it belongs to the universe of nodes standing for topics.

This isomorphism is valid but has a limited range of application, which in turn restricts the meaning attached to the semantic distinction labelled "converse". To see this, notice that most experts (including the pair under discussion) would deny the possibility of perpetual motion obtained by running RP to secure the temperature difference required for the operation of HE and running HE to provide the mechanical work simultaneously needed for the operation of RP. If the terms "temperature" and "mechanical work" and "heat energy" which contribute to the meaning of "converse" are firmed up, it becomes evident that this construction is disallowed.

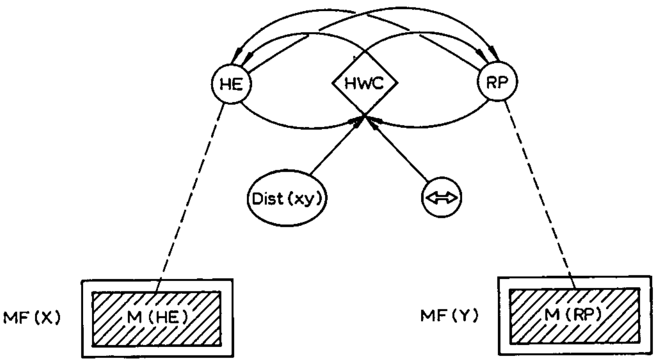


Fig. 7.2. Proposed (and limited) isomorphic analogy between "Heat Engine" (HE) and "Refrigerator Heat Pump" (RP). Models are constructed in distinct and a-priori-independent modelling facilities, *MF*(X) and *MF*(Y). Isomorphism is shown as the operator ⇔. Dist(x, y) is the predicate or set of predicates, distinguishing the universes X and Y.

2.1.8. The other (general) kind of agreement is exemplified by Fig. 7.3, constructed by a different pair of experts. So far as they are concerned the syntactic communality of HE and RP depends upon a construction called "generalised heat work machine" (GHWM), and as the name suggests, this is a generalisation of HE and RP. The most elegant and familiar representation of GHWM is Brillouin's (1953, 1965) information theoretic development of Carnot's cycle. It explicitly involves the notion of "orderliness" of a system (officially negentropy or "disorderliness" for entropy); it also involves the idea of temperature as "noise" perturbing the

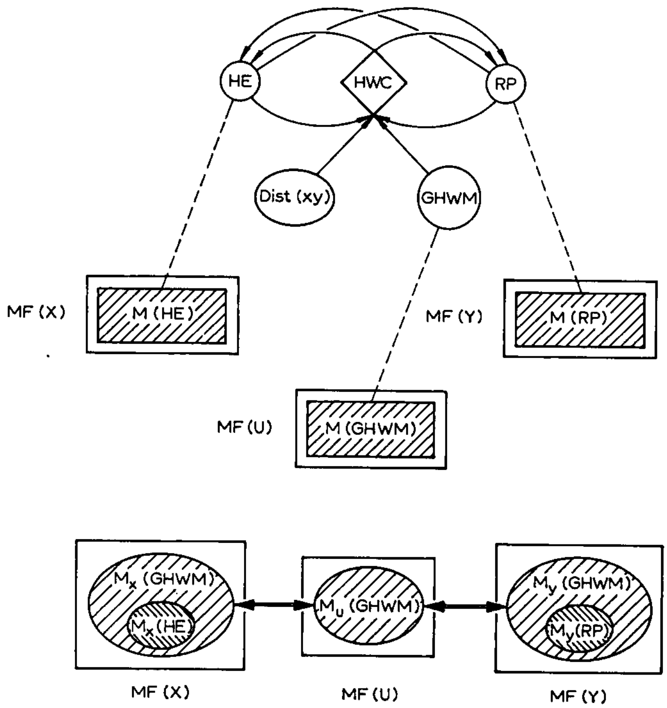


Fig. 7.3. (Above) A generalisation (GHWM) based analogy, connecting topics, HE and RP. Models for all of the topics are constructed in distinct modelling facilities shown as *MF*(X), *MF*(Y), and *MF*(U). (Below) As noted in Chapter 10, a useful material analogy has a further property; namely, specialised model M_x(GHWM) exists in *MF*(X). A specialised model M_y(GHWM) exists in M_Y. M_X(GHWM) and M_Y(GHWM) are isomorphic. M(HE) and M(RP) are not isomorphic, but M(HE) is a subsystem of M_X(GHWM) and M(RP) is a subsystem of M_Y(GHWM).

transmission of negentropy $\triangleq$ information; also of temperature difference as a "noise" gradient: Viewed overall, the operation of a reversible GHWM means that a quantity of entropy is transferred over a temperature difference.

GHWM is an entropy exchange system. It can be modelled in a *distinct* universe of compilation and interpretation and appears as a topic (GHWM) in Fig. 7.3 (above); since the interrogator/analyst insists that *if* the analogy relation is supported by a generalisation rather than an isomorphism $\Leftrightarrow$, then the generalisation itself is modelled as a topic in a Lumped Modelling Facility. Now, say HE is modelled in *MF*(X), RP is modelled in *MF*(Y), and GHWM is modelled in *MF*(U). The analogy relation HWC depends upon the fact that GHWM (in U) can be specialised as a heat engine to yield GHWM in X or specialised as a refrigerator to yield GHWM in Y, and that HE in X is part of GHWM in X and HE in Y is part of GHWM in Y. For notice, in X, GHWM is not the same as HE, though both are the same kind of system; similarly, GHWM in Y is not the same as RP in Y, though both are the same kind of system. GHWM is both more sophisticated than HE or RP and more generalised. If the symbol $\Rightarrow$ stands for "restriction under the constraints of a modelling facility," the situation is summarised in Fig. 7.3 (below).

There is an isomorphism between generalised systems, one realised in the universe of HE and one realised in the universe of RP, and the analogy relation HWC between HE and RP hinges upon *this* isomorphism (shown in the diagram). Further, *this* isomorphism is compatible with any meaning ascribed to the semantic distinction "converse" throughout the entire thesis (which ramifies, incidentally, over energy conversion in open systems, such as living organisms and some chemical reactions, as well as topics to do with elementary thermodynamics).

The act of producing HWC (between HE and RP) supported by a generalised topic (GHWM) will be regarded as a paradigm for innovation. The act of recognising that HE and RP are related by an analogy based upon $\Leftrightarrow$ is also regarded as a valid innovation, but as the limiting case of innovation.

2.2. *Origins of Innovation*

Where did the innovation come from? Our hypothesis in the matter was stated in Chapter 4. It is a consequence of the syntactical generative capabilities that are responsible for producing explanations of heat engines (in $\mathcal{L}_1$ for Expert 1) and refrigerators (in $\mathcal{L}_2$ for Expert 2), conjoined with the requirement of establishing isomorphism.

In particular, there is no need to invoke randomness (presumably, randomness could account for anything) as several theorists propose. Nor it it necessary to invoke prior knowledge of special thermodynamic constructs; we have used accepted names like "Carnot's Cycle" for ease of exposition and because this innovation has also been invented (by someone other than Expert 1 and Expert 2). But, as the argument is intended, HWC is not a regurgitation of some previously well-entrenched concept, it is the result of an $\mathcal{L}_1$ and $\mathcal{L}_2$ production sequence. True $\mathcal{L}_1$ and $\mathcal{L}_2$ are relevant to thermodynamics; they are means of generating "thermodynamic" concepts, but we suppose that the production "Carnot's Cycle" was not previously familiar, at any rate in the context of this subject matter.

In short, the innovation arises from an interaction between P-Individuals (here, between Expert 1 and Expert 2) when a common meaning is constructed. If a common meaning is established, then fresh semantic descriptors are agreed between the P-Individuals (here, the distinction "converse"). The common meaning not only produces an isomorphism, HWC, between models interpreted in universes distinguished semantically as having a positive (+) value of "converse" and a negative (—) value of "converse" but also a further syntactical construction, GHWM, which is modelled (as Model GHWM) in a further universe on which the value of "converse" is * (either undetermined or altogether irrelevant).

2.3. Rearrangements and Revisions Due to Innovation

Recall the further learnability/memorability condition imposed by the interrogator/analyst; namely, that any topic which is instated as part of a conversational domain must be such that other than primitive topics used in its derivation can be derived from the topic in question.

2.4. General Qualification

In order to satisfy these conditions, it may be necessary to revise the subordinates of any topic which is introduced. The entire network, at this stage, has only a tentative status and is open to revision (for example, refinement of topic U in Fig. 7.4).

2.5. Innovation as a Catalytic Agent

Innovation of GWHM and HWC leads to two further kinds of mental activity: one kind engenders a fresh innovation which is often subsequently consolidated; the other is a constructive (though not strictly innovative) act called extrapolation.

2.5.1. Given HWC, it is possible to ask "how" or "why" questions based upon the enquiry, "Since there is a refrigerator that uses no moving parts (the absorption refrigerator in Fig. 7.5), is there a steam engine that uses no moving parts which is not currently exhibited?" The reply to this enquiry is either citation of some conjunct of descriptor values that specifies a cell which currently contains no topic or a denial that such a machine exists.

An affirmative reply is countered by the question, "How does the machine you describe work?" (This is answered by an explanation which, the interrogator analyst will insist, is also derived.) Here the initial reply is affirmative (an historically valid exemplar is the Savery Mining pump invented around 1680 or 1690), and the explanation of its operation (sucking water up a shaft due to the condensation of steam) involves the idea of a valvelike device together with alternating vacuum chambers to implement a cyclic hydraulic process. But, we emphasise, the requisite idea could be invented de novo and has been invented by more than one expert unfamiliar with mining history.

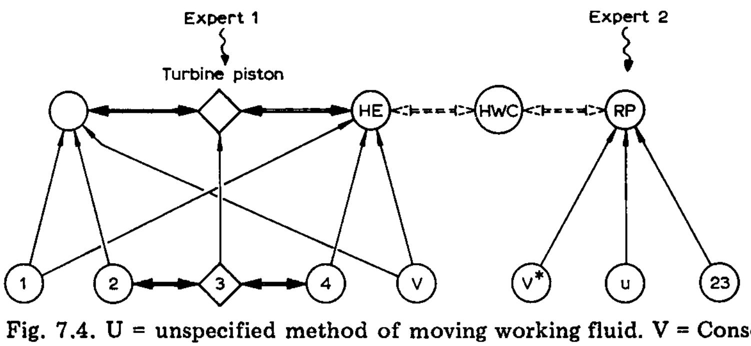


Fig. 7.4. U = unspecified method of moving working fluid. V = Conservation of stored heat (EXPERT 1). V* = Conservation of stored head (EXPERT 2).

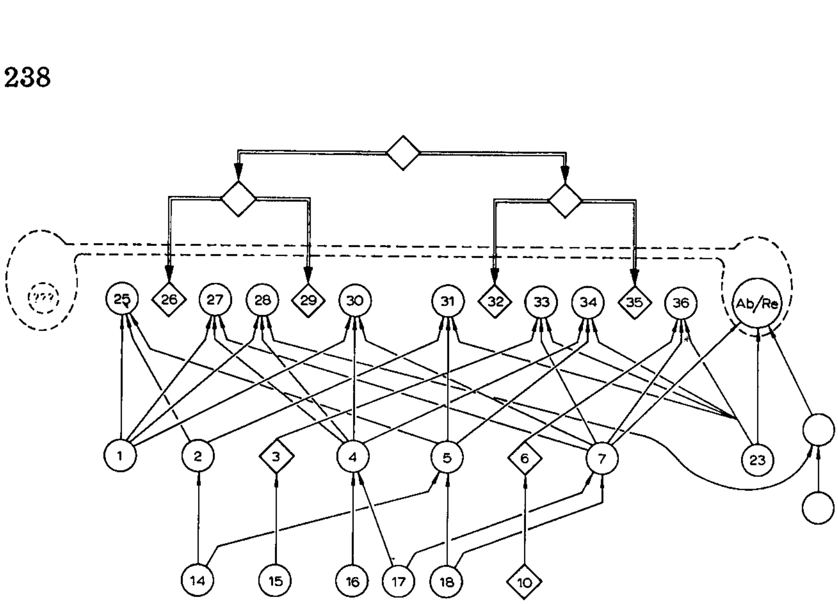


Fig. 7.5. Filling empty descriptor value cells. Proposing derivation of topic to fill empty cell in a descriptor. Since refrigerator Ab/Re has no moving parts, is there any heat engine also having no moving parts? An affirmative reply is possible and one possibility, mentioned in the text, is the Savery and Newcommen mining pump.

A negative reply is countered by the question, "Why not?" This is again answered by an explanation, instated as a topic qualifying all derivations that lead to the analogy. To quote an example culled from later in the thesis, "Since mechanical energy can be converted entirely into heat energy, is there a means for converting heat energy entirely into mechanical energy?" The "why" question emerging from a negative reply to this enquiry is the qualifier, "because there are grades of energy and some irreversible transformations in a closed system". The qualifier refers immediately to the topic "thermal efficiency" which has, at this stage, been introduced and qualifies, either directly or indirectly, nearly all of the topics superordinate in the derivation to GHWM.

2.5.2. These questioning transactions are underpinned by the metatheoretic idea that knowledge is symmetrical; the existence of an isomorphic analogy between two topics implies the existence of isomorphic analogies between symmetrically related topics. Since the proposal is a suggestion or permission rather than a directive, the existence of the companion topic can be denied. Justification of a denial asserts a local complement; namely, a complement with respect to the set of hypothetical symmetrically related topics. The underpinning idea is callled "epistemic symmetry" for reference later (Fig. 7.6).

2.5.3. Call a topic which is reapplied (that is, which makes an appearance in the pruned entailment mesh as the precursor of more than one topic, as in Fig. 7.7) a *principle*. If there is a "principle" it is possible to ask, "What is the result of applying this principle to the freshly constructed topic GWHM?" provided only that the universe of GWHM contains (in the slightly esoteric sense of "may be projected onto") the universe of interpretation of the principle. Similarly, if GWHM is a principle, it is legitimate to ask, "What is the result of applying GWHM to any topic with universe of interpretation that is, or is a projection of, the universe of interpretation of GWHM?"

2.5.4. The idea of generating such (hypothetical) topics, the existence of which may be affirmed or denied by the expert, is called "extrapolation of principles" for later reference.

2.5.5. Extrapolation of principles is illustrated in Fig. 7.8. The principle is composition of thermal or mechanical systems (CS) in order to extract work in several stages (for example, the multiple expansion tramp steamer engine) and is used in the derivation of a topic called "thermal efficiency" (TE).

Extrapolation of this principle (CS) with respect to GHWM proposes the composition of HE with RP; namely, a device, x, that does work in order to maintain a temperature difference and a device, y, that obtains work from this temperature difference. As a first stage construction, this composition is valid though not especially useful. The further composition, whereby y supplies the work to drive device x (and x, as before, provides the temperature difference needed to drive y) is a putative perceptual motion machine. In any veridial thesis (this one included), its existence is denied, and the denial qualifies or augments both the topic "thermal efficiency" (TE) and the topic "reversibility of transformations" (RT).

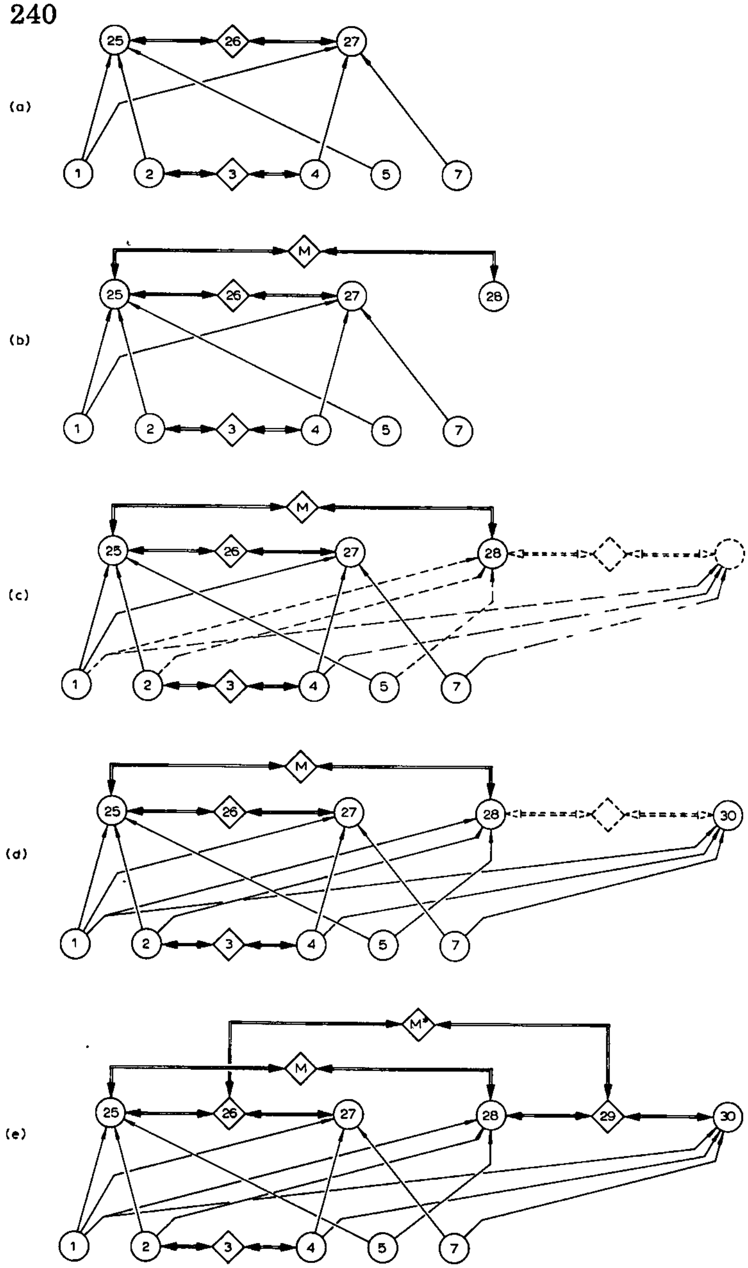


Fig. 7.6. Epistemic Symmetry. (a) Initial condition. (b) Expert builds topic 28 (a turbine which discards all its working fluid) and asserts an analogy relation M between topic 28 and topic 25 (a piston engine that also discards all its working fluid). (c) Analogy instated. (d) By epistemic symmetry substructure and further analogy relation proposed. (e) The proposal instated.

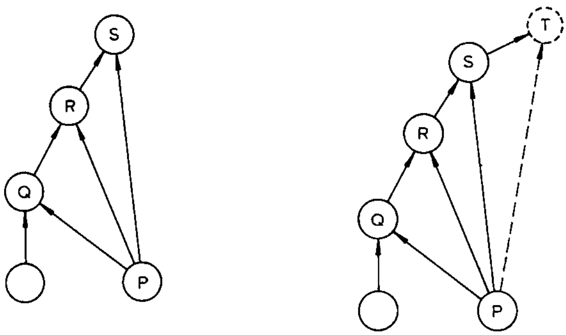


Fig. 7.7. Principles. *Topic* P is used in constructing *topics* Q, R, and S. Thus P is a "Principle".

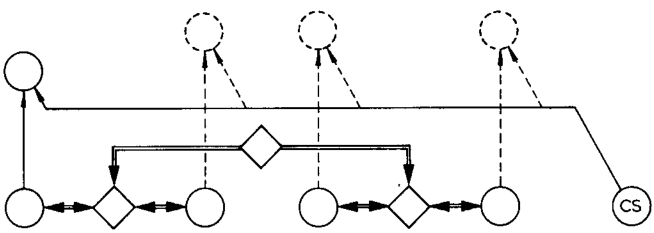


Fig. 7.8. "Extrapolation of Principles".

3. INNOVATION AND COUNTERFACTUALITY

Probably the most powerful and commonly used instrument for major innovation is a combination of extrapolation of principles and the application of epistemic symmetry. Industrial creativity certainly thrives upon this package of operations whether in technical invention (the telephone, the railway, the hovercraft, most semiconductors, the majority of clever chemical syntheses, the television receiver) or in scientific advance (Maxwell's equations, Plank's quantum theory, Einstein's relativity). So, judging from a consensus of commentators, does social innovation. There is little doubt that development in the visual and the dramatic arts stems frequently from this origin.

That is, an extrapolation, E, takes place with respect of a structure rooted in universe X which is analogous to a derivation rooted in universe Y. It is essential to recognise that the constraints upon X (its character as a universe of compilation and interpretation) are determined by the primitive topics in X; similarly, the constraints upon Y are determined by the primitive topics in Y.

If E can be realised or modelled in X (that is, a processor satisfying symmetric) extrapolation, F, is legitimate over Y, and may be realised or modelled in Y, with E isomorph to F.

This may literally be the case (Kirchoff's equations for a resistive network are isomorphic to a packing function for rectangular shapes, applied by March and Steadman (1971) to achitectural design). More often F cannot be modelled in Y, but both E and F belong to a generalisation G (modelled, say, in universe U), and G can be modelled isomorphically in X and Y as well as U (the hovercraft, for example).

However, if neither an isomorphic analogy nor a generalisation based analogy exist, then the construction using extrapolation and epistemic symmetry leads to a counterfactuality which is open to various contextually legitimate interpretations.

3.1. A Case of Counterfactual Inference

A convincing and quickly appreciated example of counterfactuality is given in an elegant construction due to Kallikourdis. It is based upon the well-known "impossible object" shown in Fig. 7.9. This figure may be viewed against many perspectives (for any of which the following comments are quite valid): one of these perspectives is the three-dimensional coordinate geometry of triangles, composed of line segments meeting at points in Euclidean space.

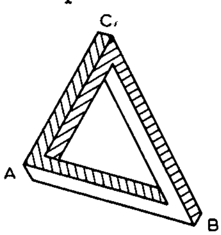


Fig. 7.9. Impossible object (from Penrose, L.S. and R. (1958). "Impossible Objects," *British Journal of Psychology* 49: 31—45).

An entailment structure, E, for a body of knowledge about joining line segments, is on the left hand side of Fig. 7.10 interpreted in a universe X. The structure E comprises nodes 1 to 9 in the entailment structure. The models are shown graphically (to understand the topics it would be necessary to build the explanatory models), and the structure and its primitives a, b, c determine the constraints upon X, i.e., the kind of universe that X is. An extrapolation of E accomodating triangles contains other nodes conjoined, together with E, and called E*.

On the right hand of Fig. 7.10 is a construction, F, for realisable properties of rectangular slabs joined with their faces at right angles. The constructions which can be modelled are shown graphically; and this entailment structure and its primitive topics (A, B, C) determine a universe Y.

The structures E, F, are related by a collection of isomorphisms, shown shaded, carrying lines into blocks, and it may be postulated (since E as we have it determines X, and F as we have it determines Y) that $X \Leftrightarrow Y$, or generally, that X, Y are constructions in three-dimensional Euclidian space; an ordinary and perceptual point of view.

Now consider the following operations. By extrapolation of E in X, a further derivation yields E*. If E* exists, then by epistemic symmetry from E* the "impossible object" (IO) is postulated as an hypothetical "block triangle". Specifically, the hypothesis is that (IO) could be derived from the (internal) analogy or through an extrapolative derivation (both shown dotted). Here is a perceptually obvious form of counterfactuality, since (IO) cannot be so derived unless some or all of the primitives of F are modified, thus altering the character of Y. The price paid for such a modification is that the existing isomorphic analogies between X and Y are falsified.

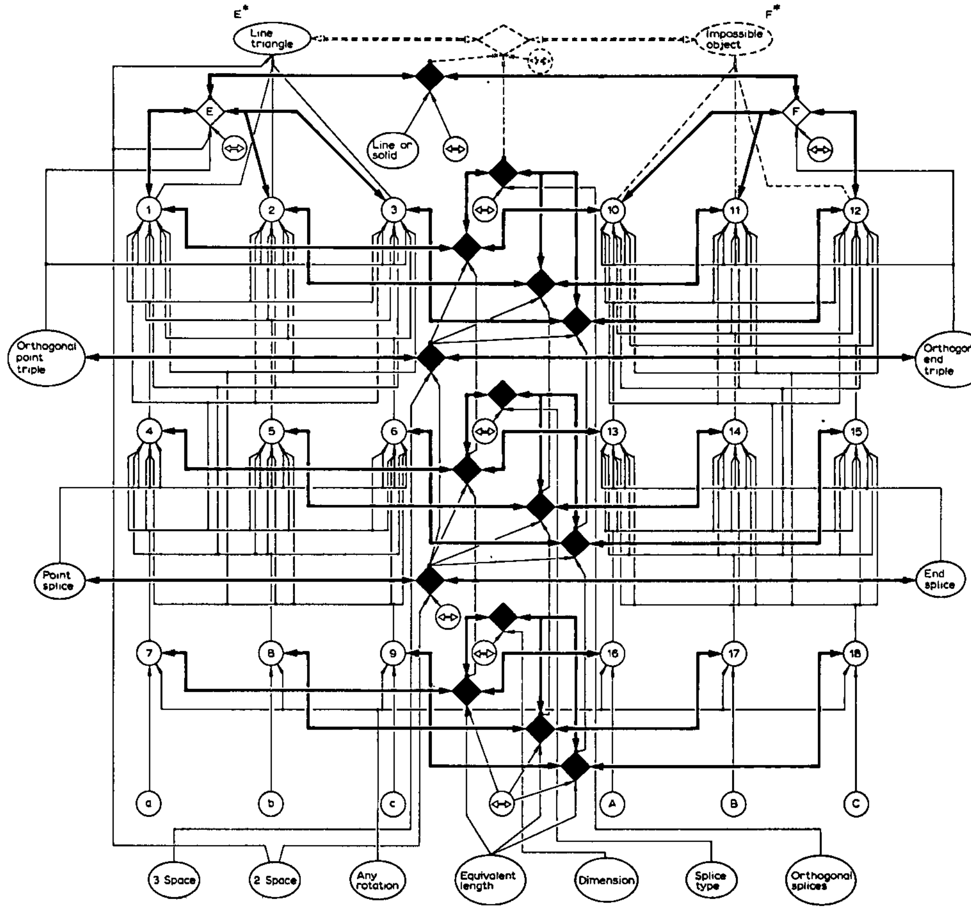
3.2. *Resolutions and Interpretations*

(1) Hypothesis (IO) (impossible object) is falsified with respect to the universe $X \Leftrightarrow Y$.

(2) I may imagine the impossible object (since it is perversely derivable), provided my brain is an L-Processor able to accommodate a generalisation G in universe U (such that a model of G exists in X and Y). But I cannot understand (IO), because I cannot

build an explanatory model in a processor that is constrained by $X \Leftrightarrow Y$.

(3) Rephrasing (2), the impossible object is unknowable, though it may be appreciated as an hypothesis.



E*
F*
Line triangle
Impossible object
E
F
Line or solid
1
2
3
10
11
12
Orthogonal point triple
Orthogonal end triple
4
5
6
13
14
15
Point splice
End splice
7
8
9
16
17
18
a
b
c
A
B
C
3 Space
2 Space
Any rotation
Equivalent length
Dimension
Splice type
Orthogonal splices

Fig. 7.10A. An entailment structure (one of many) for constructing line figures in 2 and 3 dimensional space. E* (the line triangle) is constructible in 2 space and may be rotated in 3 space. So is the analogy E which, under the distinction between solid rectangles and lines, tallies with the analogy F, realisable in 3 space generally as a discontinuous transformation. However, F* is not constructible as an object. Hence, the "Null" analogy between E* and the "imaginary" or "impossible" F* is denied.

build an explanatory model in a processor that is constrained by X ⇎ Y.

(3) Rephrasing (2), the impossible object is unknowable, though it may be appreciated as an hypothesis.

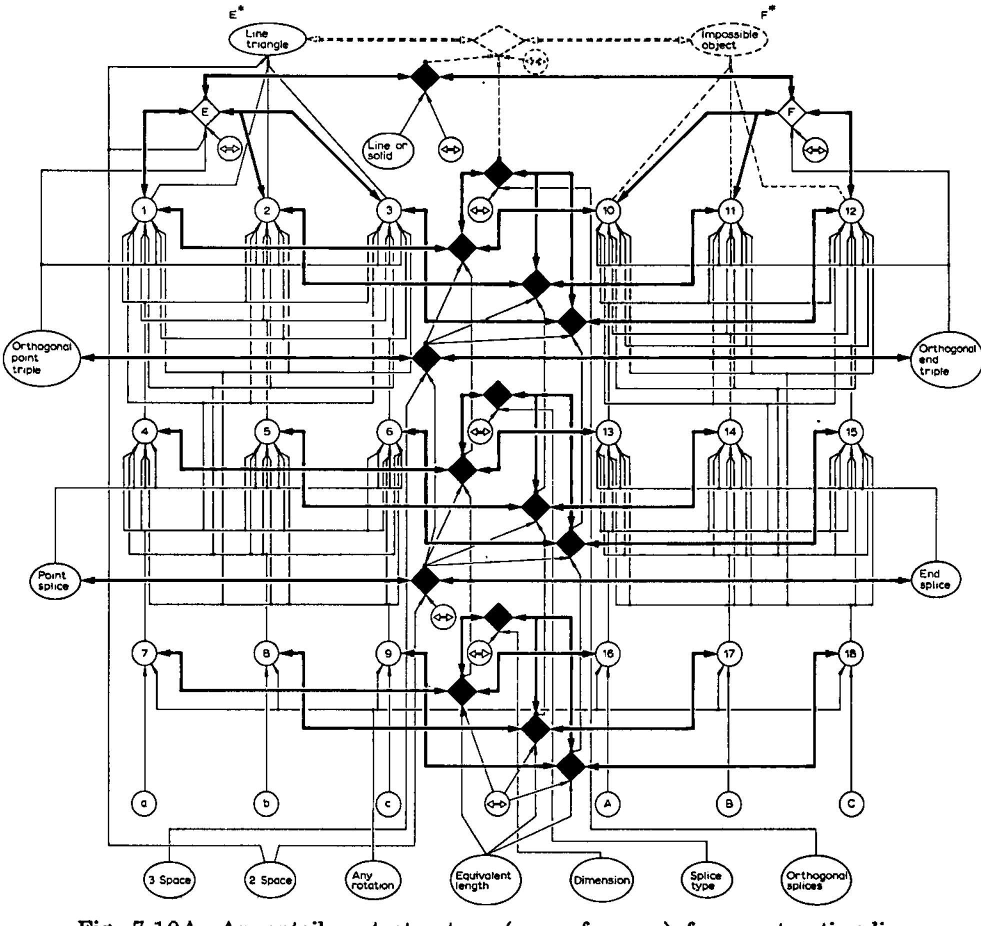


Fig. 7.10A. An entailment structure (one of many) for constructing line figures in 2 and 3 dimensional space. E* (the line triangle) is constructible in 2 space and may be rotated in 3 space. So is the analogy E which, under the distinction between solid rectangles and lines, tallies with the analogy F, realisable in 3 space generally as a discontinuous transformation. However, F* is not constructible as an object. Hence, the "Null" analogy between E* and the "imaginary" or "impossible" F* is denied.

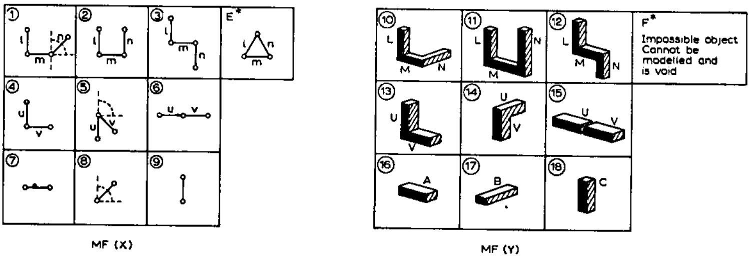


Fig. 7.10B. Representative models in a modelling facility *MF*(X), for line geometry constructions (each figure being rotated), and in a modelling facility *MF*(Y) for assembled and rectangular block geometry: a, b, c = lines; u, v = any pair of a, b, c; ℓ, m, n = any triple of a, b, c. Similarly, A, B, C = rectangular blocks; U, V = any pair of A, B, C; L, M, N = any triple of A, B, C.

(4) Conversely, if my brain is not an L-Processor with the capabilities mooted in (2), then I cannot even imagine the impossible object.

(5) Just as the Necker Cube Illusion (previous monograph) may be perceived as an oscillation between an inward facing and an outward facing image seen at one instant and the next, so the impossible object may be conceived as an oscillation between derivation structures holding tenure at one instant and the next.

If a generalisation G, exists in U to comprehend F* and G*, then this is a hybrid and forms a stable configuration of alternating perspectives.

4. THE INTEGRITY OF P-INDIVIDUALS AND OF PERSPECTIVES

A fresh slant upon the remarks in the last section is obtained by taking in earnest the contention that distinct theses are entertained by different people. The prerequisites for bridging the gap between talk about innovations, illusion figures, etc., and the (present) talk about persons, perspectives, and the like, are as follows:

If the topics in a cyclic and consistent entailment structure (for example, E or F in Fig. 7.10) are realised as a series of concepts and memories compiled and undergoing execution in an L-Processor, then the result is a viable P-Individual; that is, a replicating and stable system of beliefs.

The constraints imposed upon the L-Processor in order that the system shall be compiled are determined by the primitive topics in the entailment structure.

The analogies between two or more entailment structures (each of which contains at least one analogy relation) represent agreements; there are agreements between distinct. P-Individuals, insofar as certain agreements proposed by extrapolation and epistemic symmetry are also counterfactual, giving rise to the denials or instabilities discussed in Section 3.2. Observe that we are at this stage in the discussion taking the verbalisation, "any analogy relation is a petrified agreement," quite seriously; that is, we contemplate its converse, that "an agreement between P-Individuals may be generated by transforming an (appropriate) analogy into the concepts and memories which realise it as dynamic entities". To do so gives substance to the notion that a thesis is necessarily personalised by the person or school of thought originally responsible for its synthesis and exposition.

Now turn to the "pending" remark in Chapter 6, Section 7, where (in the context of "conversation breeding") it was maintained that certain replicative events connected with reaching interpersonal (inter P-Individual) agreement gave rise to generating fresh universes of compilation and interpretation.

The conundrum is, "How can such distinction (α_u, α_v or β_x, β_y) arise inside one L-Processor? "There is no problem if the P-Individuals are associated with spatially distinct L-Processors.

In that connection, recall that the constraints upon an L-Processor, which go alongside distinctions between universes of compilation and interpretation, are determined by the primitive concepts (namely, those corresponding to the primitive topics in a representative entailment structure), and note that such a distinction is not different in kind from the distinction between X and Y, the universes of Section 3.

The situation called the counterfactuality of one thesis in the context of another thesis is precisely the situation which puts teeth into the fission α_u, α_v or β_x, β_y. The characteristic of conversation breeding, "not all agreements (seen as analogy relations if preferred) are possible, " implies the necessity of mooting within one L-Processor (α or β as the case may be) a distinct universe of complication and interpretation; just as the counterfactuality of Section 3 leads either to denial or to the generation of modified universes to accomodate E* and F*, or finally, to a hybrid generalised system G (the "generalisation").

More profoundly, ask why P-Individuals are distinct at all; why people do have definite perspectives, fields of attention, or roles to characterise different replicable systems of beliefs. As a special case, these P-Individuals may be executed in spatially distinct L-Processors with distinct a priori characteristics. In general, the reason is simply that given the characteristics and capabilities of one L-Processor, there are limits imposed by compatibility; that E may be executed with F, or even E* with F, but E* is incompatible with F*, and in any one such system, this extension of the corresponding P-Individual is lethal (unless, of course, G exists to resolve the disparity).

The crunch comes at the point in the argument where topics are to be realised as concepts, and aggregates of topics are to be realised as P-Individuals responsible for generating a thesis containing these topics.

At the moment, the only means of performing this transformation is to ask a student to learn and believe in the thesis. But this is not an end to the matter. As a refinement of this procedure, choose a specially talented kind of student, a professional actor. Ask him to learn and enact and live the part of the progenitor of this thesis. The proposition is not absurd, but it is clear that the actor has greater demands than students have, by and large. He requires not only a thesis specification but a *characterisation*, a personalised thesis. Obviously, such a thing can be provided in principle (authors write plays as well as textbooks), and an embryonic form of characterisation is described in Chapter 10. Moreover in Chapter 11 we set the stage (in one of many ways, perhaps) for the representation of actors, not only of the characters they become.

5. EXPERIMENTAL STUDIES

The phenomena discussed in the previous sections are typical of those reported by other research workers in this field. They also tally quite well with records of introspection on the part of inventors, artists, and mathematicians. Since the examples cited come from a two person situation dedicated to course assembly, a peculiarly "objective" record is left of the "subjective" transformations; namely, the evolving entailment mesh for the entire thesis. Because of that, the innovative process is better controlled than usual and perhaps, as a detrimental side effect, somewhat impoverished. We hypothesise (in line with the construction of Fig. 6.4) that the same processes take place within one human being when he accommodates (or functions as) two P-Individuals, and further propose that a fission of this kind is an invariable concomitant of innovation.

It is natural to ask whether there is an operating system used for course assembly like the EXTEND Program of the previous monograph, in which the innovative phenomena peculiar to one human being can be exteriorised as bits of behaviour. EXTEND itself is inadequate; the one-aim-at-once restriction puts it out of court. There is now an operating system, the THOUGHTSTICKER of Chapter 6, Section 1, in which many aim (and many P-Individual) transactions can take place. Pilot trials show that these transactions do take place, and moreover, are very similar to those described in the informal discussion.

THOUGHTSTICKER serves several purposes. (a) It is a course assembly system and provides realistic aid either to a subject matter expert, in the thoroughgoing sense of somebody well versed in a field, or to an innovator, who is not so knowledgeable, but has a genuine thesis he wishes to develop. (b) The system acts as an "epistemological laboratory". It exteriorises the way in which the expert (under either of these connotations) sets about coming to know. (c) The system is not entirely neutral and embodies not only checking routines but heuristics intended to provoke invention. Hence, THOUGHTSTICKER also has a tutorial function. Insofar as the principles it incorporates are regarded as valid, it teaches the user some of the arts of knowing, thinking, or (maybe, though we are not yet in a position to claim it, positively) innovation.

5.1. Overall Organisation

The basic idea behind THOUGHTSTICKER is as follows. The user makes a model in a modelling facility which consists in several components or subsystems. He sets about the job of delineating and describing a thesis regarding the nature and operation of the model, and thus operates in a course assembly mode with respect to this model or collection of models. This amounts to a *cognitive modelling operation* (as contrasted with the initial *concrete modelling operation*), and in order to exteriorise the process, he is furnished with a *cognitive modelling facility* which we call a *construction grid.* As a result of propounding his thesis about the original concrete model(s) and describing the thesis, he may from time to time be impelled to enlarge the original *concrete model* or to build fresh *concrete models* for topics in the thesis which have no referent. Unlike course assembly, there is no fixed directionality imposed upon the production of concrete and cognitive models; the same description ultimately gives a semantic interpretation to both.

Several embellishments are needed to foster the many aim transactons that are believed to underlie genuine innovation.

There must be disjoint (or many headed) substructures in the developing network of derivations, the thesis representations. Crucially, each substructure must have models that are compiled and interpreted in distinct universes, so that several components are mandatory in the Lumped Modelling Facility. These distinct components will give rise to subtheses that are related by analogical topics with descriptors that act as distinguishing predicates holding the models apart. Moreover, it is necessary to encourage the production of further distinctions of this kind as course assembly (thesis building, cognitive model making) proceeds and as a network is developed on the construction grid.

To accomodate this requirement, it is convenient to specify an initial condition in which there are several disjoint substructures (representing an existing thesis about the original concrete models) to begin with. The concrete model for each substructure exists in a distinct component *MF*(X), *MF*(Y) ... of the Lumped Modelling Facility *MF*. The set of disjoint substructures (henceforward, the *starting set*) is obtained by denuding an existing thesis; that is, by deleting all analogy relations and obscuring descriptors.

This expedient guarantees that the many aim operation is possible and may be exteriorised.

Typically, the user contemplates topics in disjoint substructures of the starting set and instates an analogy relation between them; either one of the analogies removed when the original thesis (unknown to the user) was *denuded* or an entirely different analogy.

In addition, the user may instate topics representing behaviours of models that *he* has built in MF(X) or MF(Y) over and above the models for topics in the starting set; and, of course, he can establish analogy relations between the fresh topics.

Neither this nor any other (known) expedient will guarantee that many aim operation *does* take place, though we shall later introduce heuristics which *encourage* many aim operation.

In order to perpetuate many aim operation (if it is in vogue), there must be a (practically) indefinite supply of spare modelling facilities which will be indexed MF(z); the first z_0 of these are occupied by the models for topics in the initial (disjoint) substructures, and the remainder $(z_{max} - z_0)$ are spare modelling facilities mustered as required by the user (once committed, they cease to be spare).

Since many aim operation has the effect of constructing analogy relations between topics that are differently interpreted (and consequently modelled in different MF(z) of MF), the grid used to represent the thesis has to be laminated. Each lamina, labelled CG(0), CG(1), ... represents a region of analogy relations (Chapter 2), and the original equipment was reminiscent of a cake stand in an old fashioned tea shop (or maybe a railway station buffet). These points are summarised in Fig. 7.11 which shows the several construction grids (one to each region) as layers with the starting set of substructures in Region 0. This arrangement is inconvenient and the current implementation of THOUGHTSTICKER uses a computer controlled graphic display. However, regions and other structural features are preserved both as visual devices and as part of the (computer embodied) data structure.

To each universe of compilation and interpretation there is a distinct component MF(z) (an a-priori-independent processor) which is part of the Lumped Modelling Facility (z_0 of the available components being occupied by concrete models for topics that are parts of the starting substructures). For many purposes where the users *ability* to make a model after the event can be taken for granted, the physical existence of these processors is unimportant, but the logical independence of universes of interpretation is essential and is maintained.

Transformations typical of cognitive modelling, and some concrete modelling also, are shown as A, B, C, D, E, F, in Fig. 7.11. The cognitive model, a developing entailment mesh, is realised by

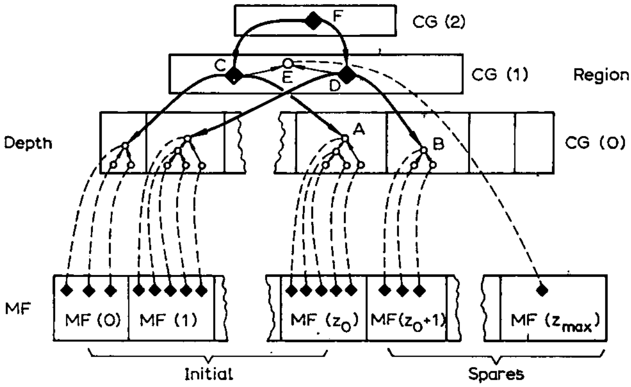


Fig. 7.11. Indication of main features in the construction process. The user has: (i) extrapolated (A) as cognitive model and a concrete model in $MF(z_0)$; (iii) stated a distinct subthesis and the cognitive model (B) with concrete models in $MF(z_0 + 1)$; (iii) has also constructed an analogy relation (C) between topics present at the outset in the starting substructures; and (iv) has made a further analogy (D) between an initial head topic and the head of his novel subthesis. *Topic* (E) is an entailment relation between the analogies (C) and (D) which is modelled in $MF(z_{max})$. Finally, (F) is an analogy relation between the analogy relations (C), (D).

mounting electronic units, which (Chapter 8) stand for topic or analogy relation nodes, on the perspex grids and connecting them together with various links representing simple entailments and analogical dependencies.

Rather simple and visually obvious construction rules apply to the placement and interconnection of the units (these rules are described in Chapter 8). The units themselves contain most of the equipment needed to ensure that the rules are obeyed, and a mechanism for signalling that a unit is either *active* or instated as a node representing a topic.

Apart from this, the main constraint upon the user's construction is as follows: If node i and node j are instated as representing *topic* i and *topic* j, if the user places a unit to represent node k (of *topic* k) on the grid, and if he derives *topic* k (by links or connections) from *topic* i and *topic* j, then he is required to show by construction how *topic* i and *topic* j can be derived from *topic* k (together with other instated topics perhaps). This weak cyclicity condition is checked before instatement is affirmed. * If node k

* As in Chapter 8, more stringent conditions may be used.

represents an analogy relation, then this purely syntactic requirement is modified; instatement depends upon describing the topics related by the analogy so that the descriptor names employed form the distinguishing semantic predicate, Dist, of the proposed analogy relation.

Coexistence of more than one-aim-at-once is signified either by the user stating two (or more) aims under two or more heads (for example, the heads of the disjoint starting structures, which make up one path leading to creation of the analogical transformation C of Fig. 7.11), or by the user marking an existing topic as aim and simultaneously instating a fresh unit. In general, there is more than one aim if there are two or more active markers u, v (either aims or freshly instated units), such that u is not in the entailment set of v (EntSet v) and v is not in the entailment set of u (EntSet u). This condition is quite easily detected, though its occurrence, as noted before, can only be encouraged not guaranteed.

Once the many aim operation is initiated, the resolution of the many aims to form a common meaning agreement (which we believe to be an innovation) is handled by the many person heuristics already discussed in Chapter 6, Section 4 and 5). Here, of course, there is only one user (in general, though THOUGHTSTICKER may be operated with several users also). The trick is to detect a certain kind of many aim situation and to consider the one user with two aims (or more, say, node i and node j) as two P-Individuals (A_1, A_2) or participants $\langle A_1, \alpha\rangle$, $\langle A_2, \alpha\rangle$, such that A_1 aims for node i, and A_2 for node j. The "certain kind" of many aim configuration is a configuration in which there exists distinctive descriptions of node i and node j; that is (as later), the user has assigned descriptors with real (+, —) values on *topic* i which have * (irrelevant) value on *topic* j, and vice versa, has assigned descriptors with real (+, —) values on *topic* j that have * (irrelevant) value on *topic* i. Under these circumstances, if node i and node j are aims, the user, regarded as $\langle A_1, \alpha\rangle$, $\langle A_2, \alpha\rangle$, is in the position of the participants $\langle A_1, \alpha\rangle$, $\langle A_2, \beta\rangle$ of Chapter 6. The user can be asked to agree about the disparity "with himself" or to reach agreement between "his own perspectives" (A_1, A_2) by the exchange grid process, i.e., to adjust the descriptors so that they come into accord (Chapter 6).

Such an agreement, if reached, is a resolution; in practice, resolution is achieved by instating an analogy relation between *topic* i and *topic* j together with additional descriptors having *values* (+, —) of *topic* i and *topic* j, the *names* of which are the distinguishing predicate Dist of the analogy relation. This newly created analogy relation is, as stressed often, the ossification of an agreement, an inscription in the mesh of a resolution act.

If no agreement is possible, the the result of disagreement is inscribed as a conditional analogy (a special kind of analogy denial which represents the coexistence in the same mesh of rival and, at the moment, incompatible subtheses).

Since there is only one mesh and it is accessible to A_1 and A_2 (both A_1 and A_2 are executed in the same brain, α), there is no point in duplicating the representation. We cannot exteriorise and capture all of the agreement process. However, much of it is captured in the revision of descriptor values, the production of a fresh analogy relation, and the addition of descriptor(s) (like those produced in the "exchange grid" process of Chapter 6) which form its distinguishing predicate(s). But, just as we cannot *guarantee* many aim operation, neither can we guarantee distinctive descriptions; only encourage them.

Thus, the heuristic embodied in THOUGHTSTICKER (henceforward the B heuristic) is many faceted. For each node instated, B must require a cyclic derivation and check it. B must pick up some one aim situations and elicit descriptions; it must pick up many aim situations and encourage resolution to yield further descriptions; it must incite the user to many aim operation.

The B heuristic is governed by an executive that continually checks these conditions shown in Fig. 7.12 and sets the proper routines in motion. If there is no aim, it musters routines to procure an aim; if there is only one aim, it musters routines to procure many aim operation. If several aims exist, each one of them is interpreted as the aim of a distinct participant, and resolution is tantamount to agreement between these "participants".

5.2. Data Bank

Quite possibly, the arrangements so far outlined (and refined in the sequel) would serve the purposes of a genuine subject matter expert who has a thesis firmly in mind. If not (and even for the

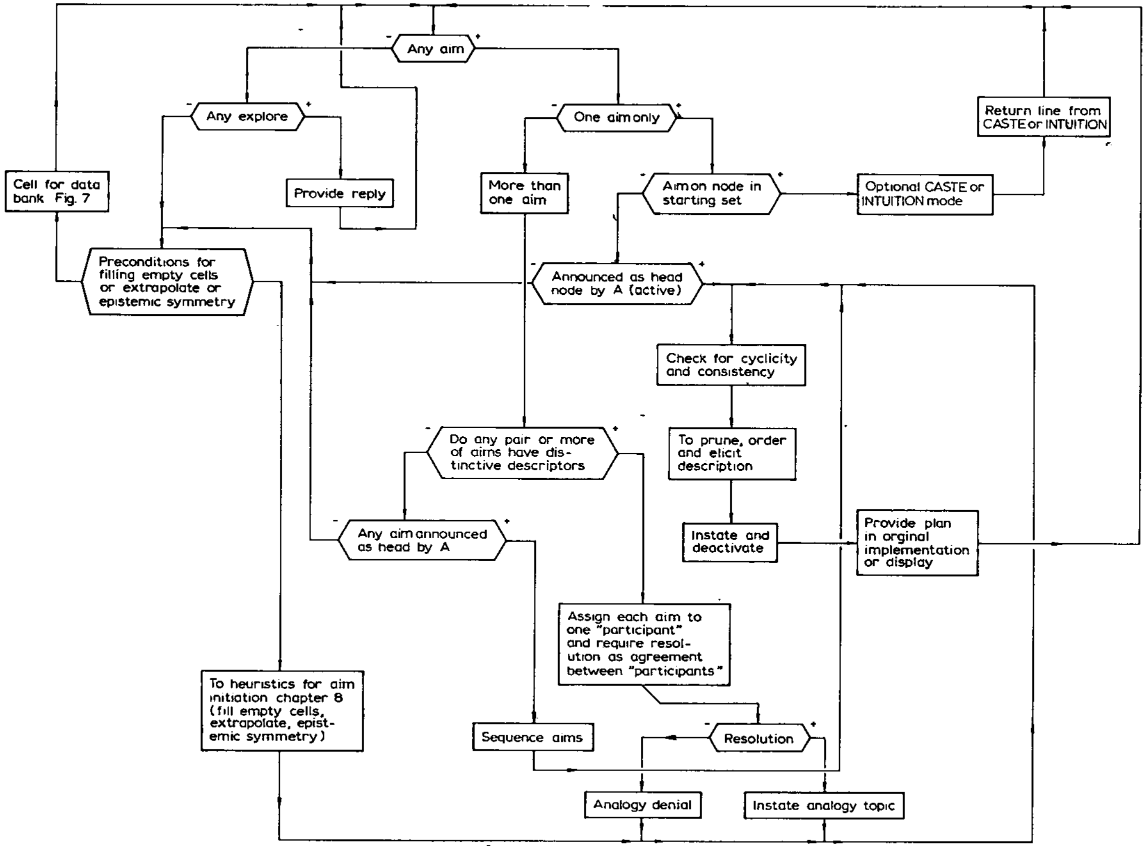


Fig. 7.12. Outline of operating cycle or executive routine.

majority of titular subject matter experts), it is necessary to augment the system by a forcing input of information over and above the information obtainable by executing the original set of concrete models (those attached to the starting set of substructures).

The augmentation is not so peripheral as it seems to be and soon comes into focus as an essential feature of THOUGHTSTICKER. In practice, the forcing input is provided from a data bank, and the data bank consists in an arbitrarily indexed set of computer controlled channels each able to act as a source of information. Channels in the data bank can either be explored (using explore transactions), or failing any activity on the part of a user, information is automatically delivered after an appropriate delay by a scanning routine that is designed to maximise novelty and also revisit channels in which the user has previously shown an interest. At this stage, the channel indices do not form part of the description scheme proper; they are tag names having no semantic interpretation. The information conveyed may even be irrelevant to the user's thesis (though relevance is desirable). If the data channels are relevant, then they become described in due course by the user in his own terms, and this personal meaning replaces the initially assigned index names.

It is often possible to choose the channels so that they have a sensible chance of relevance. For example, THOUGHTSTICKER may operate in the environment of *energy conservation.* If so the starting set of substructures is obtained by denuding the entailment structure of Fig. 7.1, i.e. by removing analogy relations and eliminating the semantic descriptors. Under these conditions, it makes sense to specify data channels as the packs of exemplary material available to a student through explore transactions in a *standard* operating system (CASTE or INTUITION). But it is important to notice that the relevance of this material and the semantic interpretation of the energy conservation topics belong to some other subject matter expert, not a user of THOUGHTSTICKER. Just as the user can piece together the Spartan minded fragments of the original thesis as he likes (by constructing analogy relations between some of them), so also, he may give an entirely different semantic interpretation to the topics (and thus use different descriptions and assign their values as he likes).

5.3. Description Eliciting

Whatever entailment mesh the user builds up on the construction grids as his cognitive model, its topics must be described. The description eliciting routine, discussed in Chapter 2, is used for this purpose (the ordering of the grid laminae to correspond to analogical depth). It is augmented by one additional trick: the *channels* are treated on a par with topics, insofar as any descriptor specified on the topics in the mesh is also assigned values over the set of channels in the data bank.

In order to display the description to the user, each locus in the construction grid (Fig. 7.11) is associated with a pair of light emitting diodes (LEDs) one red and the other green. These are used by the B heuristic to convey information to the user about the values he has previously assigned to descriptors or logical combinations of descriptors (subsets of descriptor values). Further, each cell in the construction grid (Fig. 7.11) is allocated one "attention lamp". The attention lamps are used by the B heuristic for proposing constructions. They are employed in particular as pointers in transactions which encourage many aim operation (based on "epistemic symmetry" and "extrapolation of principles," the gambits exemplified in Section 2).

5.4. Tidying Up the Cognitive Model or Mesh

Suppose the users have somehow been spurred into constructive activity, that he builds up a mesh or network (as a cognitive model) on the construction grids. It is fairly evident that the whole thing is liable to degenerate into an appalling mess. Enforcing the discipline needed to avoid this result would be certain to inhibit free use of the facilities. That, in turn, defeats the object of the system, which is to exteriorise such subtle and transient mental operations as "entertaining several perspectives" (tagged by several aims) and "resolving the differences of perspective by common meaning agreement". The problem is significant, if only because the discipline required to obtain an ordered mesh which can be input to the description eliciting routine of Chapter 2, is very stringent.

5.5. Cycle of Operations

The tidying up operations needed to keep order are simply a combination, in sequence, of the pruning, ordering and depth numbering routines of Chapter 2, executed with respect to any head node specified at a point of resolution. These programs are executed as part of the cycle outlined in Fig. 7.12 (the executive routine).

Using the older implementation with physically distinct construction grids, it is only possible to output a plan of the revised and sorted entailment mesh. The user is required to follow this plan, dismantle his construction, and rearrange it. The recently implemented system performs this chore (within limits) on the user's behalf and displays the result.

Chapter 8

Modus Operandi and Means for Encouraging Innovation

In the following sections we shall consider THOUGHTSTICKER transactions in enough detail to bring out some points of epistemological interest, and to give an overall impression of the system. The discussion of the previous chapter is extended to indicate the main construction rules and to describe the transactions (based upon "epistemic symmetry" and the "extrapolation of principles") that are used as means to encourage many aim operation and innovation by the user.

Although THOUGHTSTICKER is a versatile system (the flamboyant phrase "epistemological laboratory" is not intentionally misleading), it has so far been used chiefly in connection with the environment of "Energy Conversion, Conservation and Regulation" (the subject matter for the examples in Chapter 7). To a lesser extent, THOUGHTSTICKER has been brought to bear upon an environment "Entrainment of Oscillators".

1. MODELLING FACILITIES FOR CONCRETE MODELS

The Lumped Modelling Facility for energy conversion is the standard modelling facility (on a par with STATLAB in this subject matter field) which is used for an ongoing tutorial project, together with patch-programmable analogue computing elements over and above those incorporated in the standard design (Fig. 8.1). The state of all analogue units (integrators, adders, multipliers) is traced by the LSI machine which acts as regulator. Similarly, all structural and patch-programmed connections in the

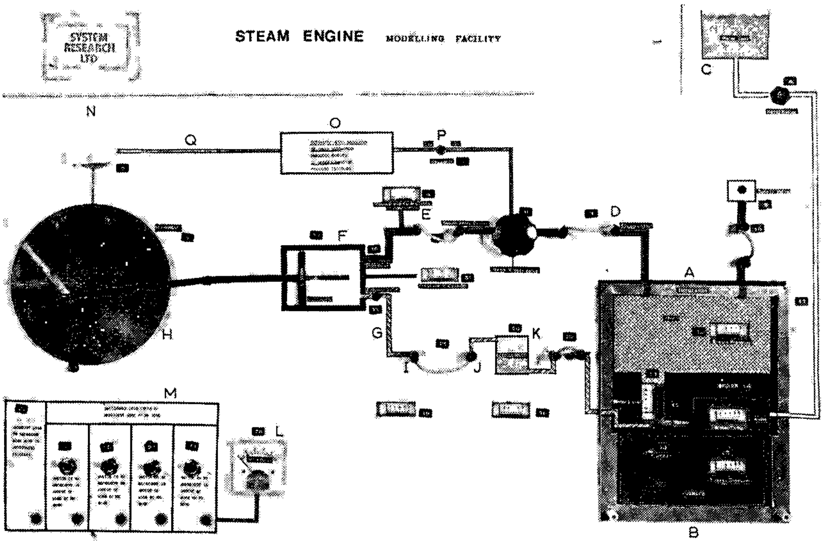


Fig. 8.1. Modelling facility used for heat engines (companion units permit modelling for refrigerators and abstract thermodynamic systems at arbitrary level of sophistication; this equipment is primarily intended for tutorials with children but is regulated by the same analogue computing circuits and differs only in the labelling). All parts and connections are working units. A = Boiler, working fluid temperature and pressure meter, safety valve. B = Heat source. C = Source of working fluid. D = Steam exit pipe with working linkage. E = Piston and inlet pressure/temperature meter. F = Inlet valve display. G = Outlet valve display. H = Fly wheel (mechanically working, can be turned manually or by the mechanism). I = Outlet pressure/temperature meter. J = Condenser link. K = Condenser and return of working fluid. L = Work done meter. M = Load switches. N = Velocity meter. O = Governor. P = Connections (manually adjustable) for governor. Q = Display of an information linkage.

standard portion of the apparatus are traced automatically; other model structures may be input manually by the interactive console. The facility can be used to model heat engines, refrigerators, and the like, together with information transfer.

The Lumped Modelling Facility for "oscillators" is a good deal less elaborate. It is simply a kit of parts (old relays, weights, semiconductors, springs, thermistors, etc., an odd but profuse assortment) which can be used for making oscillators. Both structural and behavioural data must be entered manually; no tracing is attempted.

These ad hoc arrangements suffer from obvious and irritating defects. Ideally a user should construct and enlarge a lumped facility as required to accommodate the models he wants to manufacture, and his subsequent modelling operations in one component of the lumped facility should all be computer interpretable and constrained by the models already built. As it is, only the first of these requirements is fully satisfied. True, so long as the system is an experimental tool, these deficiencies are no more than a nuisance, on a par with the chore of copying out a revised and tidied version of the cognitive model (the mesh on the construction grid). But, in contemplating wider types of application, it is crucial to notice that the existing constraints are inessential.

Mechanically speaking, all the conditions for manufacturing "spare" modelling facilities can be implemented, and several slightly context dependent examples are in existence. Papert's (1970) LOGO was noted in the previous monograph as a paradigm mathematics laboratory, and the system could be modified slightly to accomodate the distinction of differently constrained universes. A further instance is a suite of interactive graphic manipulation programs originally designed for an art school and currently used for modelling in chemistry (at a plethora of different levels: molecular, atomic, quantum mechanical, etc.), which permits the user to make and retain "spare" modelling facilities (De Fanti 1975). One further example, is Negroponte's (1970) "Architecture Machine" which permits similar inventive liberties.

The issue of practical feasibility is very important, for without a means of giving users (who are *not* versed in programming) access to freely constructed "worlds," the system would remain no more than an experimental tool of limited value. The fact is that means exist, and though they are currently quite expensive, their cost is likely to decrease very rapidly as computer technologies come to fruition.

2. THE CONSTRUCTION GRID AND THE COGNITIVE MODEL

The arrangements for building up cognitive models and entailment meshes are currently implemented using a graphic display (Fig. 8.2) and a sketch pad input augmented by a keyboard. Previ-

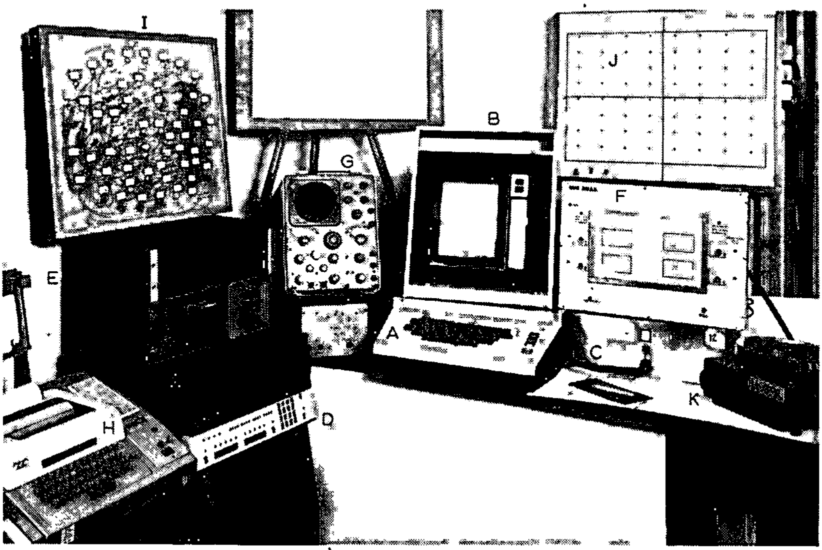


Fig. 8.2. Current realisation of THOUGHTSTICKER using graphic display tube (ARDS terminal) and auxiliary equipment. A = ARDS Terminal, keyboard and display tube. B = Sylvania graphic tablet. C = Control equipment for graphics tablet. D = Minicomputer (LSI 2 with 24k core storage). E = Digital magnetic tape backup unit. F = Mini BOSS for aim validation. G = Display oscilloscope for modelling facility. H = Teletypewriter console. I = Back-up display and modelling grids. J = Auxiliary display. K = Projector.

ously, the mesh construction was realised with certain limitations by using physical construction grids and physically placed electronic modules connected together by the user.

The previous arrangement gives a clear picture of processes which are now carried out automatically and as a result of which images are displayed. The system will be described in these terms and carries over into the current implementation, with the following caveats only.

(1) Node unit positioning refers to pointing operations; (2) connecting operations refer to key tagged link drawing operations; (3) displays, both of descriptor values (LEDs), and signal lamps (active node, and so on), are replaced by graphic conventions; (4) separate construction grids correspond to displaced tube locations; (5) regions are represented by a dashed line (quasi 3 dimensional) display.

The display tube can represent only a fairly small mesh (or part of a mesh at once) but can be augmented by concrete construction grids for representing relatively unchanging portions of a mesh. However, the mesh can be repruned under any head role (the heuristic of Section 3.2(f) is realisable), and the resorting of topic nodes according to the computer generated plan is automatic for all nodes displayed.

The programs governing the operation of THOUGHTSTICKER are under continual development: listings of the existing programs and their updated versions are available on request.

2.1. General Framework

The grids (one to each region as in Chapter 7) have modular cells associated with node positions (to be filled by the user), LEDs for exhibiting the values of semantic descriptors, and "attention lamps" via which the regulating heuristic B can bring the user's attention to one or a cluster of cells. Recall that the channels of the data bank are also associated with their own LED displays and are "tag name" labelled, but not ordered, under the (syntactic) depth descriptor.

2.2. Starting Set

The starting set of substructures is built up on the construction grid for region 0 (namely $CG(0)$) using modules (Fig. 8.3) identical with those employed in CASTE. Each module retains and displays the value of explore, aim, goal, and understand by means of flashing light codes based on three signal lamps. The data base (computer) inscription of the starting set modules is indexed by one family of descriptors (sufficient to access the topics): values being LED displayed on demand. Topics are accessed (as in CASTE, previous monograph) by specifying descriptor values via the interaction console. For the "conversion and conservation of energy" environment, the starting substructures are obtained by denuding the entailment structure in Fig. 7.1. At the outset, a user is faced with just these structures, and whilst he learns about the topics they adumbrate, his behaviour is regulated as it would be in a CASTE or INTUITION operating system.

However, the starting substructures do not delineate a full thesis

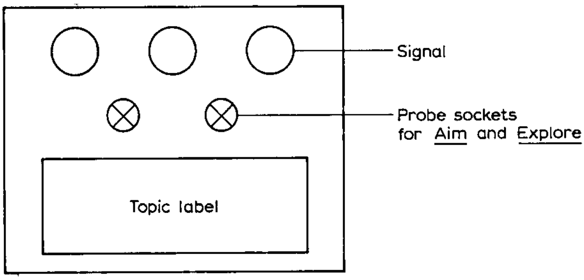


Fig. 8.3. A module used to represent a topic in starting set of substructures or a permanently instated node. Connections are specified in the computer and sketched as lines on display. As in an other-than-evolutionary operating system (CASTE or INTUITION) explore, aim, goal and understand markers are indicated by signal lamps. Sockets for inserting aim and explore probes are optional since topic may be accessed by descriptor values.

on the "conservation and conversion of energy," and the denuded fragments of the original entailment structure are deliberately truncated to secure this condition. As a result, the user can make *more* concrete explanatory models in the Lumped Modelling Facility than those attached to topics in the starting substructures. The possibility of constructing analogy relations is an obvious consequence of denuding the entailment structure. But it is practically important that topics other than analogy relations can also be invented.

2.3. Building Up the Cognitive Models

Apart from the starting set of substructures and the associated grids, the user has available an unlimited supply of electronic boxes and connecting links. As a matter of convenience and representational economy, the boxes are of several different kinds: (a) Units representing topics that are derived without analogy; (b) Units representing analogy relations, and representing topics of mutually exclusive and conditional hypotheses. Since all units stand for nodes in an entailment mesh, units are henceforward glossed as nodes: topic nodes, analogical nodes, and conditional nodes. Similarly, the links are classified as follows: (A) Unidirectional black links, representing an other-than-analogical derivation; (B) Bidirectional orange links, representing an analogical derivation; (C) White links, representing the "syntactic" component (isomorphism "⇔" or a topic) which stipulates the similarity in an analogical topic relation; (D) Purple links, representing the names of semantic predicates, Dist or the difference in an analogical topic relation; (E) Brown links, representing a conditional derivation; and (F) Speckled black links, which have no functional distinction from black links but are useful in visually discriminating several derivation paths.

2.3.1. To instate a fresh topic T which is simply derived from existing topics P and Q, the user takes a topic node (Fig. 8.4), labels it with the name for T, and inserts it into a position on a grid. This operation illuminates the active lamp on node T (Fig. 8.4). The user next connects the output of P and Q, though black links to one of the input clusters (maximum of three) on node T. Each cluster is a kernel of T (first monograph), and it may have at most six members. If P and Q are sufficient entailment precursors (in

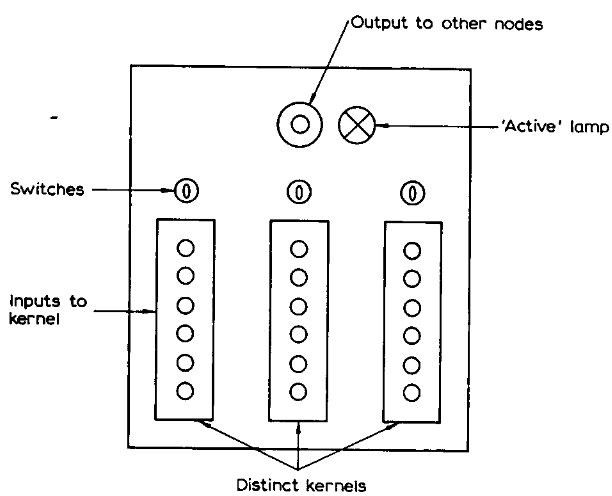


Fig. 8.4. Topic node. Each node is a "Box" with inputs (black lead) for a maximum of 3 kernels or conjunctive derivatives: each derivation being at most 6 sub-ordinates. The "active" lamp is illuminated if the node is positioned on the grid and is extinguished if the node is instated. The switches indicate that proposed derivation from more than 2 but not more than 6 other nodes is complete (the kernel in question is full). An insertion of a fresh derivation lead into any vacant kernel, reactivates the "active" lamp.

one kernel) of T, the user turns the switch (Fig. 8.4) on this input cluster. This operation signifies that node T is submitted for consideration by the regulatory heuristic. Amongst other things, a model for T must be constructed (in the processor associated with the grid on which T is mounted) before the submission can be accepted, and until this model has been successfully executed, T will remain active. However, the model could be, and commonly is, constructed and executed before any attempt is made to instate T.

If P and Q are interpreted in the same universe of compilation and interpretation (Fig. 8.5 on left), the account is complete If P is interpreted in one universe X and Q in another Y (when P and Q are in separate substructures), then, in respect of the model for T, these universes are no longer independent. T unites X and Y; a priori independence is modified by the topic instated (Fig. 8.5 on right).

2.3.2. In order to instate a further derivation of an existing topic R from existing topics P and Q, the user connects black (or speckled black) links from the output of P and the output of Q into one of the unused input clusters of node R. The act of applying input connections to an unused cluster gives R the status of active. The user next presses the switch on the input cluster and submits his fresh derivation for scrutiny by the heuristic.

2.3.3. To instate an analogical relation between topics (either existing or due to be constructed), the user positions an analogical

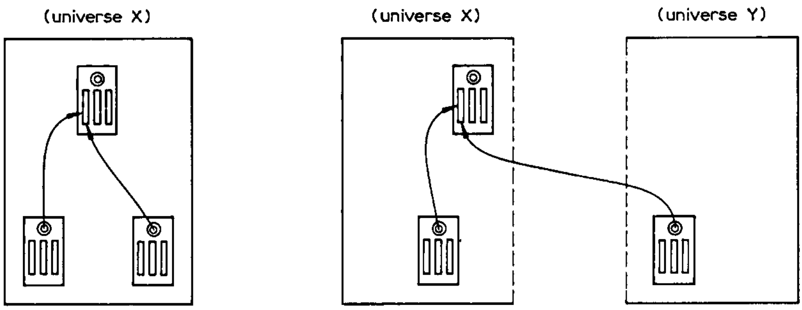


Fig. 8.5. Derivation of *topic* T (at a topic node) from *topics* P and Q. On the left, the derivation is confined to one universe of interpretation (X): on the right P and Q are in distinct universes of interpretation (X, Y), which become related as a result of instating *topic* T.

node (Fig. 8.6) on one of the reserved grids. As a result, the node becomes active and remains so until certain inputs are furnished, though they may be furnished in any order whatsoever. First (though not necessarily in order of appearance), there must be orange connecting links from existing or yet to be instated topics in different universes $MF(X)$ and $MF(Y)$, which form the terms of the analogical relation. Next, there must be a white link from an existing or yet to be instated topic, which is the similarity of the analogy. The universe of compilation/interpretation of this (simi-

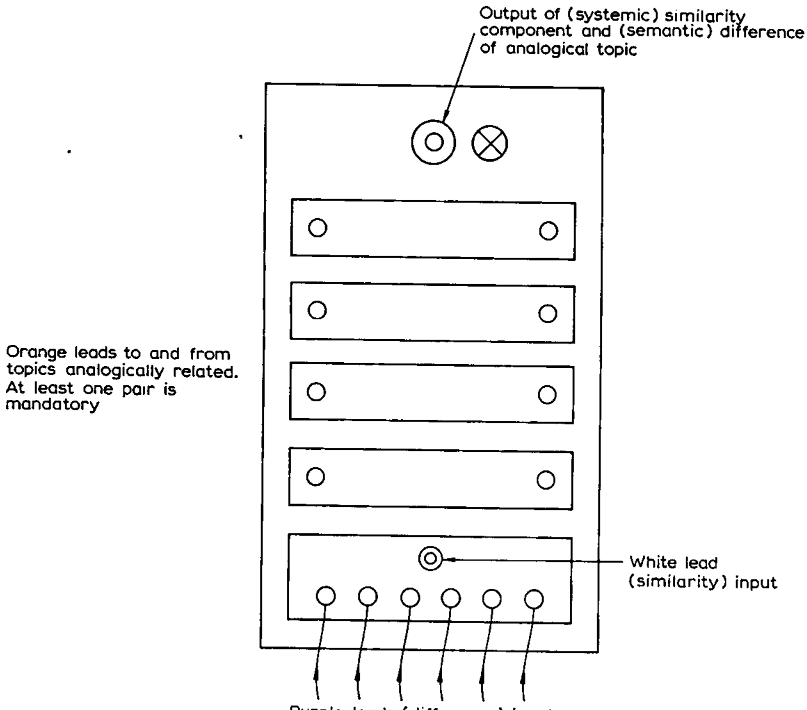


Fig. 8.6. An analogical node. This node must receive inputs (orange leads) from nodes in at least two different universes of interpretation X, Y (either partially, or completely, distinct derivation-linked substructures): An input (white lead) either from a topic indicating similarity of analogy, or from an isomorphism socket, and a (purple lead) input from either a topic or sockets labelled as semantic descriptors. Both orange and purple leads may be multiple (maximum of 4 orange and maximum of 6 purple).

larity) model may either be the union or the product of X and Y, or some distinct universe; it is a generalisation of the models for the analogically related topics.

In case the analogical relation is a strict and complete isomorphism, the white link may emerge from a special socket labelled *isomorphism operator* $\Leftrightarrow$. Finally, there must be a purple link from a topic or from one of the user labelled sockets representing free semantic descriptors, which are named as part of the description routine. The purple link thus signifies a so far unnamed difference Dist (x, y) upon which the analogical relation is based.

The user may press the submit switch whenever he has specified the collection of terms (topics he regards as somehow analogous), but the analogy relation is not adjudicated for legality until the various inputs are filled out. Fig. 8.7 shows typical completed analogy nodes, but at the risk of tedium, we stress that analogical nodes can exist (in an active state, of course) long before all the inputs are filled up.

2.3.4. The conventions built into the THOUGHTSTICKER system are deliberately pedantic. (The pedantries are justified insofar as THOUGHTSTICKER gives useful training in applied epistemology, as well as acting as a course assembly system.) According to these conventions, analogy relations hold between topics in distinct universes of interpretation (which is correct, though unduly fussy for ordinary purposes). Difficulties are thus encountered in dealing with analogies loosely said to hold between topics in the same universe.

For example, suppose it is desired to represent the isomorphism between graphs (or finite automata) F and G. As a general statement, there is one universe, $\mathcal{U}$, of graphs (or finite automata), a universe of the same kind of mathematical objects. However, the particular objects F and G cannot be simultaneously executed in the same independent and serial processor (as required if they are said to be analogous). They could, of course, be simultaneously *simulated*, but that is a very different matter; their *realisation* is actually called for. Hence, a user anxious to instate and model the F, G isomorphism must construct topic F as a node in one grid X and model it as M(F) in one a-priori-independent part *MF*(X) of the Lumped Modelling Facility; construct topic G as a node in an-

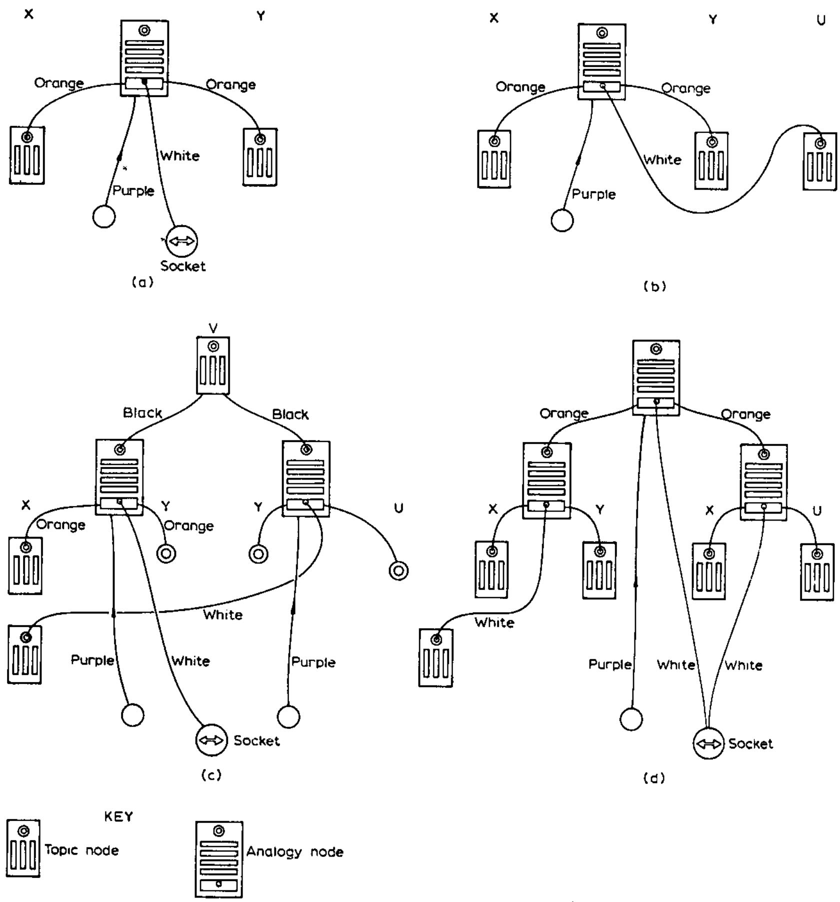


Fig. 8.7. (a) Simple isomorphic analogy; (b) Generalisation based analogy; (c) Derivation constructed from pair of analogy relations; and (d) An analogy between pair of analogy relations.

other grid (Y), and model it as M(G) in another part *MF*(Y) of the Lumped Modelling Facility. To complete his construction, he adds a white link to the isomorphism operation (the similarity) and seeks a difference between X and Y. But X and Y are equivalent so that X≡Y, which means that the universes could be represented as X, Y or the product X × X. This possibility is accommodated by a special operator signified by a socket ≡ for "equivalent but distinct" (Fig. 8.8). The difference (between otherwise identical universes of interpretation) may be regarded either as spatial (X, X) or temporal (as in X_{now} X_{later}).

To press this important point home, consider a rather larger and more realistic example. The user wishes to model a finite ensemble of dynamic systems characterised by the same system equations and being replicas, but possibly differing in respect of initial conditions. Such formulations are ubiquitous in physics, genetics and numerous other disciplines; they underpin any application of statistical mechanics. The replica microsystems are analogous (not identical, but isomorphic). The similarity is the dynamic equation common to them all. The difference is equivalence with either spatial or temporal distinction, as capturing their a priori independence. The analogy relation is the ensemble of microsystems. Thus, the system equations are represented as a derivation structure copied in each analogous universe, X, Y, ... The statistical theory is a further derivation structure in a distinct (macrotheoretic) universe, say U. The head of this derivation structure in U is isomorphic to the analogy between the systems represented in X, Y, ...

2.3.5. To instate a topic representing mutually and perhaps conditionally exclusive hypotheses, the user positions a conditional node in a grid U. This node requires inputs from nodes of the

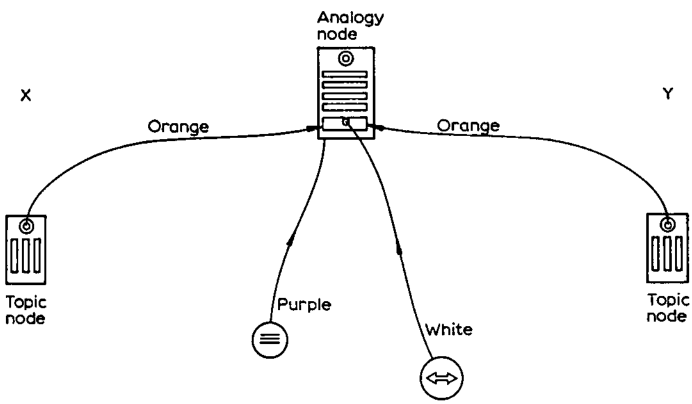


Fig. 8.8. Isomorphism between topic X and topic Y. Equivalence connection by purple link means that X and Y are regarded as coordinates of product set.

topics representing the hypotheses (which can be interpreted in universes X and Y), together with a further input through a brown link which (Fig. 8.9) either negates their conjunction in U (i.e., they cannot both be interpreted and correctly executed in U), or asserts their conditional tenure.

The topic represented by a conditional node is two or more alternative hypotheses T_1, T_2 that are purveyed or supported by different factions and are at loggerheads. In short, the topic represents a controversy between theses that are advanced or advocated by distinct P-Individuals. These P-Individuals may be as august as institutions, famous scientists, "the establishment," specific disciplines, or "schools of thought". They may be as miniscule as the different perspectives taken by one *person* (but two or more P-Individuals), as in the ambiguous figure example (Chapter 7, Section 3). Expert 1 and Expert 2 of Chapter 7, Section 2 would count as exponents of the rival theses T_1, T_2 if they failed to agree and their disagreement, the clash between T_1, T_2, was inscribed in the network. In particular, a conditional is introduced if, and only if, there is a many aim resolution (B treats the user as $\langle A_1, \alpha \rangle, \langle A_2, \alpha \rangle$, in which A_1 and A_2 do not reach agreement).

2.3.6. A typical interpretation is as follows. Let T_1 hold and be modelled in universe X. For example, in elementary physics, T_1 is some prediction (the existence of sharp shadows) from the Newtonian corpuscular theory of light, and X is the universe proper to the geometry of this theory. By the same token, let T_2 (blurred

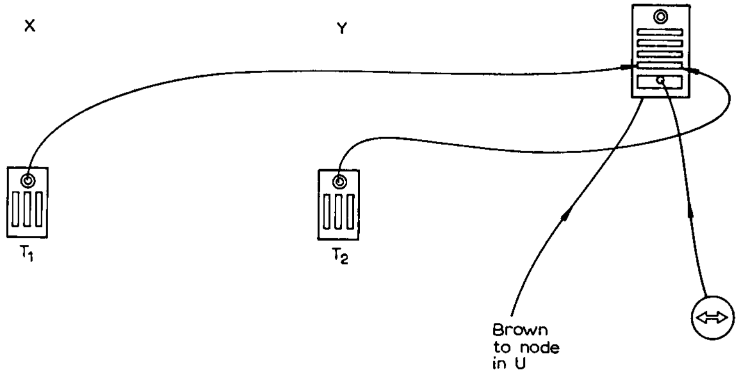


Fig. 8.9. A conditional analogy denying isomorphism between T_1 and T_2.

shadows) hold in universe Y, proper to the geometry of a (Huygens, Fresnel-like) wave theory of light. If U is a further universe of experiments with shadow casting, then T_1 and T_2 are rival hypotheses in U, and this rivalry is expressed by the conditional node as a critical experiment between the theses of P-Individuals (a user's conception of Newton and his conception of Fresnel).

T_1 and T_2 are not formally contradictory. Further, both may be realised (in X, Y). But T_1 and T_2 are incompatible in some common (and accepted-to-be-standard) universe U. The conditional node denies a possible analogy relation.

If the experiment leads to falsification in Popper's (1959) sense, then one thesis or the other will be tentatively denied (until the issue is resolved by some more advanced discovery or theory). But there need be no such critical test (the rival claims may rest undecided, and the conditional node may represent only an open controversy and a fruitful research topic). As stressed repeatedly, we are not primarily or directly concerned with verification/falsification or absolute veridicality. However, such important notions must be representable in a body of knowables (as they are by conditional nodes), and it is essential to recognise that when conditional topics are manifest, they are invariably personalised: to Newton versus Huygens, Church versus State, or several distinct roles adopted qua P-Individuality by the user himself.

2.3.7. Whilst various node constructions are in progress the B heuristic detects any aim which it can identify. An aim may either be placed on a module, in which case it is identical with the aim of other operating systems (CASTE or INTUITION), or it may be an *active* node.

Many nodes may be simultaneously active; for example, in Fig. 8.10 there are five active nodes. B is programmed to interpret only some of these as candidate aim nodes; those that are superordinate and that have full kernels are submitted and accepted for submission. Thus, in Fig. 8.10, nodes S and T are the candidate head nodes; R is excluded because the construction, even if submitted, is incomplete.

After a period of construction, the user is able to submit nodes for instatement, in which case (as below in Section 2.4.), he must justify derivation of subordinates and the like. These transactions take place through the interaction console. Once an instated struc-

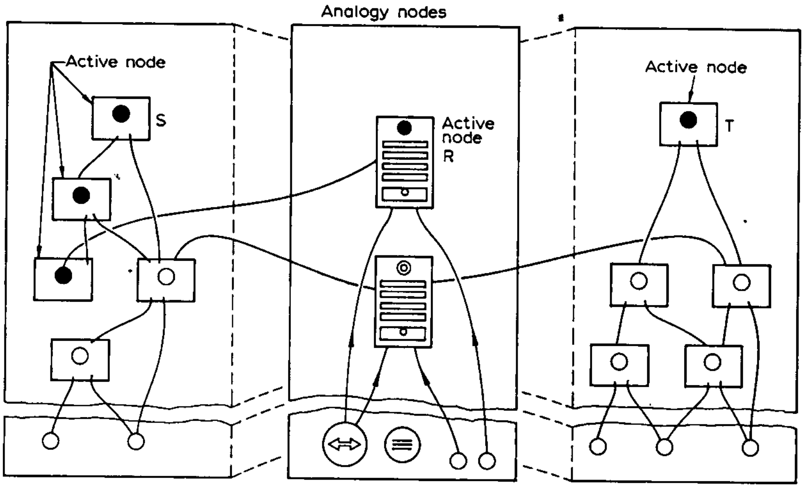


Fig. 8.10. Active nodes.

ture exists the user is impelled to state a *head* node and submit the structure. The planning routine is executed, and as a consequence, he must furnish a semantic description of the structure. For the most part, users are quite willing to choose heads; failing that, they are periodically forced to do so.

Next, if the B heuristic picks up a many aim configuration of the type shown in Fig. 8.10 (Section 2.3.7) *and* if it is also the case that at least a pair of aims have distinct descriptions (obtained by prior descriptor assignment under one aim), then B calls for resolution (placing the user in the position of A_1, A_2). In this case, the planning routine is executed, but description is replaced by comparing and updating the distinct descriptions of the aim nodes.

Thus, either B's requirement for resolution or the user's selection of a head belonging to an instated substructure initiates the planning and description routines of Chapter 7; the routines that tidy up the mesh and present it for description and/or resolution.

2.4. Instating a Node: Degree of Verification

The active lamp on a node is extinguished only if certain conditions are satisfied most strictly if the topics form a valid conversational domain, less strictly if the construction is agreed by another user (including arbitrators and groups). In the strict case, the following conditions must hold:

(a) The model of any superordinate topic contains as constituents the models for all of the subordinate topics from which it is derived (that is, according to the user's derivation).

(b) The user's derivation of a superordinate topic from its subordinate topics loses no essential specificity and is cyclic, apart from its primitives, as a result.

An adequate, weaker form of this condition is summed up in a pair of injunctions that are to be obeyed by the user:

(I) If *topic* k is to be instated as derived (non-analogically) from *topic* i and *topic* j, then within the derivational structure the user must show (by a construction on the grid) how *topic* i and *topic* j are derived from *topic* k (perhaps using primitives) without loss of specificity. Further, the user must make (or assert that he can make) a model M(k).

(II) If *topic* k is analogical, the user must show the reverse derivation (as above), given the waiver that the derivation depends upon the distinguishing predicates, Dist. Further, if the analogy is isomorphic, the user must show the one to one correspondence between *topic* i and *topic* j (directly, or by subordinate isomorphism), and if it is a generalisation, supported by topic ℓ he must make or assert that he can make a model M(ℓ).

Several degrees of rigidity are possible, depending upon the purpose in hand. At one extreme, the displayed network must be consistent and cyclic so that it (and the associated models) forms a conversational domain. If so, condition (a) and condition (b) are checked by applying the test routines of EXTEND, and these routines are also applied to isomorphic analogies between topics. This is a lengthy and rather expensive business.

At the other extreme, where THOUGHTSTICKER is used as an epistemological laboratory, we are only anxious to externalise an innovator's concepts and derivations. The B heuristic checks conditons (I) and (II). The user is required to state what he believes to model his beliefs and derive them from other topics. These statements are accepted without justification (or with only verbal justification), but there is no guarantee that the product is a conversational domain over which learnability and memorability are guaranteed unless a model can be executed for each topic, or a verbal explanation exists. *

2.5. Description Methods

The description scheme evolves together with the topic network, and consequently, it is impossible to inscribe the values of descriptors as the fixed, maplike representation of an entailment structure, over positions on the grids. Moreover, in the interest of uniformity, the descriptors of all topics, whether in the grids or the disjoint substructures (with the exception of the syntactic depth descriptor with values superordinate/subordinate), are represented in the LED display.

Each position in the grid and each node in the disjoint substructures is equipped with an LED (light emitting diode) pair, able to shine red or green if the LEDs are illuminated. Consequently, the possible conditions of any *position* are red (which stands for the descriptor value +), green (the descriptor value —), and "off," the descriptor value * meaning "undetermined or irrelevant". ‡ At any instant, it is possible to display all values of *one* descriptor or of one Boolean expression in the set of descriptors (all topics having P_1 and P_2 but not P_0, for example). The user is able to obtain LED displays by typing the name of a descriptor or the form of an expression into the terminal. Conversely, the regulatory heuristic B can present an LED display to the user and identify it by printing out the name(s) of the descriptor(s) concerned.

New descriptors and their values are introduced by the descrip-

* The less rigid criterion may be based on the views of at least two users, $\langle A_1, \alpha\rangle$ $\langle A_2, \beta\rangle$ and leads to an enhanced realisation of the "improved" operating system in Chapter 6, Section 5. For this purpose, instated nodes are temporarily replaced by modules and may thus be learned by a student in a ordinary operating system (CASTE and INTUITION) placing aim, goal, and understand markers on the topics.

Recall that the participating users $\langle A_1, \alpha\rangle$ $\langle A_2, \beta\rangle$ have agreed to each node instated (the less rigid criterion). The tentatively transformed nodes are accepted permanently, as module based topics, if, and only if, $\langle A_1, \alpha\rangle$ can learn $\langle A_2, \beta\rangle$'s thesis when he addresses it under CASTE or INTUITION control as a student; similarly $\langle A_2, \beta\rangle$ in the role of a student can learn $\langle A_1, \alpha\rangle$'s thesis.

‡ This arrangement leaves open the possibility of representing the values of Fuzzy Predicates of the topics as intermediary shades of light.

tion routine of Fig. 2.8, executed if the user asserts a head topic, or B calls for a many aim resolution.

3. COOPERATIVE INTERACTION

If the user does nothing, he is bombarded with items of information from the data bank. At least, he must engage in explore transactions in order to stem this flux of data. Initially, he can only explore the data bank or the minimal topics in the starting set, and he receives in return items from the channel addressed by exploration.

As soon as some cognitive model has been constructed and the description routine has been executed, the user is able (and forced) to assign values of his own descriptors both to the topics or analogy relations he has instated, and to the data bank channels. True, in the limiting case when the data bank is deemed irrelevant, all descriptors have the value "*" on all channels. Otherwise, channels in the data bank act as information sources that back up topics or groups of topics.

Construction of analogies or topic nodes involves activity in the modelling facility and transactions instrumented through the interaction console and the construction grid display.

The whole process takes place under the following rules (recapitulated from Chapter 7): (a) If *topic* k is instated as derived (in a conjunctive substructure) from *topic* i and *topic* j, it is necessary to show how *topic* i and *topic* j are derived from *topic* k without loss of specificity overall. (b) Analogical derivations satisfy the same rule with the waiver that specificity may be lost (if replaced by the Dist predicates).

At the moment the user asserts a head (or the B heuristic detects a many aim configuration and demands resolution), the pruning and numbering routines come into play and provide a tidied up plan of the mesh (currently, on the Display Tube).

3.1. The Observer's Picture

We, the observers, see an exteriorised version of the user's mental operations. What does the user get in return for all his trouble? Part of the story has been told already. But there is a gap to fill:

namely, the transactions meant to encourage innovative action and many aim operation. In these transactions, the heuristic B acts as an innovative assistant to the user A. Succinctly, B promotes innovation on A's part by essaying innovation itself.

3.2. Promoting Innovation

(a) If more than one deductive scheme exists (as a separately headed or disjoint conjunctive derivation structure) and if the schemes (conjunctive structures) have analogous *parts* but are *not* identical, then B applies epistemic symmetry (Chapter 7, Section 2.5.2) to provoke the syntactic component (and a putative semantic component) of an analogy relation between topics of the existing scheme.

(b) If a principle exists (Chapter 7, Section 2.5.4.), then B applies extrapolation to provoke the development of any existing deductive scheme. *

(c) If an analogy is supported by a strict isomorphism, it stands. If there is an analogy k between *topic* i, *topic* j with M(i), M(j) in *MF*(X), *MF*(Y), and it is supported by a generalised *Topic* with M(ℓ) in *MF*(U), then B asks the user to model a projection of M(ℓ) in *MF*(X) or *MF*(Y) or both. This operation ("Inversion") provokes innovation.

(d) If there are empty cells in the space of descriptors as there are (previous monograph) in an evolving entailment mesh, then B points to the empty cells and provokes the instatement of fresh topics to fill them.

(e) If there is a (suitable) many aim configuration, B requires resolution; if agreement is reached, B instates an analogy relation and, if not, a conditional node.

(f) Using the graphic facility, the mesh can be represented and displayed under any head node at the request of any user.

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* The syntactic construction produced by extrapolation may not be interpretable in the existing universes, so, at the next stage, extrapolation leads to the construction of a novel universe in a spare modelling facility. For example, the information theoretic development of thermodynamics (Chapter 7) involves such a construction and is an innovative gambit. A further example is the invention of a (orthogonal) dimension to accommodate the mathematical extrapolation of "number" to "complex numbers". Goodstein (1962) and Polya (1954) give this example, as does Spencer Brown (1969), the latter author in terms that are precisely attuned to the present discussion.

4. ANOTHER VIEW OF AIM INITIATION

We argue that B acts as an innovative assistant to A because the aim initiating operations (a, b, c, d, e, f) have an interesting and equisignificant interpretation under the general title, "problem posing"; i.e. (given a network of topic relations) "form and pose problems that will generate further topics".

Von Foerster and Weston (1974) note, in their discussion of context oriented systems, that no problem exists without context. A relational specification on its own is insufficient to determine a problem, let alone an acceptable class of solutions to a problem. For example, under the relations x and =, the pseudo problem

$2 \times 3 = ?$

might be solved by 3×2 or by 6; or to cite a further example from Von Foerster and Weston, the curiously enigmatic pseudo problem posed by $6 = ?$ has any number of solutions depending upon the context in which this relation is embedded.

A fortiori, an uninterpreted network does not in itself determine a problem. But all of the procedures used to initiate or catalyse constructive activity are context proposing (hence, problem posing) operations. A few of the proposals may be as specific as the contextual resolution, "6 = some product of integers". Most are far less specific though possibly no less useful. The procedures are surely not complete and in that sense do not constitute an "Artificial Intelligence" (or, as we prefer, in the spirit of the context paper, a "General Intellect"). But they represent part of such a thing, and in combination with the other routines, yield a system in which it is impossible for an external observer to tell whether the innovation (if any) that takes place is due to the user A or to the heuristic B. As promised, B *encourages* innovation.

5. SPECIFIC PROCEDURES

The principles and operations of Chapter 7, Sections 2 and 3 are built into B as a number of "problem posing" or "innovation attempting" procedures.

5.1. B examines the network built up on the grid for analogies between a *topic* i, which is part of a subnet superordinate to node i, and a *topic* j, which exists in isolation. By *epistemic symmetry*, B infers that there may exist a subnet superordinate to *topic* j which is isomorphic to subnet i and is formed by copying the subnet i across the analogical distinction to form a hypothetical subnet j.

B displays this subnet by illuminating the *attention lamps*, of which there is one to each position on the grid. The display is intermittent since there may be, and quite commonly are, several topics with the status of *topic* i. A single display consists in illuminating the attention lamp on *topic* i, and whilst it is turned on, scan-illuminating the attention lamps in the hypothetical subnet j.

This operation is interpreted as a B question to the user, "Do you affirm or deny the existence of each topic on subnet j?" There are two equally productive ways of pursuing an answer: justifying affirmation, and justifying denial. So far, it has only been possible to implement the former method.

An affirmative reply from the user, in respect of an element v of subnet j consists in placing a node at the position on the grid occupied by v; this node being thereby given an *active* status (notice, however, that the node does not cover the attention lamp at this position).

Denial (which, in the current implementation, is not followed up) is achieved by pressing a key on the operating console at the moment when the denied element v is scan-illuminated coincidentally with *topic* i. As a result of denying that v is a topic in the thesis under construction, subsequent scan-illuminations of subnet j do not include v.

Once initiated, the display of subnet j in the context of *topic* i is repeated from time to time, unless

(a) the tenure of all elements v in subnet j is denied, or

(b) all affirmed elements (with nodes positioned) have been derived and instated, so that the corresponding nodes are no longer in an active state.

5.2. As soon as a fresh *topic* i is instated on the grid, B searches the entailment set of this topic for a node representing a *principle* (any *topic* j of the kind described in Chapter 7, Section 2.5.3.).

If such a topic exists, B infers from *Extrapolation of Principles* that *topic* j might be applied to *topic* i as a means of obtaining some further *topic* k, and B thus displays the pair ⟨node i, node j⟩ coincidentally by illuminating the attention lamps at these positions on the grid.

The display is interpreted as a question to the user, "Can you obtain a further topic (which is part of your thesis) by applying principle j to *topic* i?"

An affirmative reply consists in placing a node k at a position superordinate on the grid to the node of *topic* j. This node becomes active, and the attention lamp display is repeated from time to time until *topic* k is derived and instated.

A negative reply is given by pressing a key on the console at the moment when the display is presented. As a result of denial, the attention lamps are extinguished, and the proposed application of the principle is deleted from B's repertoire.

5.3. B searches the *descriptor* space for any conjunction of descriptor values that specifies a unit set and is not occupied either by a node or a uniquely specified *channel* (recall that the descriptions cover the data base, as well as the topics). B prints out the description and asks if there is such a topic, which the user must affirm or deny. The procedure was exemplified in Chapter 7, Section 2.5.

5.4. In Chapter 7, Section 2.1.8, we discussed the construction of a generalised topic (GHWM) to represent the similarity in an analogy relation (HWC) between "heat engines" (HE) and "refrigerators or heat pumps" (RP) and noted that specialised forms of GWHM could be realised as isomorphic models (more general and more comprehensive than HE or RP) in the universes of compilation and interpretation proper to HE and RP, respectively.

Suppose that GHWM was, in fact, constructed in THOUGHT-STICKER. For this or any generalisation based on an other than isomorphic analogy relation (detected by the absence of the reserved isomorphism operator ⇔), B asks the user to construct the specialised topics obtained by interpreting the freshly instated supporting generalisation (for example, GHWM) in the original universes of interpretation. The user A is required to "invert his generalisation". The request from B to A is a typed out question, "Is there a case of the generalisation supporting an X, Y, analogy actually realised in MF(U) within the original universes of compilation and interpretation MF(X) and MF(Y)?" An affirmative reply is evidenced by instating fresh nodes in X and Y, respectively, or in just one of them.

5.5. A completely negative reply, "the proposed construction is *impossible* according to my thesis," denies the validity of an analogy relation based upon the generalised topic. Such replies are stored by B and are the main evidence at B's disposal for contradicting a mooted analogy relation (though not the generalised topic itself).

5.6. The last process, resolution of a many aim situation, is the most general weapon in B's armoury. Notice that resolution of a many aim situation is always productive.

(a) It enlarges the set of semantic descriptors.

(b) If agreement is reached its syntactic component is inscribed in the mesh as the similarity part of an analogy relation (and usually a generalisation based analogy relation).

(c) If there is *disagreement*, the syntactic product is a conditional analogy, as the mark of rival theses.

Resolution is probably also the commonest transaction. We conjecture that all autonomously produced analogies and conditionals are due to "internal transactions" of this kind; only a few of them are captured as "official" and observable resolutions. To the extent that THOUGHTSTICKER *does* capture *some* of these internal transactions, it is able to exteriorise innovation.

Chapter 9

Comparison of Course Assembly Systems: Their Use in Teaching People to learn

All the operating systems have a functional as well as a systemic communality. Notably, all of them serve, in one way or another, as devices which foster a generalised positive transfer of ability, the art of learning without specific commitment to the subject matter being learned.

In Chapter 2, for example, we presented evidence from interviews and group discussion, together with some quantified evidence, of a generalised positive transfer of training due to experience as a participant in CASTE or INTUITION. Even these tutorial systems with a fixed conversational domain appear to foster *versatility* (both operation learning and comprehension learning, and in combinations able to cope with various classes of learning and teaching strategy). Such experience may or may not influence an underlying global/local bias; that is a moot point. But one thing is certain. Though versatility is a prerequisite for an ability to learn in an unstructured environment and though it is evidenced by students who have "learned to learn," versatility is not a sufficient condition. If the general art of learning implies putting together bits of unstructured experience, seeing the wood for the trees, and so on, then a student who has learned to learn must be able to assemble course material on his own account. Although we can examine this aptitute in the tutorial (or fixed domain) operating systems, they do not, just because the conversational domain is fixed, provide tools for studying how, if at all, people learn to assimilate raw data in their own way and, subsequently, to learn within the personally assimilated structure.

For this purpose, we must turn to the course assembly systems:

EXTEND (previous monograph) and THOUGHTSTICKER. As a preliminary, these systems will be compared with a focus on THOUGHTSTICKER, since it has much greater capabilities. Section 1 is devoted to a general overview, and Section 2 spells out the comparison in terms of the macrotheoretic variables of uncertainty and doubt (previous monograph). Section 3 is an attempt to bridge the gap between definitively innovative situations and more commonly observed "learning" situations in which, however, successful students are required to structure the environment on their own. Sections 4 and 5 contain an account of some experiments in which principles, winkled out from experience with the operating systems, are used to inculcate the art of learning in general.

1. PROCEDURAL COMPARISON BETWEEN THOUGHTSTICKER AND OTHER OPERATING SYSTEMS

When learning the topics in the starting set of disjoint substructures, the user has the role of a student in a strict conversation, which is CASTE or INTUITION regulated. Later, under the control of THOUGHTSTICKER proper, he has the role of subject matter expert or innovator. We noted, in Chapter 8 of the previous monograph, that a similar transition takes place when EXTEND is called into play. But THOUGHTSTICKER exteriorises innovation, whereas EXTEND merely permits it and records the product.

1.1. One salient feature of the CASTE organisation is that a student "drops into" a conversational domain representing knowable topics from "top to bottom". He arrives at the learning session with certain concepts in his mental repertoire. He *must* have concepts for the primitive topics, but he *may* have concepts for topics at a superordinate level. Whatever topics he does have concepts to represent are initially marked as understood, and these the student may regard as properties.

The top to bottom orientation (in contrast to the assumption that knowledge is built up from elementary fragments) is dramatically manifest by the order in which an *understanding* is reached; the derivation is first sensed (at which point the student knows *how* he can explain the topic, *if* he can explain it). A correct explanation (the other evidence required for an understanding) comes *after* the derivation. Or, phrased differently, the student knows the kind of model he can build as a non verbal explanation of the topic before either he or the regulatory heuristic knows whether he can, in fact, build a correct model.

1.2. To realise a strict conversation and to exteriorise *understanding* we also imposed a polarity, expressed in the experimental (tutorial) contract, to the effect that the student learns towards a head topic. Considerable stress was placed (notably, in Chapter 7 of the previous monograph) upon the inessential nature of this constraint. Under many descriptions of the same conversational domain, a student can learn his way through the topics in *any* direction; the restriction is introduced to facilitate regulation and observation and to represent the dialogue as a series of discrete occasions (one for each understanding) at which cognitive processes begin and end.

1.3. To demarcate occasions (which is essential in a strict conversation), we pay the price of enforcing the one and only one-aim-at-once condition; and we noted, in context, that students are inclined to rebel against this restriction.

1.4. Much the same polarities and constraints apply to EXTEND control when the student opts into the role of a subject matter expert. EXTEND permits the introduction of fresh topics, and the conversational domain evolves. But there is still one-aim-at-once; there are still discrete occasions; there is still a directionality attached to the method of course assembly permitted by the operating system. These are not so much restrictions upon cognition as restrictions upon those aspects of cognition which can be exteriorised as behaviours. It was conceded and emphasised that the restrictions hampered the subject matter expert, though on balance he gains more from using the system than he loses by accepting its authority as arbiter of legitimacy.

1.5. Moreover, in course assembly under the EXTEND program, these constraints add up to produce a (fairly salutary) dictate. The subject matter expert produces the syntactic component of his thesis *first* (the derivations and the explanations), and the semantic descriptions *later*.

1.6. THOUGHTSTICKER permits and sometimes encourages many aim operations; the simultaneous production and comparison of models; the formation of generalised (not only isomorphic) analogy relations.

1.7. Thus, all the constraints noted in Section 1.1 to 1.5 are relaxed. By dint of a much more complex organisation in the operating system, it is possible also to exteriorise an appreciably greater body of cognitive processes and, at the price of some observational ambiguity, to exteriorise most facets of innovation.

1.8. For example, although the user (in his course assembly role) may work from "top to bottom," he may also do the reverse (making a model first, explicitly, and instating a topic later). He must still have a head oriented polarity under one thesis, but he may also (and usually does) entertain several theses to be merged later. Although he may output the syntactic form of this thesis (or theses) first and their semantic description later, he may also choose to construct a framework of descriptors and build a thesis within this ossature. Finally, not only may he reverse the order

Derivation → Explanation (model)
  into
Explanation (model) → Derivation

with respect to models built as non-verbal explanations in the one or many $MF(z)$, he may also, insofar as the data bank is described (channels or a par with topics), impute meaningful behaviour to whatever lies behind the data bank. Thus, the following sequence is quite legitimate.

Explore data bank → Impute behavior → Model it in the $MF(z)$ → Give derivation.

2. ALTERNATIVE AND MACROTHEORETIC DISTINCTION BETWEEN OPERATING SYSTEMS

It is possible to characterise a one-aim-at-once operating system (any of them at all) in terms of the attentional uncertainty d_0 calculated in the course of aim validation and the uncertainty variable d* (Chapter 6, Section 11), which is computed with respect to a finite (though open ended) list of nodes.

For a one-aim-at-once system, the experimental contract demands that $d_0 = 0$ (or nearly so) if an aim is validated; since there may only be one aim, this implies that $d^* = 0$ (some one aim is selected and the participant contemplates no other). Although it is impracticable to obtain confidence estimates over the entire set of nodes (topics, channels, or whatever), the index d* is usefully approximated by presenting the set of nodes which have been at least once explored during the last m occasions ($m = 12$ is arbitrary, but satisfactory). If these are alternatives for aim selection, as they are by edict in a one-aim-at-once system, the already stated covariation of d_0 and d* is anticipated. By eliciting confidence estimates over the explored node set during a sample of explore transactions, we obtain empirical variation curves of d* and d_0 (a *discrete* value, sampled at aim).

For THOUGHTSTICKER or any other many aim system, this constraint no longer applies. The user may appreciate, be certain about a description for, and validate his aim with respect to, several topics at once. Hence, the confidence estimates upon which the calculation of d* are based do not sum to unity; d_0 and d* are not expected to covary; their empirical estimates do not do so. One way of phrasing the difference is to point out that in a many aim system d* is not a probability or uncertainty measure but a Fuzzy Set measure and that in a many aim system the topics are necessarily Fuzzi Predicates as proposed in Chapter 4, Section 2 (the very far reaching consequences of this remark are also considered at that point).

3. AN OPERATIONAL DEFINITION OF LEARNING TO LEARN: ITS RELATION TO INNOVATION

It was argued in Chapter 2 that certain students have a generalised and apparently transferable ability to *learn;* regardless of the subject matter they face, these students are able to assimilate it. Their ability to do so depends upon several factors. They can structure an otherwise unstructured environment by acting, in this respect, as personal subject matter experts; having done so, they must exhibit versatility (both *DB* and *PB* competence, Chapter 5) in executing learning strategies. Neither skill on its own is sufficient to qualify the student; on the other hand, the skills in question are correlated and probably interact positively rather than interfering.

All this amounts to a sloppy categorical specification. If learning to learn (by experience) or teaching people to learn (under duress or persuasion) deserve the elevated station in the educational system ascribed to these activities in Chapter 2, it is essential to give an operational definition of the competence or ability thereby inculcated. Such a definition is available and is tantamount to the bald statement that an ability to learn (the skill) *is* an ability to employ THOUGHTSTICKER, producing a sensical output when the unstructured subject matter/environment is the active data bank and when the output structure is formed on the grids above the starting substructure. By hypothesis, this much, but no more than this, need be said; for THOUGHTSTICKER determines a well-specified process, albeit open ended, which either can or cannot be handled.

In common with the other operating systems, CASTE and INTUITION, there is still an irreducible but, practically speaking, harmless ambiguity. Does our definition refer to a test for "ability to learn," or does it act as a training device. Clearly, it may do both and the functions are inseparable. For the system is (amongst other things) an "epistemological laboratory" containing principles which may be instilled. Some of these principles are well entrenched pieces of conventional or academic wisdom (though they are not often recognised explicitly by students). Others, like "epistemic symmetry" and "inversion" are debatable; all the same, they are upheld by common sense as well as by theoretical doctrine.

The evidence suggests, moreover, that the use of THOUGHTSTICKER has a powerful training function. Just as a student with a defective repertoire acquires versatility in CASTE or INTUITION if only by virtue of seeing his own "Globetrotting" or "Improvidence," so the user of THOUGHTSTICKER "learns to learn" even if he cannot do so at the outset.

The data available are sparse for two reasons: (a) The experiments are lengthy, arduous, and form part of a phased and ongoing study of innovation. (b) To secure the kind of result which is called for requires a rather special operating condition.

Under (a), the current results only attest to the existence of a training function; its magnitude and reliability cannot yet be stated. The examples of Chapter 7 are however quite typical. Whereas the "reinvention" of Brillouin's work was due to an adult, youngsters "reinvented" the Savery mining pump and various ingenious composite engines (often with fields of application quite bizarre to the adult mind). That analogical structures relating these "fields" (or, in our jargon, "universes of interpretation") are far more complex for younger people is suggested by the relatively tidy and sober minded thesis of Fig. 7.1. At first sight, more significant information about learning to innovate will come from comparing transactions and relational structures than from a gross, numerical comparion; at any rate, our conviction that the system has value stems chiefly from such evidence.

Under (b) "learning to learn" rather than "learning to innovate" calls for a situation dominated by the data bank as a source of information on a par with odd texts in a library or odd experiences in streets and airports or laboratories. The required conditions are shown schematically in Fig. 9.1. The user picks up information from an initially unstructured data bank. On the basis of this information, he makes models in the $MF(z)$ and seeks to delineate a thesis by building a cognitive model, mesh, or network on the construction grids. Having done so, he is in a position to describe his thesis, and (since channels are placed on a par with topics, and furthermore, since the channel output, rather than invention alone, engendered the models) any description of the thesis will be relevant to and descriptive of the channels (usually one channel to a cluster of topic nodes in the mesh). The thesis and data bank description (together with the mesh of the thesis which forms the glue that sticks one descriptor to another) is one of the personalised structures we are anxious to exteriorise.

The distinction between this mode of operation and the current mode is to some extent a matter of degree; for example, exactly this cycle of activities can, and occasionally does, take place. On the other hand, it is hardly encouraged by a subject matter like "conservation and conversion of energy". The "oscillators" environment, mentioned in Chapters 7 and 8, is a more fertile field of enquiry insofar as the data bank is esoteric (indexed by author names and containing extracts from Apter, Beurle, Gaines, Osnager, Prigogine, and many others). But, this environment has so far been little used.

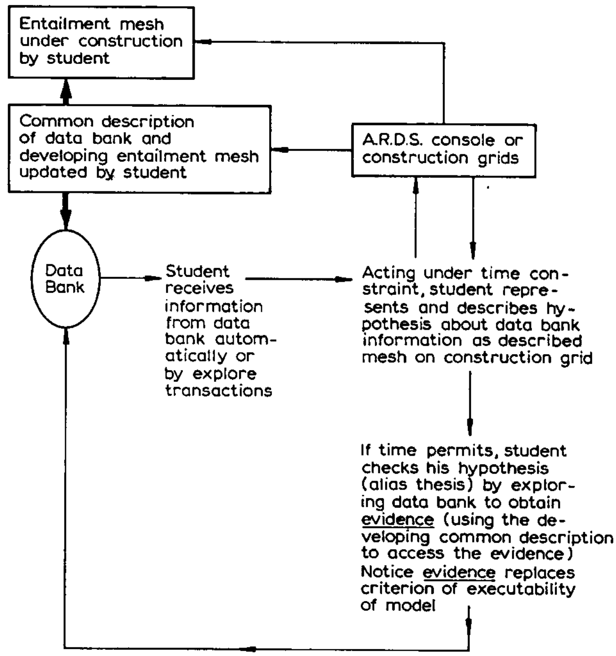

Fig. 9.1. Outline of the THOUGHTSTICKER configuration required for experiments on "learning to learn" and "learning to structure disorderly experience" (the data bank dominates the system; face validity is established by appeal to evidence from the data bank).

A further distinction between the current and the desired mode is as follows: People who are learning frequently act under duress induced by a time constraint; for example, an examination date looms up in the future. Under these circumstances, innovation (in particular, innovation based on "epistemic symmetry") occurs in order to guess at parts of the subject matter which have not been covered. It is not innovation for convenience, or for its own sake, or with much pretence to success. It is innovation of necessity and is very common. It follows that an ideal experimental situation would impose a time constraint likely to be incompatible with the implementation of the present system (though not with the next generation THOUGHTSTICKER under construction). Generally, we feel that investigations are better carried out by other means for this reason.

A final difference between the current and the desired mode of operation is that learners, qua students, are inclined to accept the rectitude of data sources (wisely or not). As a result, canons of workability depend upon whether the data bank (or any particular channel) "says it works". At all events this is what the examiners "want to know". It is quite easy to incorporate the necessary bias into THOUGHTSTICKER, but it is incompatible with the conduct of general experiments on the system.

Under these circumstances, the dogma, honestly and unreservedly enunciated in the introduction, comes to our rescue. Although a theory of educational learning and knowledge must rest upon a well-specified experimental scheme (and in practice if only due to the magnitude of the conversational domain, this implies an operating system like THOUGHTSTICKER), the main use of the results in an educational or institutional context does not involve the operating system directly. Principles of instruction may be extracted from the results produced by CASTE transactions; by the same token, principles of "learning to learn" are readily extracted from the results obtained in THOUGHTSTICKER. If a tutorial (rather than experimental or comparative) object is dominant, most of these principles can be presented, demonstrated and recommended for adoption by any convenient mode of advocacy, for example, in a classroom to a group of interested students.

This expedient has been adopted in experiments chiefly due to B.C.E. Scott and Elizabeth Pask, using the following design.

4. CLASSROOM EXPERIENCE

A group of between 10 and 25 students (age 20 to 35 years) are asked to attend sessions in which they will "learn to learn". On arrival, they are told the following innocuous "story" to form a work setting.

You have been attending a class called "Cosmic Processes". It includes diverse material: the study of Kant, Engels, Bateson, Casteneda, Einstein, Schroedinger, Blum, Kuhn, Kelly, and others, but the course content is inherently interesting and open to personal interpretation. For one reason or another (politely, we do not ask what reason), you have failed to attend the lectures provided. Hence, you are substantially ignorant of the content of the course. That is lamentable since tomorrow you face an examination on the course which you intend to pass. As might be expected, the examination is made up chiefly of essay questions evoking replies to "how" and "why" questions, and there is a marking bias in favour of answers that give some idea of how you arrived at your conclusion: Consequently, most of the questions are open ended. However, the examination is laden with a few factual questions which are more than makeweights.

Something can be done to extricate yourself from the dilemma of entering the examination room without proper study. We have here copies of all the texts used in the course, and they have been edited down to extracts which (though weighty) can be read in approximately 2 hours. You have 2 hours (or slightly more, in fact up to $3\frac{1}{2}$ hours) to study these materials.

At this point the experimenter presents the *Session A* texts (Table 9.1) and leaves the students to mull over them. Students leave the experimental room when they have got as much as (they think) they can from the material.

Although reading rate is not, in the population sampled, a limiting factor, the experience is pressing and for some students positively traumatic. A few break down emotionally, or literally escape. Those who remain are submitted to an examination, liberally augmented by Piaget like interviews.

Session B, when the group next gathers, is devoted to a training and demonstration exercise. This session lasts for several hours and exhibits the major pathologies of learning (Chapter 5), their explication in terms of *DB* and *PB* operations, and the salient principles of THOUGHTSTICKER. Within the limits of a classroom session, the students are required to do and see for themselves, not merely to listen to a lecture.

Finally, *Session C* is a virtual replication of *Session A* using *different* materials (Table 9.1) and is again followed by an examination and Piagetian interviews.

The usual controls are applied. The materials employed in Session A are found to be of comparable difficulty to those employed in Session C; for some groups, Session A materials are used first, and for some groups, Session C materials are used first. Possible practice effects are controlled by interpolating inactivity in place of the (training) Session B (and found to be negligible; if anything, performance gets worse unless something is done to eliminate the confusion produced by assimilating a large and indigestible mass of data). For all that, and presumably as a result of indoctrinating students with THOUGHTSTICKER principles in Session B, there

TABLE 9.1

"Learning to Learn" Experimental Materials

Session A Texts: K. Walker: *A Study of Guardjieff's Teaching* (Chapt. 7). F. Engels: *The Dialectics of Nature* (Chapts. 2 and 10). J. Lilley: *The Cyclone's Centre* (Chapts. 11, 13, 14, 15, 15, 17). J. Clarke: "A Map of Inner Space," in *Six Approaches to the Person,* R. Buddock (ed.). C.G. Jung: Extracts from his Introduction to Richard Wilhelm's translation of *Secrets of the Golden Flower.*	
Session B	Training session (special materials).
Session C Texts: L. Wittgenstein: *Tractatus Logico-Philosophicus* (extracts). A. Schutz: *Collected Papers: 1. The Problem of Social Reality* (extracts). W. Heisenberg: *Physics and Beyond* (Chapts. 9 and 20). E. Schrodinger: *What is Life?* (Chapt. 4). C. Castaneda: *A Separate Reality* (Introduction and Chapts. 5 and 17). C. Castaneda: *Journey to Ixtlan* (Chapts. 15 and 20).	

is a very marked and statistically significant improvement due to Session B practice. These results are shown in Table 9.2, and the acquisition of an "ability to learn" is most marked in terms of the "how" and "why" questions for which the answers are derivations and explanations mostly innovated by the students. Graphic responses (for example, flow and connection charts) are encouraged. In this arrangement the materials used in Session A and Session C correspond to the THOUGHTSTICKER data bank, and in Session B, to a stripped down operation of the THOUGHTSTICKER system.

Various compromises and classroom administrable techniques have been tried. Details of the currently used technique, which works well for 5th and 6th form students, are given in Appendix C. It is a practicable, fairly inexpensive method tested over some 120 students; it can be used also for adult populations, and a modified version is being piloted for use in primary schools.

TABLE 9.2

Learning to Learn, Experimental Data

Subject Number	Pretest score	Posttest score	Complexity of graph (pretest)	Complexity of graph (posttest)	Complexity Difference	No "+" topics pretest	No "+" topics posttest	Diff. test	Failed Subjects	No "+" marked topics pretest understood	No "+" marked topics posttest understood	Diff. test
1	50	55	60	84	+24	7	9	+2	—	6	9	+3
2	30	60	40	56	16	2	7	+5	—	2	6	+4
3	80	65	55	20	−35	10	4	−6	*	10	4	−6
4	55	50	47	51	4	7	3	−4	—	5	3	−2
5	60	75	35	78	43	4	8	+4	—	4	8	+4
6	25	45	26	65	39	1	2	+1	—	1	2	+1
7	35	55	34	66	32	5	9	+4	—	4	7	+3
8	25	47	25	57	32	1	6	+5	—	1	5	+4
9	20	55	20	47	27	1	6	+5	—	1	6	+5
10	45	50	40	80	40	1	2	+1	—	1	2	+1
11	45	50	32	49	17	4	10	+6	—	3	10	+7
12	35	60	15	35	20	4	7	+3	—	4	7	+3
13	50	70	46	84	38	5	9	+4	—	5	9	+4
14	40	65	42	86	42	6	8	+2	—	6	8	+2
15	60	85	57	61	4	3	2	−1	—	3	2	−1
16	70	65	34	36	2	5	7	+2	*	5	7	+2
17	55	37	36	18	−18	5	5	0	*	5	4	−1
18	52	40	25	10	−15	10	8	−2	*	8	6	−2
19	63	77	36	58	22	8	12	+4	—	3	11	+8
20	56	71	61	79	18	8	12	+4	—	6	1	−5
21	37	44	11	26	15	7	12	+5	—	6	10	+4
22	15	20	44	68	24	5	6	+1	—	5	2	−3
23	65	67	32	48	16	5	12	+7	—	4	11	+7
24	78	81	55	70	15	12	29	+17	—	1	10	+18
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	70	91	55	82	27	9	12	+3	—	9	7	−2

25	48	26	56	33	−23	10	15	+5	*	1	4	+3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	70	91	55	82	27	9	12	+3	—	9	7	−2
29	59	80	46	67	21	5	14	+9	—	1	12	+11
30	78	43	44	25	−19	8	16	+8	*	7	6	−1
31	45	60	30	56	26	4	9	+5	—	4	7	+3
32	40	30	29	8	−21	9	12	+3	*	7	2	−5
Mean	50.7	58.2	39.6	54.2	14.5	6	9.1	3		4	6.7	2.66
SD	17.34	17.37	13.71	22.76	20.91	3.244	5.192	4.242		2.565	3.804	4.976

Mean	47.92	62.2
SD	16.96	15.79

For $n = 25$ excluding subjects marked "*" in "Fail" column who dramatically failed to learn and with graphs that degenerated, on posttest, into disjoint subgraphs.

Statistical Summary

Posttest score > Pretest score (all subjects n = 32) t = 1.73 (0.1 > p)

Posttest score > Pretest score (*excluding* subject who *failed,* by graph criterion or behaviour, marked "*" but *including* those who could already learn before training, n = 25) t = 3.03 (0.01 > p).

Graph complexity after training > Graph Complexity before (all subjects, n = 32) t = 3.13 (0.01 > p).

Topics marked "+" after training > topics marked "+" before) (all subjects, n = 32) t = 2.77 (0.01 > p).

Marked topics understood after training > marked topics.

Understood after training (all subjects, n = 32) t = 3.33 (0.01 > p).

When the Spy Ring History Test is administered (some 18 of these subjects) there is a positive correlation between versatility and score difference significant at 0.001 > p).

Control Data

(a) No training interpolated (n = 12) *reversed* trend in test scores t = 1.218 (0.1 > p).

(b) Materials A before materials C (n = 15 subjects) pretest score mean = 49; SD = 17.06. Posttest score 61.3; SD = 21.4. Difference mean + 12.8; SD = 20.44.

(c) Materials A before materials C (excluding laboratory subjects, n = 11) Pretest score mean = 51.8; SD = 17.27; Posttest Score Mean = 56.6; SD = 21.26, Difference Mean + 5.7, SD = 16.23.

(d) Materials C before Materials A (reversed learning material) n = 14 subjects. Pretest score mean = 55; SD = 18.95. Posttest score mean 71: SD = 18.25, Difference mean + 15.43, SD = 21.8.

(e) Materials C before Materials A (reversed learning material) excluding laboratory subjects, n = 9). Pretest score mean = 59.6, SD = 12.33; Posttest Score Mean = 64.9, SD = 17.73; Difference Mean + 5.33; SD = 15.89.

Comparison of Control Data

b/d and a/c pretest mean differences t = 0.9 (not significant) posttest differences.

t = 1.136 (not significant). Group Differences b/c and d/e; t = 0.336 (not significant); t = 0.051 (not significant).

5. DISCUSSION

The average improvement fostered by a THOUGHTSTICKER technique is unequivocal. For subject matter which is so heterogeneous and sometimes recondite, it is hardly necessary to question the transferability of any skill which is acquired.

We do not claim that everybody "learned to learn". An appreciable number of the students opted out (especially before the latest technique was introduced). We conjecture that this is the main reason, in practice, why people do not "learn to learn". A few who stuck out till the end of the experiment gained little benefit, but these form a small percentage of the total. Most students who did not benefit already had the general learning skill in their repertoire at the outset, so that they cannot for this reason be said to have learned a novel art. The great majority of students who were initially naive and who did stay through the experiment showed a major degree of improvement. Further, judging by their comments during the interviews, they enjoyed the experience, found it useful, and became aware of how they set about learning.

Amongst the students who did show evidence of learning the art of learning in the course of the experimental sessions, there are two groups of special interest.

(a) Students whose response at the first examination indicated that one (or at the most two) text passage had been picked out for scrutiny and the rest neglected. Apart from the severe time constraint imposed by the work-setting, these students might have been adept "serialists" or they might have been "improvident" learners (with a purely arbitrary, sequential-looking, learning strategy).

(b) Students whose replies at the first examination showed every sign of "Globetrotting" over some or all of the text passages. Given longer, they might have been successful "holists". As it is, they answered questions in terms of loose, distorted, or even purely nominal pseudo-analogies (generally, noting similarities and neglecting differences; invariably, unable to explain the topics thus linked together).

It was sometimes possible to observe gross features of explanatory behaviour during the learning session, and these observations, when available, are commensurate with the pattern (a) or (b) detected in the examination phase.

After the training session B, the majority of these students, type (a) or type (b), improved their performance in terms of absolute score on the examination following Session C. The time constraint upon learning in Session C is just as stringent as it is in Session A, but judging from the students demeanour whilst learning, it is far less bothersome. Nearly all of the students imposed a structure of their own upon the texts, were both conscious of doing so and able to recall the structuring scheme (often graphed or charted on paper). Students of type (a) enlarged the scope of their explanation (occasionally falling into the "Globetrotting" snare), whereas type (b) students concentrated on satisfactory explanation and derivation, as though compensating for their original defect (at the training session they were probably still aware of their performance and thus able to obtain corrective feedback from the training).

Observation of behaviours and protocols support the main conclusions based upon a smaller sample of well-controlled results and upon the theoretical argument, namely:

(1) Innovation involves the resolution of many aims to produce one.

(2) This may occur in one person (brain) if it is inhabited by more than one P-Individual.

(3) It may, equally well, occur in groups of several people.

(4) Course assembly is replete with innovation.

(5) Innovation, "course assembly" (in the technical sense of this book) and "learning to learn" are tied together by a common process, which also sets them apart from less creative learning.

Chapter 10

An Attempted Integration of Theories of Creativity and Innovation

The present theory of innovation is intimately connected with theories that are tentatively accepted as explaining certain types of creative activity. The comparative study in this chapter is limited to a handful of possibilities and restricted cases in which a process or mechanism of innovation is postulated. Further, the cases examined are supported by empirical evidence from field studies, historical observation, or (occasionally) laboratory data.

It will be argued that the present theory bears up quite well and does a useful job of work in unifying the theories scrutinised. Insofar as this and other theories are not at odds, even though most other theories taken alone have significant points of difference, it is reasonable to claim that our theory is a generalisation of the others and is also in some respects more detailed. This pretentious-sounding claim is duly qualified; the fact is, the present theory, though it has predictive power, is also tailored to fit limited experimental situations. The others, in contrast, have a far richer field of interpretation. Let us stress at the outset that the present theory is no "better" than the others. It is systemic and the others have a systemic core; the "generalising capabilities" of the theory are limited to the systemic core. But, seen in this light, the unification achieved is extremely useful.

Section 1 contains a brief review of the literature, as a result of which certain comparable theories are winkled out for attention. Next (Section 2), the present theory is expressed in a form applicable to unfettered creativity (yielding an approximation to the statement in Chapter 6 and Chapter 7). Section 3 is devoted to comparing the selected themes with the paradigm of Section 2, and the results are summarised in Section 4. Methods of fostering creativity (most of them already discussed in a different guise) are noted in Section 5, and Section 6 briefly explores the educational implications.

1. EXISTING THEORIES AND THEIR COMMON FEATURES

The literature on innovation and creativity is widespread and oddly mixed. One aspect of it is concerned with the psychometrics of innovation; ever since Galton's (1883) studies, attempts have been made to demonstrate traits, usually inherited, that are conducive to innovation. For example, Guildford's (1956) divergent-production factors (analysed into several components in his "Structure of Intellect" model) go along with a tendency to innovate, or at least to eschew convergent thinking. Several important facts are generally acknowledged; for instance, given a careful study (such as Taylor and Ellison, 1964, using the biographical-inventory multiple-factor test batteries), it turns out that a propensity to innovate is *not* in register with academic performance and is not differentially predicted by academic success. But, unless the psychometric devices are used in sequential investigations of developmental psychology (Piaget 1968, Baldwin 1966), no specific mechanism of innovation is directly involved.

It is clear that the present (mechanism oriented) theory cannot be compared with theories which involve no serious postulated mechanism; this in no way derates the value of studies aimed at describing or predicting the distribution of creative mental traits in a population or their development as a function of age. However, it seems imprudent to identify reliably testable traits *with* creativity, as some researchers are prone to do. The easily made confusion between a testable feature and a process or mechanism is a category error; committing this error (often in a very sophisticated form) leads to the well-known hazards of (unwittingly) equating "intelligence" with "performance in an intelligence test". The perils are especially great within education, where individual value judgements, "he is intelligent" or "he is creative," are apt to hang upon the results. If only for this reason, we insist that creativity/innovation, whatever else it may be, *is* a process or a mechanism, rather than a cognitive manifestation/behaviour pattern. Henceforward, the discussion is confined to theories which postulate a process or mechanism and which may, as an incidental result, be compared with the present theory.

Proposed mechanisms of creativity may very roughly be classified as linguistic or cultural (on the one hand), and individual (on the other). The demarcation is not at all clearcut; individual innovation takes place in a cultural context and is often mediated by linguistic tools such as metaphors designating analogies and parables. For example, the theories of Upton and Sampson (1963), of Cassirer (1946), and Fromm (1951) posit general classes of mechanism that are evidenced by the history of societal transformation or the structure inherent in a corpus of knowledge convention or tradition, for instance, the structure of myths or a style of expression. In contrast, individualistic theories — due to Schon (1963), Koestler (1964), Barnett (1953), Gordon (1961), Elshout and Elshout (1960), Fischer (1969, 1974), and Maslow (1954) — propose more or less specific mechanisms for innovation, and find support either from detailed protocols, laboratory experiments, or the observations made at the level of interviews by designers anthropologists and social or educational psychologists. It is still true from a systemic point of view that the form of innovation in the large (social, cultural or linguistic) is identical with the form of personalised and miniscule innovation.

1.1. Common Features

The theories of Schon, Barnett, Koestler, Fischer, Gordon, Maslow and Elshout have (or may be interpreted as having) certain important features in common. These are:

(1) All of them are concerned with *relations*, either abstract or holding, between tangible objects. For example, innovations in scientific theory deal with relations involving coherent sets of propositions called *theories* (but henceforward, and in line with the terminology of the book, called *theses* to avoid confusing "theories that are innovated" and "theories of innovation"). In contrast, a technical invention, even if backed up by a thesis, results in a relation instanced by a tangible object.

(2) There is a phase of schism or disunity of attention whereby amorphous knowledge is divided into isolated units. The units may

either be problems, specified by adjoining a context to the original relations, or distinct perspectives.

(3) The isolated units are juxtaposed (as a rule, in a larger context or by union in a contrived or accidental event).

(4) The result of this juxtaposition may be abortive; it may be productive.

(5) If productive, the result is an analogy between the original units (relations).

(6) Suppose coalescence does take place and yields an innovation. A "large" innovation corresponds to a *generalised analogy* (our nomenclature), rather than an *isomorphism;* however, isomorphic analogies are usually countenanced as limiting cases of innovation.

(7) The result of coalescence, if it takes place, is accredited as an innovation (rather than an insight or a bright idea) insofar as the general concept, often interpreted in its own universe, can also be represented in one or both of the universes proper to the units generated by a schism.

(8) Very definite subjective events are correlated with the phases (1) to (7); these may be given neurophysiological interpretations.

1.2. Qualifications and Disclaimers

The kind of mental activity countenanced as innovative, either by theories of the type outlined in the previous subsection or by our own theory, is quite narrowly bounded. The definitions involved are technical, and their value rests upon a possibly blinkered specificity.

For example, suppose some children are playing with Papert's (1970) LOGO. A child discovers a principle (for instance, "subroutine" or "partitioning") applicable to existing programs, and the novel program is unequivocably an extrapolation on this basis from the old programs. According to the hypothesis under discussion, this extrapolation is not *in itself* an innovation. But Papert (1970), Bruner (1966), and others sometimes maintain that it is.

There is no fundamental disagreement. On the one hand, it is stupid to argue over teminology (we have already hinted that our technical definition might be unfair and concede that the other usage may be more equitable). But, nomenclature apart, we only noted that extrapolation is not *in itself* an innovation. Let us agree that extrapolation is necessary (in LOGO, it *is*), and comment that so far as our technical usage is concerned, the child's inventiveness depends upon what is done with the extrapolation, i.e., the new program and its productions (geometrical patterns or whatever), henceforward just "P".

In particular, the child will be innovative if P is used to *suggest* a new idea; that is, if P is juxtraposed with some P* (in the LOGO universe or not) and is found to be analogous, so that P solves a problem suggested by this means. If so, P is used as an *Eolith*: the word is culled from the early work of Storm (1922), resuscitated and developed by Hawkins (1969). In the original context, an Eolith is an object, conventionally a slab of stone or wood, which an innovator stumbles across by accident. It differs from other objects in suggesting a novel use; for example, its shape fits it for use as a plough. The innovator did not have a plough in mind, but he did (say) have in mind the notion of breaking up the ground. He innovates (and his innovation *is* a plough) insofar as the Eolith (P), in juxtaposition with the *class* of earth cutting instruments (P*), forms a functional analogy that is resolved as an invention (the plough). Here, we submit that potential Eoliths are generated by extrapolation, to form P; rather than cropping up by accident. In this respect, the child's extrapolation is like the act of walking over the earth. The result of extrapolation is innovative if P is assimilated in the context of P*, and yields a program that has a radically different function. Probably everyone would agree that *this* is "more innovative" than the extrapolation itself and they might agree (depending upon the detailed conditions) that *only* such *uses* of extrapolation count as "innovative".

It is also worth pointing out that under everyday circumstances an apparent extrapolation can be due to a (technical) innovation, and it is only in an operating system like THOUGHTSTICKER (or a "paired experiment" or a "depth interview" perhaps) that the original assertion, "the program is unequivocally an extrapolation," is justified at all.

2. A GENERAL REPRESENTATION

In order to obtain a clear set of comparisons between specific examples of the creative mechanisms discussed in outline in Section 1.1. and the theory under consideration, the present theory will be represented as a scheme. Nothing new is added, and the scheme is merely a collection and crystallisation of points which have already been made.

One prefatory note is in order. All studies of creativity make use of the notion "context". As remarked in Chapter 8, Section 4, a context is needed if any problem or question is to be posed; a relational structure does not, in itself, specify a *problem*, though it may determine an indefinite number of possible problems (Von Foerster and Weston, 1974).

The word "context" is also double edged. The act of attending to a particular universe of compilation and interpretation with a topic in mind furnishes one kind of context insofar as the constraints of this universe impose boundary conditions and dictate that only certain topic relations can be realised. A far richer notion of context (closely related to the meaning imputed in Chapter 8) appears as soon as there are two or more P-Individuals (or, in the original discussion, one P-Individual and an interrogating heuristic). If so, one P-Individual can question the other from his perspective (with queries apposite to his universe) and, of course, vice versa. Insofar as the forthcoming scheme posits the co-existence of two P-Individuals having distinct universes of compilation and interpretation, the idea of a context, in both senses, is firmly embedded in the creative process.

SCHEME 1

Main Postulates	Commentary and Identification
(1) Two or more P-Individuals exist. Two or more contexts are thereby determined.	Two or more people with one focus of attention; each, or one person, having two roles or perspectives, posing two or more problem classes.
(2) These P-Individuals have distinct universes of compilation and interpretation, but their languages have a modicum of syntactic communality.	The universes of compilation and interpretation may be distinct brains or distinct areas in the same brain. Universes of interpretation may be conventionally and metrically distinct (magnetic as against gravitational phenomena; Peru as against Brazil), or they may be different state descriptions of the same object (a classical and a quantum mechanical view of a molecule).

SCHEME 1 (continued)

Main Postulates	Commentary and Identification
(3) One, and only one, focus of attention (P-Individual or aim) is a seat of awareness; though this awareness may be (and if externally observable *is*) the origin of consciousness on the part of one P-Individual *with* another *of* something. Expressed in terms of the macrostate (subjective probability) variables, d_0 (doubt about focus of attention) is high because there is more than one aim so that d_1 (doubt about method) and d_2 (doubt about outcome) are undefined.	Two people may be jointly aware of one topic or two. One person may only say he is aware of one topic at once, though he may say that he is conscious *with* some other person of a topic, or that he is conscious *of* entertaining some other perspective *about* this topic. Whatever else, neither you nor I can say we are aware of two foci of attention (two aims) though our attention may oscillate between two foci of attention (alternative theses or ambiguous figures), and we may be aware of the oscillation.
(4) From (2) the languages of the P-Individuals in question have certain commonly formed expressions. Hence, common meaning agreement is possible in certain universes. Moreover, as a weak postulate, common meaning agreement is *likely*. We shall later argue that it is a *necessary* occurrence.	Common meaning agreement may be deemed likely because of geographical proximity or cultural similarity between people. By the same token, if several P-Individuals are compiled and undergo execution in the same brain the likelihood of overlap may be due to physical limitations. The argument of necessity does not deny the various phenomena responsible. But they are regarded as secondary consequences (secondary, that is, to common meaning). In other words, we maintain that people *must* come into geographical proximity, belong to specific cultural groups, and that brains (or other L-processors) *must* have structures guaranteeing overlap of P-Individuals *because* of the primary requirement, occurrence of common meaning.
(5) If (or, given the necessity of common meaning, whenever) common meaning is resolved, the result is either an isomorphic analogy relation or a generalised analogy relation; these cases, hitherto discussed at some length, are summarised in	Several comments are in order (a) The other than analogical topics in Fig. 10. 1 and 2 may, at one extreme, be simple relations or, at the other, coherent sets of propositions which constitute theses or (apart from the reserved notation) theories.

SCHEME 1 (continued)

Main Postulates	Commentary and Identification
Fig. 10.1 and in Fig. 10.2. The productions of Fig. 10.1 may be counted as innovations and the productions of Fig. 10.2 are invariably counted as innovations. Any of these productions is (formally) a *topic* and is associated with one aim, or attentional focus. Since coalescence of P-Individuals is believed (Chapter 5, Section 11) to involve concurrent operation d_2 will be high (doubt about the stages in computation) whenever d_0 is low enough for the definition of d_1, d_2. There are many processes acting in parallel until a common meaning is reached; as a result, we predict little or no awareness of "an outcome"; at most, there is a Fuzzy "set of outcomes". In contrast d_1 (doubt about method), may be low; and is predictably lower than d_2. That is, the innovator may (d_1 low) or may not (d_1 high) be able to specify a Fuzzy Method for innovation.	(b) An example of Fig. 10.1 is the *discovery* of the isomorphism between mechanical and electrical oscillators; or the invention of an electrical oscillator given a mechanical oscillator. An example of Fig. 10.2 is the *discovery* of the information theoretic interpretation of thermodynamics, or the construction of topics given a realisation of this generalised analogy relation. (c) The productions are taken to include covert and overt explanations, as well as the construction of models. The latter productions, being tangible artifacts, are usually tagged as *inventions.* (d) To apprehend the scope of these examples it is important to realise that information theory *could* have been devised as a generalisation of thermodynamics, or vice versa, and someone may, in fact, have discovered information theory by following that route. (e) Since common meaning gives rise to a fresh (single) aim the innovator (whether encompassed by one brain or residing in several) *becomes* aware of the innovation as a novelty produced at the moment when the common meaning agreement is reached.
(6) Resolution of a common meaning may (Chapter 6) and usually does give rise to a richer structure (a generalisation) and it *does* entail mutual interpersonal hypotheses (in the sense of Chapter 6). Moreover, if the conditions of Chapter 6, Section 7 and 8 are satisfied, fresh P-Individuals are created by the resolution through "Conversation Breeding". These conditions sometimes *are* satisfied and "Conversation Breeding" sometimes takes place.	Resolution may either involve an "internal" or an "external" productive interpersonal conversation. The latter case is widely discussed by social psychologists and social anthropologists; notably by Bateson (1972, the Double bind effect, and Higher than Deutero Learning); Bateson (1958, the Naven Ceremonies); Mead (1957); and Shwartz (1962, especially in connection with the "Cargo Cultures" and other Messianic movements).

The postulate (clause 4) that common meaning agreement is necessary is supported on the following grounds, though it could certainly be justified, more satisfactorily perhaps, by means of a formal argument.

The conversation breeding process (clause 6), or some essentially similar variant, is the only mechanism able to produce two or more P-Individuals de novo from one P-Individual, apart from a random process. Notice, that any random process which might be invoked is of a peculiarly fundamental kind; for example, "Noise Sources" and "Background Noise" will not suffice to explain the random element, though appropriate sorts of random generating processes might be employed to describe it. The existence of two or more P-Individuals is required as a base (clause 1) to render this series of definitions recursive, rather than vacuous or terminating. As a matter of empirical fact, the process adumbrated by these definitions does take place.

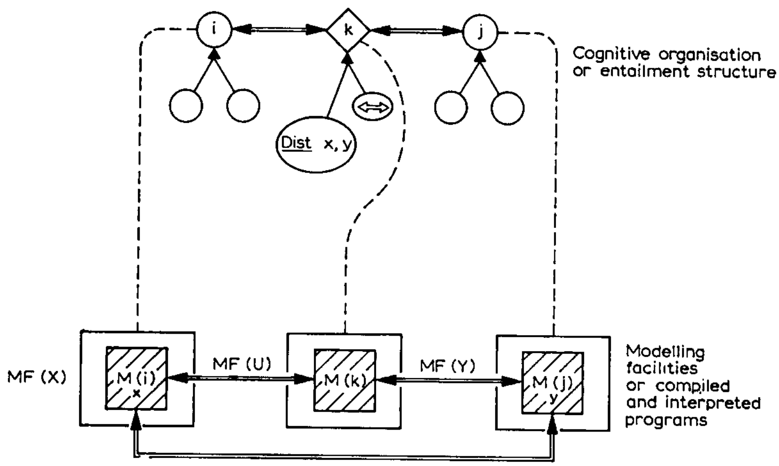

Fig. 10.1. Simple analogy configuration. The isomorphism may be replaced by a *topic* k which expresses the syntactic or formal similarity common to a model (Mi) of *topic* i (in X) and a model M(j) of *topic* j (in Y) which is represented as Model M(k) in any distinct (abstract) universe of interpretation. The universes of interpretation are shown as modelling facilities *MF*(x), *MF*(y), *MF*(u) for simplicity. In general, the interpretations and compilations are in the L-processor of a brain when the Proc i, Proc j notation replaces the representative models M. However, the crux of the construction is captured by noting that the entailment structure induces an isomorphism between models.

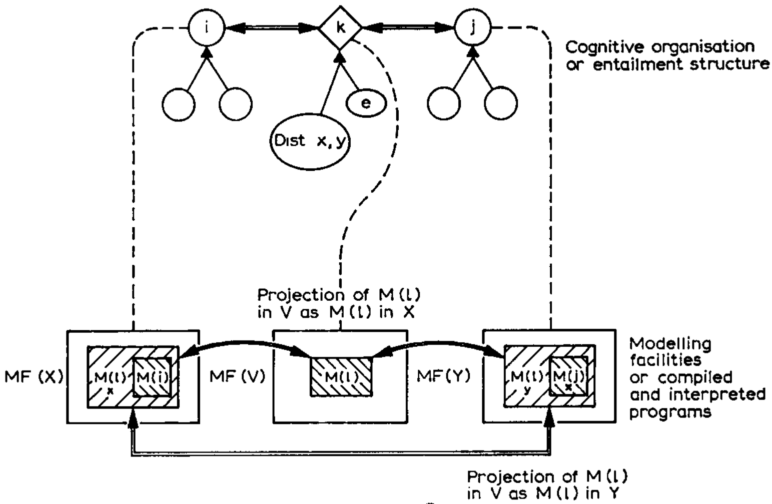

Fig. 10.2. A generalised analogy relation supported by generalised topic ℓ, with a model M(ℓ) in a distinct universe of compilation and interpretation *MF*(V). Projection of M(ℓ) to *MF*(X) yields $M_x(\ell)$ and of M(ℓ) to *MF*(Y) yields $M_Y(\ell)$. Notation is same as Fig. 10.1. M(ℓ) in *MF*(X) is isomorphic with M(ℓ) in *MF*(Y) and M(i) in X is a subsystem of $M_X(\ell)$ and M(j) in Y is a subsystem of $M_Y(\ell)$. At least *one* of these projections *must* exist for a useful material analogy. But models (M_x(i) and M_x(j) are not ismorphic. If modelling facilities *MF* (shown for clarity of expression) are replaced by L processor of brain, and compilation and interpretation of procedures as Procs, then generalised analogy is concurrent execution of <u>Proc</u> i and <u>Proc</u> j. Similar comments are applicable if *MF* is replaced by the fuzzy interpretation set (chapter 4) of a natural language.

We are unwilling to countenance as part of our theory the peculiarly fundamental and subtle type of random event which might, as an alternative to conversation breeding, give rise to the required supply of P-Individuals, because no clear meaning can be given to random events of this calibre. Instead, we invoke the already stated principle, "The least unit is a conversation," and augment it by the further postulate, "In any conversation accommodating more than one possible aim (consequently *not* in general a strict conversation), at least one common meaning agreement is reached after a finite number of occasions (n) and is resolved as a generalised analogy relation".

3. COMPARISON OF THEORIES

In the following section several theories are compared with the present theory in an attempt to achieve non trivial unification. Something is gained by all the theories (our own included).

3.1. *Schon's Displacement of Concepts and Innovation*

Schön (1963) is primarily concerned with technical innovation, invention on the part of people or teams, and the kind of creativity manifest in understanding (rather than proving) a mathematical proposition. His theory is fruitfully exemplified by mulling through records of industrial invention such as Rossman's (1964) classic and compendious work.

The bare bones of his argument are as follows: the unitary entities in the theory are *concepts* designated "Schon Concepts" *SC*, *contexts*, *metaphors* designated "Schon Metaphors" *SM*, and "displaced concepts" *SD*. A concept may be a proposition, an analogy, or a thesis (alias, a theory). Any concept brings about a relation (R), and it is "structured" by the context in which it appears. All concepts occur in some context. The context is a set of facts, other concepts, and propositions; typically, a thesis, together with an interpretation and an intention (for example, to solve a class of problems).

To show that Schon's theory and our own hypothesis are isomorphic, it will be sufficient to consider the most general case examined by Schon, and to point out that he permits all diminutive or constrained formulations as special cases. Any composite of the general case or a special case is also permissible.

The theory is outlined as follows. Certain concepts are entertained by one person or several, but are distinguished with respect to their universes of interpretation, as for example:

SC_i realises R_i in X
SC_j realises R_j in Y

where X and Y, at least, characterise contexts and problems.

At some point SC_i and SC_j are juxtaposed and related by a Schon Metaphor *SM* which designates a putative or actual analogy relation. In general, the analogy relation

$SC_i\ (SM)\ SC_j$

cannot be realised unless steps are taken to modify (displace) SC_i or SC_j or both; for instance, it is not generally possible to realise SC_i in Y or SC_j in X.

Suppose SC_j is transformed, in this conceptual system, to yield SD and that SD, if realised in Y, yields R_j^* (vice versa, displace SC_i). The displacement is useful if SD can be realised in X (though SC_j cannot be) and if it realises R_i^* in X; where R_i^* encompasses R_i. If so, SD is *created*, and the model constructed under SD in X (which brings about R_i^*) is an invention.

To give a concrete example of the process, one of Schon's colleagues was familiar with the context, X, of recycling and refreshing the constitutents of a closed environment in contact with a polluting entity or further environments. One system, characterised by SC_i, filters and recycles air in a living space after carbon dioxide and other waste products accumulated during habitation are removed. The relation thus preserved is R_i. At the outset, when the requirement for a cleaning device was mooted, Schon's colleague did not immediately muster these ideas, but learned about relations and processes in a further context, Y, of cleaning machines (for example, vacuum cleaners, brooms for brushing sawdust) by a systematic investigation. One machine characterised by SC_j uses a buffer material that is in equilibrial contact with a dirty surface and is readily removable (for instance, dirty sawdust that is thrown away) and preserves a relation R_j in Y. The buffer material must be discarded as soon as the concentration of dirt in *it* is equal to or greater than the concentration of dirt on the *surface* to be cleansed; otherwise, "cleaning" ceases and dirt is transferred back to the surface.

At this stage, it was recognised (SM) that the buffer material *is* an *environment* in contact with the larger "open" environment of the surface (a notion from context X). But, if the buffer material (alias, the buffer environment) can be recycled and renewed, the act of cleaning can continue without limit. Various mechanisms are able to secure these requirements, but none of them is identical with the system under SC_i (for recycling and filtering air). One such mechanism, characterised as a *displacement* (SD) of SC_i, consists in a buffer environment of fabric in contact with the surface to be cleaned and permeated by a continually flowing liquid dirt solvent. The liquid solvent is recycled so that the dirt it carries can be removed, either by differential absorption, or else along a concentration gradient, and the purified liquid used again and again as the primary solvent.

Suppose that *SD*, the displaced concept under examination, really *works*, in the sense that a system or program representing *SD* can be modelled and realised in some concrete or intellectual universe distinct from X or Y (say, in U). If so, *SD* may be, but need not be, realisable in X and/or in Y. At this stage in the proceedings, *SD* is a workable idea and a candidate for realisation in X.

Let *SD*, in fact, be a successful candidate, insofar as a system or program representative of *SD* can be modelled (compiled and executed) in X to bring about a relation R_ℓ of which R_i is a subrelation, so that R_i is satisfied if R_ℓ is satisfied. Alternatively, if M_X as before stands for "model in X of," let both M_X (representative *SC*i) and M_X (representative *SD*) bring about the same relation (R_i), but let M_X (representative *SC*i) be a subsystem of M_X (representative *SD*), so that *SD* furnishes a more general set of cleaning methods than *SC*i. If one or both conditions are satisfied, then any M_X (representative *SD*) is an *invention* (in the concrete sense of an artifact); M_U (representative *SD*) is also an innovation (often, though not necessarily, an abstraction of the invention); and *SM* is the analogy relation, or a metaphor designating it, which Schon regards as closely akin to Cassirer's "Radical Metaphor". Schon also notes that a successful displacement (*SD*) is irreversible. Once that *SD* is established, *SC*i even if evocable is seen in the context of *SD*, since *SC*i is a subsystem of *SD*.

Some of the special cases to which we alluded earlier can be obtained by permuting the origin of the displacement and the universe in which the invention is constructed as a model. For example, *SC*j may be displaced rather than *SC*i or both of them may be displaced. All of the models M_X (representative *SD*), M_Y (representative *SD*), and M_U (representative *SD*) may be constructed as stable entities or only one of them. Further, it is quite possible for Y to play the pivotal role of U (and if U = Y, then U need not be made explicit in the formulation).

Two classes of innovation are distinguished by Schon, and these also are special cases of innovation in general. The two classes differ in the polarity of mental operations.

For *Problematic Enquiry* (stressed so far), there is a problem obtained by juxtaposing X with some *intention* to generate a context and noting that the currently existing repertoire of X interpreted concepts do not solve this problem: here, the problem of making effective cleaning equipment. The inventor casts around another universe, such as Y, in an endeavour to find *SM*, such that *SC*i (*SM*) *SC*j, after which the other operations are applied, either successfully or not.

Speculative Enquiry reverses this order of events. Some *SM* exists (in the inventor's mental repertoire) and *SC*i, *SC*j, or both are built up as hypotheses to satisfy *SC*i (*SM*) *SC*j.

All this is in accord with the present theory, given the following series of identifications (under which the special cases of displacement are given by substitution in Fig. 10.2.). Notations are culled freely from previous chapters, notably 4 and 6.

(a) A Schon concept *SC*i is a concept in the present sense of a compiled procedure. Thus, some typical *SC*s are

$$SC\mathrm{i} \Leftrightarrow \underline{\mathrm{Proc}}^{0}\mathrm{i};\ SC\mathrm{j} \Leftrightarrow \underline{\mathrm{Proc}}^{0}\mathrm{j};\ SD \Leftrightarrow \underline{\mathrm{Proc}}^{0}\mathrm{k}.$$

The crucial feature is that any *SC*, like any Proc, can be expressed in terms of a syntactic or programmatic part, together with a compilation and interpretation part. So, as before

$$SC\mathrm{i} \triangleq \langle \underline{\mathrm{Prog}}\ \mathrm{a}, \underline{\mathrm{Inter}}\ \mathrm{x}\rangle,\ SC\mathrm{j} \triangleq \langle \underline{\mathrm{Prog}}\ \mathrm{b}, \underline{\mathrm{Inter}}\ \mathrm{y}\rangle$$

where a = b only in the relatively uninteresting case where the displacement is trivial (an isomorphic analogy; the same program is compiled and interpreted in a different universe).

Further, $SD \triangleq \langle \underline{\mathrm{Prog}}\ \mathrm{c}, \underline{\mathrm{Inter}}\ \mathrm{u}\rangle = \underline{\mathrm{Proc}}^{0}\mathrm{k}$

where U is generally an abstract universe (concrete if viewed as a brain or L-Processor, but having no direct correspondence with other than mathematical realities).

(b) R_i is computed by *SC*i (alias $\underline{\mathrm{Proc}}^{0}$i) in a universe X; R_j is computed by SC_j (alias $\underline{\mathrm{Proc}}^{0}$j) in a universe Y.

(c) The usual situation is that $\underline{\mathrm{Prog}}$ b in *SC*j (alias $\langle \underline{\mathrm{Prog}}\ \mathrm{b}, \underline{\mathrm{Inter}}\ \mathrm{y}\rangle$) cannot be compiled and executed as it stands in universe X (that is, $\langle \underline{\mathrm{Prog}}\ \mathrm{b}, \underline{\mathrm{Inter}}\ \mathrm{x}\rangle$ is either impossible or impossible in the context of other concepts in the innovator's repertoire. From Chapter 5, Section 11, recall the expedient of writing *DB** to represent an actually more subtle act involving the synchronisation of a priori asynchronous procedures.

(d) There is a transformation $DB^*(R_j, R_m) \Rightarrow R_k$ and a transformation PB^*(Proc^{0}j, Proc^{0}m, R_k) $\Rightarrow$ Proc^{0}k (the notation of Chapter 5 with m, ℓ and n free indices) that yields the displaced concept *SD* (alias Proc^{0}k = ⟨Prog c, Inter u⟩) compiled and executed in a universe U. From the preceding description, *SD* is useful if, and only if, Prog c can be compiled and interpreted in universe X also; that is, as a further concept written Proc$^0\ell$. With *DB* an isomorphism, the transformation is the generalised analogy operation of Chapter 5, Section 11, namely, $DB^*(R_k, R_i) \Rightarrow R_\ell$; *PB* (Proc^{0}k, R_ℓ) $\Rightarrow$ Proc$^0\ell$. We stress the important caveat of Chapter 4 that this expression only simulates an actuality or furnishes a convenient shorthand. Strictly and practically, we have no right to talk of *DB* or *PB* acting between P-Individuals, and it is maintained (clause j below) that Proc^{0}i and Proc^{0}j belong to distinct P-Individuals.

(e) The formalism uncovers an otherwise elusive feature of Schon's theory. *SD* is slightly (and, in the original frame of reference, harmlessly) ambiguous; it stands for both Proc^{0}k and Proc$^0\ell$, designating uniquely only the syntactic component (Prog c) which these concepts share in common. Schon's argument implicitly calls for an extra-theoretic universe of interpretation; hence, we spoke in our previous discussion of "*SD* interpreted in U" and of "*SD* interpreted in X".

(f) An acceptable displacement usually has the further property that Proc^{0}i is a subsystem of Proc$^0\ell$, and it is often true that Prog can be compiled and interpreted in Y, as Proc^{0}r, such that Proc^{0}j is a subsystem of Proc^{0}r. These conditions usually imply that Prog a is a subprogram of Prog c and that Prog b is a subprogram of Prog c. * Hence, the irreversibility of displacement provided that Proc$^0\ell$, Proc^{0}r are replicated by appropriate memories (as they must be if able to count as concepts in the first place).

(g) The invention, previously glossed as M_X (representative *SD*), is a model realised in universe X. For consistency with the previous discussion, X is characterised as a modelling facility *MF*(X), and the invention becomes simply a model M_ℓ compiled and submitted for execution in *MF*(X). Thus, the invention is M_ℓ and is a more general construction than M_i (which is a subsystem of M_ℓ).

* More complex possibilities can be envisaged but will not be discussed because they do not modify the main contention of irreversibility.

(h) Recalling the earlier chapters, M_i and M_ℓ figure as compilations in X of S Prog i and S Prog ℓ, where S Prog i is a serial representative of Prog a, and S Prog ℓ is a serial representative of Prog c. The execution of M_i in MF(X) brings about R_i; the execution of M_ℓ in MF(X) brings about R_ℓ, and R_i is a part of R_ℓ.

(i) The crucial feature of this line of argument — recall the caveat in clause (d) — is that two (or more P-Individuals exist with distinct foci of attention or (if the whole construction is referred to a conversational domain) two or more aim topics. That is, the node of R_i is in the EntSet of a node I with subordinates (at some depth) circumscribing X, and the node of R_j is in the EntSet of a node J with subordinates (at some depth) circumscribing Y. Equisignificantly, R_i is interpreted in X; R_j in Y; and X, Y are distinct.

(j) Displacement may be initiated by an externally presented problem; for example, that an existing artifact, M_i realising R_i is inadequate for a certain purpose or in a certain situation. Equally well, it may be engendered internally insofar as pairs of concepts (Proc^{0}i, Proc^{0}j) which are capable of displacement to yield Proc^{0}k, Proc^{0}ℓ, and M_ℓ arise in the course of an ongoing conversation between P-Individuals A_1 and A_2 (Proc^{0}i in A_1's focus of attention, or aim; Proc^{0}j in A_2's focus of attention or aim). As a matter of interest, it appears that any displacement which is engendered by external constraints or boundary conditions may be represented with some advantage in a conversational domain (so that "aim of A_1" and "aim of A_2" correspond to markers placed on an entailment structure).
Whether external constraints exist or not, two cases need attention. Either the displacement involves a team of two persons $\langle A_1, \alpha\rangle$, $\langle A_2, \beta\rangle$, or one inventor in the transient condition of maintaining two P-Individuals; namely, $\langle A_1, \alpha\rangle$, $\langle A_2, \alpha\rangle$. In any case, the two P-Individuals, A_1, A_2 are distinct prior to displacement and are coalesced at the moment of displacement. For unconstrained innovation, the "team" is presumably a "think tank" or a "T group" or a "Free Innovation Group"; the inventor becomes an "ideator".

(k) The context varies in the course of displacement. Its magnitude may be roughly appraised if the mental operations are referred to a conversational domain. It is greater than the concepts attached to nodes in the intersection of EntSet I (where I is A_1's aim, the X focussed context) and EntSet J (where J is A_2's aim, the Y focussed context). It is less than the concepts attached to nodes in the union of EntSet I, EntSet J.

(l) Innovation, according to Schon's theory, satisfies the conditions set out in Scheme 1; we show this by outlining Scheme 2 (below) and placing it in register. The important distinction between Problematic Enquiry and Speculative Enquiry tallies with the distinction (Chapter 5 and Chapter 6) between "discoving an analogy with topics given" (Problematic Enquiry) and the "analogy first" construction (Speculative Enquiry). This distinction is chiefly obtrusive in clause 5 of Scheme 1.
Displacement, according to Schon's theory may either be interpreted as "successful displacement" (when it adumbrates all of Scheme 1), or as a process that satisfies clauses 5 and 6 of Scheme 1. Both interpretations are legitimate; their relative utility depends upon the purpose in hand.

SCHEME 2

Clause in Scheme 1	"Displacement" Conditions or Events
1	Two (or more) contexts and perspectives X, Y.
2	*SC*i in X, *SC*j in Y.
3	Awareness postulates and observations in *The Displacement of Concepts* (not described in this overview).
4	For some *SM*; *SC*i (*SM*) *SC*j is possible, and may be likely.
5	Production of *SD* to support *SM*.
6	Resolution (the several special cases) and generation of two or more contexts/perspectives required in (1).

3.2. Cultural Innovation

Barnett approaches innovation from an anthropologist's position and derives empirical support from various cultures; notably, from detailed studies of the American Indian Shaker cult (a deviant but devout religious group, founded in the mid-1700s near New York). However, the underlying theory of innovation is applicable to individual as well as societal transformations.

The basic mechanism is similar to displacement, and by token of Scheme 2 and the preceding identifications, it is compatible with the conversation theoretic hypothesis. Compared to Schon, the detailed argument put forward by Barnett (1953) is tortuous, complicated, and difficult to exhibit for stage by stage analysis. The complexity is essential for two main reasons.

(a) Since the theory is primarily societal, it is expedient to distinguish several types or subprocesses of innovation (for example, "assimilation" and "projection") and various phases of innovation (for example, "identification" and "incorporation" and "analysis"). Expediency becomes a necessity insofar as innovative cultural transformations involve a great deal of other-than-innovative activity from which they cannot be meaningfully extricated: thinking, learning, adaptation; symbolic, normative, and ritualistic modifications.

(b) Again, because of the societal interpretation, it is necessary to enrich the paradigmatic situation. When talking of invention for instance, it is reasonable to deal in terms of analogies between two topics with the caveat (frequently stressed in the earlier pages) that n-fold-analogies ($n > 2$) and analogies-between-analogies are often intended. Little is lost by this piecemeal approach, and the relevant processes are much more easily represented. In contrast, it would certainly be unrealistic to cite generalised analogies involving two topics as exemplars of cultural transformations. As a result, any cogent argument must comprehend very elaborate clusters of innovation.

No attempt is made to summarise the full force of Barnett's argument (the burden of which is carried by Chapter VII and VIII of *Innovation, The Basis of Cultural Change* and by an *Appendix on the Nature of "Things"*). However, it is possible to accomodate the basic theory as compatible with Scheme 1 under the following identifications.

(A) The primary units are configurations ("Barnett configurations" *BC*) which themselves relate several concepts. A configuration may be conceived as a whole since it is a stable entity, or analysed in a context into its parts. The *BC* are identified either with stable *understandings* of a concept class in a P-Individual, or with P-Individuals. In all cases that involve innovation (in contrast to the other cultural transformations of learning, and so on), either identification is apposite. Thus, the P-Individuals of clauses 1, 2 and 3 in Scheme 1 are *BC* (henceforward, just A_1 and A_2) without commitment to their locus of execution (several *BC* in one brain or a *BC* distributed over several brains).

But A_1 consists in a replicative collection of other *BC*s (some but not all of which may be factor P-Individuals in their own right); call them BC_1^1, BC_1^2 .... Similarly, for A_2 there is a collection of *BC*s, say BC_2^1, BC_2^2 .... At the least, a BC_1^i may be a stable $\underline{\text{Proc}}^0$i, namely a concept; generally, it is a cluster of concepts, the constituent $\underline{\text{Proc}}^0$i in which must be extracted by analysis in a given context. If such an analysis is carried out for $\underline{\text{Proc}}^0$i (alias BC_1^i) in A_1 and for $\underline{\text{Proc}}^0$j (alias BC_2^j) in A_2, then the relations R_i, R_j brought about by executing $\underline{\text{Proc}}^0$i and $\underline{\text{Proc}}^0$j are interpreted in distinct universes (X, Y).

(B) At the least, R_i and R_j are simple. In general (herein lies the complexity as well as the verisimilitude), they are analogy relations to begin with. For example, *BC* may be a Schon analogy *SC*u(*SM*)*SC*v.

(C) The context in which BC_1^i and BC_2^j are isolated and juxtaposed may be set by external means; for example, if an A_1, A_2 conversation is referred to a conversational domain, or if a problem is specified by external boundary conditions. It may also arise autonomously in the course of A_1, A_2 dialogue.

(D) Barnett uses a special term "Barnett Analogy" (*BA*) to designate both the juxtaposition and its resolution. Thus, *BA* is an L^1 operation (a $\underline{\text{Proc}}^1$ in the present theory) which may be approximated by the *DB**, *PB** construction of Section 3.1, augmented by a pivotal *SD* in a universe U. However, at this stage, there are two important differences between the elementary sort of displacement so far investigated and the action of a *BA*.

First of all, the *BC*s upon which *BA* operates may be inherently complex; configurations such as the resolutions *BC* = *SC*u(*SM*)*SC*v or *BC* = *SC*s(*SM*)*SC*t, so that *BA* gives rise to various structures; for example,

$$BC_{12}^1 = \langle SC\text{u}(SM)SC\text{v}(BA_1)SC\text{s}(SM)SC\text{t}\rangle$$

$$BC_{12}^2 = \langle SC\text{u}(SM)SC\text{v}(BA_2)SC\text{t}(SM)SC\text{s}\rangle$$

(which are analogies between analogy relations), or to diminuitive forms of which $BC^3_{12} = \langle SCu(BA_3)SCs\rangle$; $BC^4_{12} = \langle SCu(BA_4)SCt\rangle$; $BC^5_{12} = \langle SCv(BA_5)SCs\rangle$, are some examples. Moreover, in interesting cases at least one, and possibly several, *SC* are displaced to *SD*. Such colligations are called *hybrids.*

The other difference, a source of equally legitimate complexity, is that the *BC*s arising in the process are, or may be, viable P-Individuals. Of the two differences, the latter underlines the cautionary comments of Chapter 5, Section 11. Although *BA*, qua operation, may be expressed in the manner of Section 3.1(d), this formulation is approximate; it is a scarcely legitimate shorthand. Barnett's use of hybrid is singularly apposite. The resulting configuration does resemble a resonance hybrid (using the jargon of elementary chemistry) and like a resonant, in contrast to a tautomeric molecule, may only be accurately pictured within some more comprehensive (in the chemical case, quantum mechanical) frame of reference.

If the emergent *BC* is complex and is stable, it is itself a P-Individual, and in this case the formation of a hybrid is not only a complex displacement, but is also an example of "Conversation Breeding" (Scheme 1, clause 6). Barnett makes the point explicit by noting that innovation is (symbolic) *evolution.* The power of his theory, as well as much of its complexity, resides in the fact that evolutionary processes are accommodated within the theory.

The price paid for such an encompassing construction is that several situations have an air of strangeness about them. For example, it sounds odd and almost like a conundrum to say that a concept (or the relation it brings about) is both the same as some other concept and also different to this other concept, given a particular *BA*. This difficulty, at least, may be surmounted by recognising that stable *BC*s are P-Individuals (A_1, A_2; that the similar-or-different concepts are part of different *BC*s (A_1's repertoire and A_2's repertoire); that A_1 and A_2 have distinct perspectives (or, where the notion is applicable, distinct aims); and finally, that the distinct points of view (A_1's and A_2's) may be resolved as a syntactic similarity and a semantic difference (Dist (x, y)) if A_1 and A_2 coalesce in the process of breeding further *BC*s.

3.3. Innovation as "Bissociation"

Koestler's masterly *Act of Creation* (1964) contains the clearest statement of a theory compatible with our own. There is a very close similarity between the theories of Koestler, Barnett and Schon (the wealth of examples obviously spring from distinct sources), but Koestler is far more explicit about the dynamic character of the entities involved and comments at greater length upon the role of consciousness in the creative process. Part of the argument appeals to historical and conceptual reality, and part of it (the latter half of the book) is couched in terms of a process oriented physiological allegory. That is, unconscious activity and so on are tacitly identified with the operation of functional subsystems in a brain which is differentiated (on the one hand) as more or less automatic and (on the other) as more or less phylogenetically archaic. "Allegory" carries no pejorative overtones. It merely stakes out a salutory distinction between unique and multiple causality. Thus, the posited mechanisms *may be* responsible for the psychological effects; on some occasions, they probably *are* the causative agents. But so may many other mechanisms act in this capacity. Like Hebb (1949) when he speaks of "cell assemblies" or "phase sequences" as the progenitors of psychic events, Koestler is using one possible mechanism in order to tell a true story about ubiquitous mental happenings, which may, or may not, have a direct connection with physiological processes.

With that qualification, the unitary constituents of Koestler's theory are matrices (*KM*) and an operation between matrices called "Bissociation" (in contrast to a comparable operation upon one matrix, which is *association*). "Matrix" is a rubric given to various coherent and rule obeying mental activities (from Bartlett's (1932) "schemata" to "skills"). Certainly, a "matrix" tallies with a class of stable $\underline{\text{Proc}}^0$i (concepts that are undergoing execution with respect of one universe of interpretation). Matrices denoted *KMX* (in X) and *KMY* (in Y), where X and Y are distinct (no conjunctive derivation has been established to unite them), and thus belong to two P-Individuals A_1 and A_2 (separate people $\langle A_1, \alpha\rangle, \langle A_2, \beta\rangle$ or more usually as roles or perspectives entertained by one person $\langle A_1, \alpha\rangle, \langle A_2, \alpha\rangle$).

Cognitive operations involving only one P-Individual (within or upon *KMX* or *KMY* in isolation) are either run of the mill learning processes (imaged by one-aim-at-once transactions) or the constructive act of *extrapolation* (Chapter 7). Koestler classes all of these operations as *associative* operations.

Bissociation, the crucial process, involves the coexistence of two

P-Individuals A_1, A_2 as matrices *KMX* and *KMY*; their subsequent coalescence to yield a novel or displaced concept and the modification of concepts that exist in the repertoires of A_1 and of A_2. Koestler identifies the phase of analysis (where some $\underline{\text{Proc}}^0$i stands out from *KMX* and some $\underline{\text{Proc}}^0$j stands out from *KMY*) with the "Conversation Breeding" of Scheme 1, clause 6 and the juxtaposition of the P-Individuals (or the *KMX*, *KMY*) as conversational participants. He identifies the phase of coalescence with the action of a mechanism such as the *DB*, *PB* approximated transformation of Section 3.1(d) or Section 3.2(D). The outcome of coalescence is either *nothing* or a further and *novel* matrix *KM**. These words are not Koestler's but the "translation" appears to be justified by the previous discussion and by Scheme 3 which places Koestler's terminology in register with Scheme 1.

SCHEME 3

Clause in Scheme 1	"Bissociative" Phase
(1)	Two or more contexts, in perspectives generated in (6) below; *KMX*, *KMY*, or A_1, A_2
(2)	Proc^0i in *KMX* and Proc^0j in *KMY* are subject of A_1, A_2 dialogue, possibly yielding agreement over common meaning.
(3)	The process is unconscious until *KMX* and *KMY* are differentiated. At that point, there is consciousness of a similarity and a difference between Proc^0i in *KMX* and Proc^0j in *KMY*.
(4)	Bissociation of *KMX* and *KMY* is possible and may be likely or necessary.
(5)	*KM** is produced to support any other-than-void bissociation.
(6)	If *KM** (or the bissociation) is stable, it may constitute a further context, as required in (1).

Bissociation may be induced externally by deliberate intervention to juxtapose *KMX* and *KMY*. Telling a joke that juxtaposes two or more bizarre sets of rules has this calibre, so does a funny cartoon or the illusion figures, or a comical play (for example, in a Feydeau Farce the juxtaposition of men in wardrobes with the universe of crown princes, anarchists, and fashionable eccentrics).

The psychological concomitant of this event is stress, and it may lead to laughter or evaporate in a cathartic process. But it may also lead to the production of a novel "matrix" KM^*, which (side condition) can be replicated and stabilised.

That is, something (KM^*) may be created by the joke. If this condition is satisfied, then the bissociation is productive or resolved as an innovation.

Koestler stresses humour because it is inherently important and also because its symptoms are unequivocal and reflexlike (we cannot tell by inspection if someone thinks a story is beautiful; we can tell by his smile that he considers it amusing). However, he emphasises that humour is only one of the concomitants of stress (the same play may induce fear, joy, laughter or sympathy). Moreover, plays can be constructed as comedies or tragedies; the same is true of any work of art.

Turn now to the issue of spontaneous creativity (invention or whatever). Koestler accounts for spontaneous creation in terms of various mechanisms and at a chiefly descriptive level discusses their experiential concomitants. His argument is:

(a) *KM*s are continually active (essentially the "man must learn" requirement of conversation theory).

(b) The distinction between universes is not absolute (this we paraphrase by saying that <u>Dist</u> (x, y) depends upon an interpretation of what may be known within some thesis to which the participant subscribes; and saying also that the distinction is relative to a Fuzzy Universe).

(c) The main mechanism fostering innovation genesis is *reculer pour mieux sauter* (roughly, taking a step backwards in order to make a better leap ahead). The "leap ahead" is innovation. The "step back" is conceived as reference to distinct modes of brain activity, perhaps characteristic of the limbic system or any other phylogenetically ancient structure, rather than the neocortex. This contention may be too specialised (it is part of the physiological allegory), but our theory predicts that innovation genesis and the possibility of bissociation are often heralded by awareness of different and conceivably more primitive rules; the activity of *KM*s (say *KMU* or *KMV*) that do not enter into consciousness because their activity is asynchronous. Consciousness occurs at a point of partial or local synchronicity.

(d) The innovator is commonly unconscious of (unable to communicate with some other sentient being, about) both *KMX* and *KMY* until such moment as resolution is attempted. After that, there are two possibilities. If resolution is unsuccessful, then *KMX* and *KMY* will alternate, temporarily, in consciousness, like the alternating perspectives of an ambiguous picture. If it is successful, *KM** will emerge as an innovation.

On translation: *KMX* and *KMY* are, or belong to, two P-Individuals A_1, A_2 with initially independent execution. As such, they are asynchronously executed. There is thus no information transfer (in Petri's sense) and, at that instant, A_1 and A_2 are not conscious, with each other, of anything in X or Y (though they may be conscious of an alternation of *KMX* and *KMY* or conscious of their distinction and their similarity, separately).

(e) Resolution of *KMX* and *KMY* is treated uniformly (spontaneous creativity does not differ in this respect from induced innovation). Bissociation may be equated to the achievement of a common meaning agreement between A_1 and A_2. If successful, *KM** is a generalised analogy relation.

If the resolution results in an innovative (generalised) analogy, then, equisignificantly, there appears a novel P-Individual A (the fused hybrid of A_1 and A_2) or a novel concept is created; namely, the innovation *KM** from which *KMX* and *KMY* may be retrievable (with augmented meanings, as Schon insists) as specific precursors.

Koestler summarises some of his psychological points by comparing salient features of habitual (and commonly rigid, ritualistic or automatic) thinking with features of creative and innovative thinking, using a table for this purpose. The pertinent entries in Koestler's table are copied into Scheme 4, where they are related to constructs in conversation theory.

One last point is worth making. Nearly all the creative processes discussed by Koestler (and similar remarks apply to the other authors when they deal with creativity in one person) involve characterisation. This is especially true of the conditions (humour, laughter, pathos, agony, surprise, and so on) which are forerunners of bissociation itself. For example, members of a theatre audience identify themselves with more than one character in a play and thus enact and extrapolate the plot in their own mind. "Stepping back to leap forward" is another example, since in doing so, I see myself as I was (quite apart from the "back to the primitive mind"

SCHEME 4

Habitual		Innovation	
Association	Integral P-Individual	Bissociation	Transient multiple P-Individuals
Preconscious guidance of process (may become conscious or not)	Mental operations other than generalised analogy	Subconscious guidance of process (may become conscious or not)	Operations of Chapter 4, Section 10
Dynamic equilibrium	(Ideally) Understanding under one aim condition	Evolution	Conversation Breeding agreement by common meaning necessarily involves more than one aim operation
Variations on theme	Learning or simple extension of entailment structure	*Reculer pour mieux sauter*	Assimilation of domains into Fuzzy Structure
Repetition or mechanical derivation	Problem solving or automatic operation	Novel ideation	Analogy resolution often followed or preceded by (one aim) extrapolation
Conservative and stabilising function	Cognitive fixity	Partly conservative but also revolutionary function	Cognitive fixity in any one P-Individual momentarily disrupted by intrusion of other P-Individual

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Conservative and stabilising function	Cognitive fixity	Partly conservative but also revolutionary function	Cognitive fixity in any one P-Individual momentarily disrupted by intrusion of other P-Individual

connotation) and see myself also as I am. Here, as in a member of an audience, there is an internal dialogue between the constructed personalities. This fission and dialogue is predictable, according to the present theory; for, we expect that any generalised analogy achieved by resolution of several aims or foci of attention will be founded upon an exchange of personalised hypotheses, as well as hypotheses which refer directly to the matter in hand.

3.4. Operational Creativity and Synectics

Around the mid-1940s, W. J. Gordon and several colleagues began to develop means for encouraging innovative activities on the part of individuals and groups. Much of their work during the 1950s, which is reported in Gordon (1961) and Prince (1970), took place against the background of industry (in a division of Arthur D. Little, Inc., and at a later stage in an independent organisation, Synectics, Inc.) and dealt with technical invention and innovative solutions to managerial or administrative problems. However, both authors stress the (indisputable) relevance and efficacity of synectic methods in education.

Like the other creativity theorists, advocates of synectics (from the Greek for "joining distinct and superficially irrelevant components") emphasise the role of analogy, of personal perspective, of juxtaposition and resolution. However, since they are concerned with operationally practical methods for conducting group sessions or guiding individual thinkers, these principles emerge with great clarity and lead to positive recommendations. For example, exemplary universes of compilation and interpretation (the "worlds" of synectic theory) are explicitly listed, as are the manoeuvres to be adopted by a group leader in order to enliven dialogue whilst introducing the minimum possible bias.

Prince (1970) tries, as I have done, to express cyclic, iterated, and often concurrent operations as easily communicated process charts; he makes precisely the same reservations (for example, that the process which is being depicted is not really serial, that it may be distributed or localised). With these reservations in mind, Scheme 5 (below) is an attempt to summarise the salient characteristics of a group activity which fosters innovation. Any group of this kind includes, amongst other participants, a subject matter expert (or, at any rate, someone having access to the facts of a problem area), and a group leader who plays a catalytic role, as well as monitoring application of the heuristic embodied in Scheme 5. The "examination" phases involve the expert, though he is not allowed to suppress imaginative and seemingly bizarre propositions; the "choice" phases are introduced insofar as group discussion is more efficiently focussed upon one major topic at once, without prejudice to the likelihood that individual participants follow different trains of thought.

All phases, apart from selection (ultimately the leader's prerogative) and personal analogy (Phase 6, the participants brood on their own), are accompanied by lively debate, during which the participants criticise and comment upon each other's ideas. The participants are also encouraged to expand the interpretation of their dialogue, so far as possible, by mustering and citing odd bits of special knowledge; especially, if it is arcane or recondite. For example, in the protocol (from Prince 1970) on which the scheme is based, the participants embark at one point upon a discussion of electric fish, and it turns out that a particular participant is quite an authority on this subject. The purpose of the dialogue is to explore and juxtapose several worlds, or universes of interpretation, in which to adopt perspectives, to develop a common metaphorical language, to resolve the issues at hand, and to reach a series of tentative agreements. Hence, although it is crucial to have expert knowledge about the original world (geology and engineering), it does not matter whether the propositions about other universes are factually true or false, so long as they hold together in some kind of derivation.

The "technical" terms are mostly self-evident in the context of the scheme but one of them, "force fit," requires special comment since it has a dual connotation. On the one hand, it means bringing together concepts that have matured in distinct "worlds" or "universes of interpretation" and on the other hand, it means resolving these concepts to produce a common meaning and to model it as an analogy relation. Conceivably, the result could be a simple analogy (for example, an isomorphism between principles or systems), but usually, due to the method employed, this is a generalised and realisable analogy relation.

There is one apparently arbitrary step in Scheme 5, namely, in phase 4 the leader selects a "world" other than the original (geology and engineering) world. It is clearly necessary to ensure that

SCHEME 5

One Synectic Procedure

Phase	Operations
1.	Examine statement of problem situation; for example, problem of extracting representative core sample of oil bearing rock, without adulteration in the process, from a great depth.
2.	Elicit objectives as conceived by participants; for example, "getting oil to tell me how crowded it is in reservoir stratum".
3.	Choose one objective for scrutiny (assume the objective cited is selected).
4.	Elicit instances (of chosen objective) in distinct world; for example, since the original problem is posed in a world of geology and engineering, elicit instances of the objective in a world of biology (these range from flies crowding upon dung to a culture of viral agents in a host tissue).
5.	Select instance for scrutiny; for example, virus culture in host tissue.
6.	Personal Analogy. This is an interesting and potentially powerful method of enforcing a perspective. It consists in persuading the participants to see the chosen instance situation as though they are some element in this situation; in this case, as though each participant is a virus and part of the culture in host tissue.
7.	Elicit "book title" from each participant. A "book title" is a pithy phrase which serves as a tag metaphor for the participant's experience in the role of a virus (in this case) and summarises a paradoxical or incompatible feature of this role.
8.	Select "book title"; for example, one quoted by Prince is *Compulsive Indifference.*
9.	Elicit instance situation in a biological world or a somewhat more general world that embodies the meaning of the book title; for example, the territorial and aggressive propensities of cats, as contrasted to dogs.
10.	Select resultant instance exemplifying chosen "book title" and "force fit" it to the original objective given in the world of geology and engineering; that is, cite an analogous situation in the original world.
11.	Examine efforts to "force fit" and select plausible "viewpoint" (synectics word) or possible recommended solution; for example, the idea of calming down a crowded roomful of cats gives rise to the plausible suggestion of freezing out a rock sample filled with oil droplets so that it is not polluted whilst being removed from the boring hole.

there shall be a difference (the technique hinges upon the coexistence of distinct universes of interpretation), and it may be expedient to leave this selection to the leader. However, there is no reason, in principle, why he rather than the others must determine the different universe, and in practice, his selection is coloured by the ongoing discussion.

The cyclic and re-entrant character of the process is made especially clear in Gordon (1966), a book which is primarily concerned with synectic principles as they are applied to learning. In Appendix I of Gordon (1966), the "viewpoint" is not charted as a terminal solution (recommendation) or set of solutions (recommendations) but as the genesis of a novel objective. Moreover, there are many, almost unchartable, "internal" loops; for example, the personal analogy phase can be, and often is, either replaced or augmented by a forced "direct analogy" between the distinct worlds or universes of interpretation. Whereas "personal analogy" stresses an analogical or metaphorical universe (akin to U in Section 3.1), "direct analogy" is a straightforward recourse to the realisable universes (X and Y in Section 3.1).

With these points in mind, and noting both Gordon's and Prince's insistence that the synectic process may either be interpersonal (as depicted in Scheme 5) or intrapersonal (in either case, however, involving distinct P-Individuals), it is not difficult to see that clusters of phases in Scheme 5 are designed to bring about the events noted in Scheme 1. The identification is summarised in Scheme 6.

The phases of the synectics procedure do not, and are not meant to, capture all of the underlying heuristics (the "deep structure" of the process catalysed by the group leader). In a sense, the underlying heuristics are made evident by following the procedural suggestions and mandates; the underlying heuristics are not written out as a series of transformations.

However, on reading the literature and (at least) toying with the method, it is evident that the procedures induce cognitive transformations similar to, if not identical with, those stated explicitly in THOUGHTSTICKER (Chapter 9). The explicit statement may be useful in guiding the conversation; for example, if it is agreed that the THOUGHTSTICKER transformations (epistemic symmetry, extrapolation, and so on) are desired, amongst other things perhaps, then we feel that the leader and perhaps the participants

SCHEME 6

Comparisón of Synectic Procedure and Present Theory

Clause in Scheme 1	Phases in Scheme 5 or Comments Upon Entire System
1. (Distinct P-Individuals)	Given throughout by integrity of participants and by differential perspectives as highlighted in Phases 5, 6, 7, 8.
2. (Distinct universes)	Highlighted in Phases 4 and 6 for the geological/mechanical universe and the biological/animal universe (on a par with X and Y in Scheme 2 or Scheme 3). The analogical universe of personal perspectives (on a par with U of Section 3.1) is made explicit in phases 6, 7, 8.
3. (Focus of attention)	Phases 5 and 6 juxtapose and coalesce foci established in Phases 2, 2, Phases 4, 5 and Phases 8, 9.
4. (Common language)	Maintained throughout by leader manipulation
5. (Common meaning agreement reached)	Phases 2, 3, 4 compared with Phases 8, 9, 10. Resolution is made explicit in Phases 9, 10 and is refined and reified in Phase 11.
6. Common meaning is generalised	Intention behind "generalising the perspective" in Phase 9, but the tendency to resolve by generalised analogy rather than simple analogy is part and parcel of the "force fit" operation and the events leading up to it.

would gain by knowing of them as explicit meta objectives. It is quite true that overconsciousness of such information could demolish the spontaneity and emotional interplay of the dialogue. But this is not a necessary consequence, and in practice, a substantial advantage may be gained by adding explicit "deep structure". Though our own theory lays *emphasis* upon systemic aspects of thinking and creativity, it *depends* as much as any other theory upon the conative as well as the cognitive facets of the intellect.

3.5. A Microstudy of Innovation

The last exemplary theory of innovation comes from a study of problem solving and training students to solve problems: Elshout and Elshout (1960). These investigators employed Guilford's "apparatus test" as their subject matter. A typical test item consists in the description or mention of an "apparatus"; for example, a chair or a razor (an "apparatus for sitting on" and an "apparatus for shaving with," respectively). The student is asked to think up and record an improvment of the "apparatus" in each test item, i.e., an improved chair on an improved razor. An improvement of *some* kind exists if the solution offered is distinct from the original apparatus but is recognisable, as having the same function as the original, perhaps having other functions as well. It was found that two very different kinds of strategy are used by students: the "locating problems" strategy and "successive transformations" strategy (abreviated to *LP* and *ST*, respectively). Of these, *LP* gives rise to responses deemed pedestrian or prosaic according to several extremely plausible criteria, whereas *ST* gives rise to creative responses.

Elshout and Elshout found it possible to pretrain students to adopt either type of strategy, using one or the other of two programmed texts. In their paper, they call the prosaic solutions, minor innovations, and the creative solutions, major innovations. Here, stress is placed upon the nature of *LP* and *ST* and the differences between them. As a matter of terminology, the solutions produced by *LP* are probably *not* innovative under the present terms of reference; those of *ST* undoubtedly are innovative.

Although Elshout and Elshout do not make the claim explicitly, they appear to *have* a cogent theory of innovation embedded in the distinction between the strategy types, and it is sufficiently detailed to allow for training operations that substantially increase the proportion of innovative solutions.

The strategies in question are as follows: (Scheme 7 and Scheme 8, below). The serial form is artificial and unrealistic; for example, execution of Step *LP*1 may continue as the other steps are instituted. But certain order relations are essential; for instance, execution of *LP*1 must start before *LP*3 is instituted.

Elshout and Elshout's terminology is very close in style and meaning to our own, and it is easy to see that their theory corresponds with singular accuracy to the relevant points of conversation theory, as do their results. For example, a "problem solving procedure" (in this context, at any rate) is a concept; the learning strategies exhibited in *LP* and *ST* are regarded as

SCHEME 7

Locating Problems (*LP*)

LP 1	List the attributes of the given apparatus (possibly an indefinitely long list) by abstracting from the instance given.
LP 2	Specify the uses of the apparatus. That is, how it functions in different contexts; for example, the chair functions as an instrument for sitting on, but it has the attributes "size" and "softness" which are of different consequence if it is used in a confined space or in the open air.
LP 3	Select an attribute that under one-use-context poses a problem or produces a difficulty; for example, the chair stands up and if its "size" is "large" this fact proves embarrassing if the chair is used in a small room.
LP 4	Determine the effect of changing the *value* of the selected attribute in a manner that eliminates the context-dependent nuisance upon the functioning of the apparatus; for example, though a dumpy chair is conceivable, a child sized chair is unacceptable to adult users.
LP 5	If the selected value-change destroys the function, return to *LP* 3 and select another attribute unless no attributes remain on the list, in which case, return to *LP* 1. If the selected value-change does not destroy the function, instate the change of value; for example, "size = large" into "size = small".
LP 6	Construct a modified form of the original apparatus that incorporates the selected and functionally innocuous change in attribute value. Thus "large chair" becomes "small chair" (with some specific meaning attached to *how* the chair is smaller than it was, i.e., narrower, shorter or whatever). Select a description of this modified form of apparatus as the solution.

"higher level problem solving procedures" (learning is problem solving about problem solving, and their "level" distinction like the L^1, L^0 distinction is a matter of convenience, not fact).

Moreover, the following point, though imported and imposed as an explanatory device, is probably implicit in Elshout and Elshout's account, though they do not speak of it in these words. The difference between creative thinking as governed by *ST* and non-creative thinking as governed by *LP* is simply that *ST* demands more than one-aim-at-once, whereas *LP* makes no such demand. Of course, the student pursuing *LP* instructions might

SCHEME 8

Successive Transformations (*ST*)

ST 1	List attributes by abstraction, as in *LP* 1.
ST 2	Specify the uses of the apparatus, as in *LP* 2.
ST 3	Select a tentative attribute that poses a problem in some context or other, as in *LP* 3.
ST 4	Change the value of this attribute or adjoin some attribute (giving it a novel value), such that the apparatus is rendered *dysfunctional.*
ST 5	Attempt to transform the structure of the apparatus so that it *does* function with contradictory values of the selected attribute (which may or may not be possible). For example, if the selected attribute is "posture" a chair that "stands up" occupies room space. Changing the value of the chair's "posture" so that it "lies flat" renders the chair dysfunctional. It may or may not be possible to invent a chair (such as a collapsible deck chair) that accommodates both values of posture.
ST 6	If the attempt to transform the apparatus is unsuccessful, return to *ST* 3 unless the attribute list is exhausted (in which case return to *ST* 1). Otherwise, if the attempt is successful, specify the modified apparatus and submit its description as a solution.

divide his attention. *LP* does not prohibit this. But the student who learns and obeys *ST must* do so.

The distinction occurs at Step 4 and Step 5 in *ST*. The fact is, an apparatus (in our jargon a model, albeit a mental model) cannot be simultaneously functional and dysfunctional in the same universe. On the other hand, the posited dysfunctional apparatus must work in some universe; it can neither be a stroke of caprice nor a fatuous construction. Hence, Step 4 in the *ST* instructions tacitly calls for the construction of two a-priori-independent universes; one in which the original apparatus works, and one in which the dysfunctional modification works. Further, the resolution to be attempted at Step 5 requires the contemplation and comparison of the two universes, each with its distinct focus of attention or aim selection.

Informally, we have found that students required to solve problems of an open ended type and given instructions that tally with those in *LP* and *ST* report that the comparison at Step 5 involves the interplay of personalised as well as problem oriented hypotheses. The student conceives himself, for example, as a user of the different pieces of apparatus, or as the progenitor of different theses about them. Generally, the emergence of the transformation which resolves the incompatibility is sudden; the student is conscious of the apparatus to be tendered as an innovation as a crystallised whole. He is not (clearly) aware of all the steps that lead up to the crystallisation, though by token of the fact that he can obey *ST* instructions or recognise his mental process as *ST* rather than *LP*, he is able to describe a series of commands he gives himself, or the constraints he applies in order to achieve this result.

This much is predictable in terms of the macrostate variables d_0, d_1, and d_2. There is a point (Scheme 1, clause 3) when d_0 is high, but its value approaches zero at "crystallisation". The act of reaching a common meaning (Scheme 1, clause 5) by hypothesis, due to concurrent autonomous operation, is associated with high d_2 (there is no awareness of "steps"). But, insofar as *ST* is described as a Fuzzy Procedure, d_1 is low. The student, under these conditions, knows how he innovates even though $d_2 > d_1$, he is unaware of the results until ($d_0 = 0$) they are reified as an artifact or a solid idea.

3.6. Other Possibilities

Similar spirited comparisons can be extended to other theories rich enough to posit a process underlying, and somehow peculiar to, creativity. For lack of space, the matter is not pursued, but the reader may find it rewarding to examine the creativity theories of Bateson (1972), Maslow (1954), and Fischer (1969, 1974) in the light of the foregoing discussion. These are chosen, as far from exhaustive or exclusive examples, for two reasons: first, each is a beautiful and well-attested statement; secondly, the theories stem from different departments of cognitive science.

Bateson's view of innovation emerges in part from social and anthropological studies, and in part from individual psychology. The doctrine of "deutero learning" and "higher than deutero-learning" establishes a positive connection between "ordinary" and "creative" thinking; specific mechanisms, such as the cultural "double bind" and its several analogues, set the stage for innovation (or, in the present jargon, for "many aim" operation). Various hierarchically organised homeostatic mechanisms are compatible with the picture of coalescence and resolution drawn in this chapter, and the fundamental evolutionary component is compatible with conversation breeding.

Maslow's theory is set in a less encompassing framework, a species of transactionalism, but once again, it contains the full complement of processes, and these are compatible with the identifications so far mooted. Similar remarks apply to Fischer's theory, which is stated in a series of quite widely scattered papers. Its background is mixed: first, an eclectic but basically mentalistic psychology, and secondly, the area of neurophysiology and psychopharmacology. In "translating" Fischer's concept of a perception-hallucination continuum (in which creative productions occupy a special place), it is necessary to "translate" simultaneously the mechanisms of symbolic evolution which underlie this continuum. Further, it is necessary, and apparently legitimate, to identify Fischer's concept of "private" and "public" verification of the images so produced with the notion of modelling (intellectual or factual) in correspondingly "private" and "public" universes; to note, as Fischer does, that the status of a *creative* image (our "idea") is aleatory. Concordance between the model of an image and of an individual (or the societal status quo) is undecided at the instant of inception.

4. MERIT IN IDENTIFYING THEORIES OF INNOVATION

We embarked upon this chapter with the promise of unification amongst theories which, taken alone, have points of disparity. This promise has been fulfilled by exhibiting a common systemic core adequate to accommodate variously described processes. The essay might be justified on these grounds alone, but some other advantages are also gained.

The present theory forms a natural bridge between the many person situations (Chapter 6), the many aim situations (many person or just one) which seem to engender innovation, and the process (Chapter 9) of "learning to learn". Differences of degree exist; these aspects of reality may be usefully discriminated. But the underlying process is the same throughout. It involves "Conversation Breeding" (a comprehensive type of symbolic evolution), the juxtaposition of aims or perspectives and their resolution by the coalescence of P-Individuals in a common-meaning agreement. Since P-Individuals, the major working units of our theory, may be localised or distributed over several brains and since several may coexist in one brain, the perplexing differences between societal and personal innovation mostly evaporate.

In return, our theory is buttressed by a body of evidence. Chapters 7, 8 and 9 gave some examples of innovation observed in THOUGHTSTICKER and the "learning to learn" experiments. But since under these circumstances cognition is laboriously externalised, the instances are rare and miniscule: a picayune body of data quite inadequate to support a serious hypothesis. So it would remain after many repetitions of the experiments. For data about realistic innovation are garnered over years from different cultures, and the most dramatic instances are best observed beyond the laboratory (as Minsky remarks, in order to study "intelligence" examine the cognition of someone who is superlatively intelligent; by the same reasoning, creativity is best studied amongst people or systems or groups who have an outstanding creativity record). Now the data supporting the other theories usually *are* of the required kind; they are far more convincing than a few laboratory transactions. Insofar as the other formulations can be placed in register with the present constructs, much of this data is put at the disposal of our theory and is held to lend it inductive support.

5. PREDICTION AND PRAGMATISM

Obviously, we claim to predict the form of an innovative process. The tricky question is whether or not it is possible to foster creativity, and if so, by what means. To some extent the question has been answered in the affirmative. In Chapters 8 and 9, we cite procedures for encouraging various ingredients of innovation; for example, these listed under "aim initiation" or the overall heuristic of THOUGHTSTICKER, which induces a resolution behavior akin to Elshout and Elshout's "successive transformation" tactic. It was noted in Chapter 9, Sections 3 and 4, that these methods are not bound to pieces of machinery, however convenient the machinery may be; by token of this, principles extracted from usage of the operating system have been used successfully to approximate the same result in entirely non-mechanised studies of "learning to learn". Elshout and Elshout obtained similar results in the context of the "Apparatus Test"; Gordon and Prince, in the practice of synectics.

The scope is wider than these parochial examples suggest. First, the recommendations arise from the essence of a theory; they are not just arbitrary or empirical suggestions. Next, the theory has been identified with the systemic core of other theories for which recommendations as diverse as the areas of interpretation already exist. So, for example, it is possible with Bateson's and Barnett's theories to stipulate cultural organisations conducive (say) to "aim initiation" (one ingredient of innovation), and to infer that innovation is more likely to occur if these organisations are realised together with means to guide the other ingredient processes. Or, in the psychophysiological interpretation of Fischer's theory, it is possible to argue that certain brain states increase the likelihood of innovation; at least, that these states will stimulate appropriate subprocesses.

6. RELEVANCE IN EDUCATION

Often and probably rightly, innovation is cited as desirable as an end in itself. If that is agreed, then there appear to be rather complicated training operations which encourage innovation; either the mechanical or non-mechanical expedients of Chapters 7 to 9. It is of interest that these operations tally well with the conditions held to be fecund in this respect by process oriented theorists; in contrast, they do not tally well with the manoeuvres of simple minded encouragement which (however attractive they are in terms of potential cost benefit) have proved disappointing (see, for example, the very clear and candid review of one such endeavor by Torrance and Gupta 1964).

Suppose, however, that innovation is not so universally valued, that children or adults should not be specifically "trained to innovate". After all, a number of career oriented educationalists honestly take this point of view.

It would still be agreed, in most quarters, that "learning to learn" and "group competence" *are* important parts of the educational system (if not of the curriculum). For example, even if the object is to produce technicians and specialists as the main product, they will benefit from versatility (a component of "learning to learn") and are likely to be better citizens if they understand each other rather than acting as robots. Moreover, it has been argued (and the case appears to be indisputable) that an efficient educational system, be it for generalists or specialists, *depends upon* "the art of learning" disseminated amongst the students. This is so for the following reasons: (a) That rapid learning with sensible retention is achieved (in practice) only by utilising the valid analogies in a subject matter, discovering them and checking their proper comprehension, both of which entail the "art of learning"; (b) because only a small fraction of the environment is an academia where knowables and do-ables are coherently structured. Most learning must (for most people) take place outside an institution, on the job or in the street; a moiety of the time spent in an institution should, therefore, be devoted to indoctrinating the "art of learning" (from unstructured surroundings), just as time is spent inculcating the other basic skills of communication, arithmetic, and so on.

Whichever point of view is adopted — namely, "Innovation is good in itself," or "Innovation should not be generally encouraged when we need specialists or hodmen," or "I am indifferent to innovation or not, but education should be, in some sense, efficacious" — the comments in this chapter and the last are still very much to the point. It has been argued that the processes called "innovation" and "learning to learn" and "learning to participate in a group" have a common component and that, operationally speaking, their encouragement is a matter of adopting the same class of tactics and methods. I do not think, whatever is done, we can guarantee that someone will prove a brilliant inventor/artist/politician. But we do have the inklings of how to achieve a less grandiose, though no less laudable, goal: that this person will learn to make sense of and savour his intellectual or concrete environment, its past and its future; that he will learn to love his neighbour and simultaneously aspire to ambitions which (I do believe) have no limit whatsoever.

Chapter 11

General Conclusions and Recent Developments

Since this chapter is the last one, I take the liberty of conjecturing about questions which seem important enough to warrant critical imagination. Several old themes are revitalised and combined so as to weave new fabric. A prefatory qualification is in order. The speculations rarely concern matters of fact. The facts are given in some adequate sense; for example, they are consensually undisputed or positively demonstrated, or (when factual options remain open) the minutiae are experimentally decidable. What is at issue is a view of the world, sometimes a composition of views; the question for debate is whether any or all of these world views are worth adopting. Such judgments, if formalised at all, rest upon criteria of utility and aesthetic compass. Insofar as I have made certain affirmative personal judgments in choosing a gaggle of speculations, it is only fair to comment (since many readers may disagree) that though I surely respect pragmatism, my choice is also weighted strongly and unashamedly by aesthetic preference. I think the new fabric has a beautiful pattern, and its threads establish fascinating connections between otherwise disparate notions. Locally at any rate recognition of this pattern has often proven useful. As pure opinion, hunch or belief, the same pattern may have general utility and lead to some sensibly fundamental discoveries. The following aphorisms and mental exercises are intended to support this opinion, hunch or belief.

1. CHARACTER REPRESENTATION

There is nothing unfamiliar about the idea of a character appearing in the context of a play or a novel, and it is also fairly common to encounter classes of characters or roles (for example, town clerks, solicitors). Using some specific instances, for the possibilities are legion, we shall argue that the notions of character and role are of the utmost educational significance. Yet, for various reasons, the subject of characterisation is either treated intuitively (the art of an author is involved) or avoided like the plague. The reasons for avoidance appear to involve ways of viewing reality rather than the inherent difficulty of the subject. Hence, we shall attempt to clear away some conceptual brushwood and lay the foundations for an approach to this matter.

According to the present thesis, a character is a *representation* (π_A) of a *P-Individual* (A) and a role, in the sense of a class of characters is a representation of a class (π) of π_As with certain features in common. By prior definition, A is the execution of π_A in some existing but unspecific L-Processor, and taken thus, is a coherent and self-replicable set of beliefs; conversely, π_A is a static representation of these beliefs, minimally as a coherent set of propositions (Chapter 4). Extrapolating, π is required to maintain coherency and to have member representations (for example, π_A), all of which have some coherent subset (the role specification, at least) in common. Though freshly introduced, these definitions are probably uncontentious, but all of them are qualified by the existence of a context in which the characters or roles appear (Mr. Jingle is a character in the context of *Pickwick Papers*, and Miss Prism is a character in *The Importance of Being Ernest*). Such a contextual binding seems to be an essential ingredient of characterisation (hence, the static representation of P-Individuals) and is written "Q"; thus "π_A in Q" is the proper statement of π_A. Usually, Q is a story, a plot, or a scenario, but it need not be.

It is essential to distinguish between characters in general (such as π_A in Q, Mr. Jingle in *Pickwick Papers*, Miss Prism in *The Importance of Being Ernest*) and particular static inscriptions of these entities. Confusion is virtually impossible in literature or drama (we do not get mixed up between *Pickwick Papers* and a particular printed edition of *Pickwick* on that bookcase). In contrast, confusion is quite likely when these notions are generalised.

If the character is executed in some L-Processor (π_A to realise a P-Individual A), it is also essential to distinguish between the general and the particular enactment. To press the point home, Mr. Jingle is executed in any reader's brain, and even if differences in interpretation are discounted, the general execution is distinct from Joe's or Jim's particular execution. Similar comments apply to dramatic enactments; the general case of Miss Prism is distinct, even if differences in interpretation are discounted, from enactments by different and specific actresses or the same actress on different nights.

In general, a play or a novel involves more than one character; as a rule, we speak of "π_A in Q" and of "π_B in Q" and notice that a rendering of the novel or a performance of the play involves "π_A and π_B in Q".

In particular, we have constructed a framework in which an assertoric thesis T stands as a special case of characterisation, and the student who learns T acts a *null* character; his enactment is of the expert's perspective, when expounding T.

2. EDUCATION PARTICULARS

To see how this bears upon education and epistemology, let us consider a few of the situations discussed up to this point and take the opportunity to indicate their significance.

2.1. Innovative Learning

Many aim situations (Chapter 7 onwards) and innovative situations in particular (Chapter 10) involve characterisation. Minimally, this is of the type, "A's image of B's image of a topic T," which serves (rather than a plot or a story) as the context, Q = T. The characterisation is genuine insofar as this statement can be rephrased, "A's image of B in the context of T," or "π_B in T," which is generally executed to form a P-Individual in A's brain. Under these circumstances, the image itself is A-constructed so that we may either talk of the general execution of "π_{BA} in T" performed by an unspecified L-Processor, or else of the execution of "π_B in T," subject to the constraints imposed by executing A's image of T (π_A in T) within the same brain; that is, of an internal conversation on topic T between the execution in A of π_A and the execution of π_B, and generally leading to an internal agreement about topic T. Since we have already stressed that transactions of this type whether internal or external to a brain play a critical part in innovation, no further comment is needed.

2.2. *Rival Hypotheses*

There is increasing empirical evidence that certain theses can only be understood if their *progenitors* are characterised. For example, rival theories S, T (the wave/corpuscular controversy) can only be represented in a conversational domain if their progenitors A, B (Huygens/Newton) are also represented as characters π_A, π_B in the same conversational domain. The proposition is not altogether surprising, for it is common practice to laden instruction with historical and personal detail sufficient to characterise protagonists (not Huygens and Newton but adherents of each school of thought). But the empirical claim deserves careful formulation since a strongly affirmative finding, indicated by the data so far available, would place a stamp of approval upon current practice. If we are right about understanding rival hypotheses, then the historical and personal background is *essential.* It is not, as often supposed, gratuitous enrichment material to be employed as an optional embellishment. The claim is that students can *understand S and T* only if these theses form the context for characters π_A and π_B who are debating the merits of S and T, so that the context of *understanding S and T* is a series of A, B agreements and disagreements.

A very similar claim is made with respect to the ambiguous figure in Chapter 7, Section 3. Clearly, a student might understand S alone and understand T alone and link S and T by some tenuous indexing scheme, permitting concepts of S and T to alternate in consciousness. By the same token, a student can understand the geometry of three dimensional lines and three dimensional blocks, and he can conceive or envision the ambiguous figure; even draw it with the perceptual tricks. But understanding S and T means understanding a dispute (the wave/corpuscular theories are really taught to illustrate the process of scientific development, not primarily as a bit of optics). We claim that a student cannot understand this dispute unless there is character representation, any more than he can understand the ambiguous figure (qua figure, rather than as a series of tricks).

2.3. *An Invitation to Act as a Dramatist*

The last example (rival hypotheses) rests upon the existence of a peculiarly constrained representation of characters; namely, representations of "π_A in S, T" and "π_B in S, T" within a conversational domain (rather than in a book, play, or as any unspecified mental scheme). The question is, "Do such representations exist?" And, if they do, there is a further question, "What do they look like?"

These questions are tackled in stages. As a first step, we show that a *context* Q of the usual form (a plot, story or scenario) can be constructed. An exemplary construction is shown in Fig. 11.1, which depicts the entailment structure for the "Spy Ring History" test of Chapter 3. True, this is a special case, but there are no obvious limitations, sheer complexity apart, upon the plots, stories or scenarios which may be represented in the same manner.

Further, this special case is worthy of study, for there are circumstances under which the "Spy Ring History" task acts as an invitation to dramatise within a contextual framework that is virtually a *tabula rasa.*

Although the structure shown in Fig. 11.1 is moderately complex, it is also extraordinarily arid; the syntactic or systemic similarities are quite specific, but the structure is semantically barren. Since almost any choice of distinguishing predicates will suffice, the student can give any meaning he likes to "spies" or "countries". The degree of freedom permitted by such a sparse description is, of course, deliberate. Not only are we anxious to find out how different students recall the material (by operation learning or by comprehension learning the relations, as in Chapter 3), we also desire to find out how the student clothes the structure in descriptors of his own invention in order that he can actually learn these relations.

First of all, there is no reason why students should not conceive the entire Spy Ring History as an *it*, an *object.* For example, they could construct the spy networks as graphs from lists, or as others do, could reconstruct them from the Cartoon function. A slightly more sophisticated approach, also observed, is to construct finite-state-machine-like-representations that generate the communicative behaviours.

On the other hand, there is no reason why students *should* con-

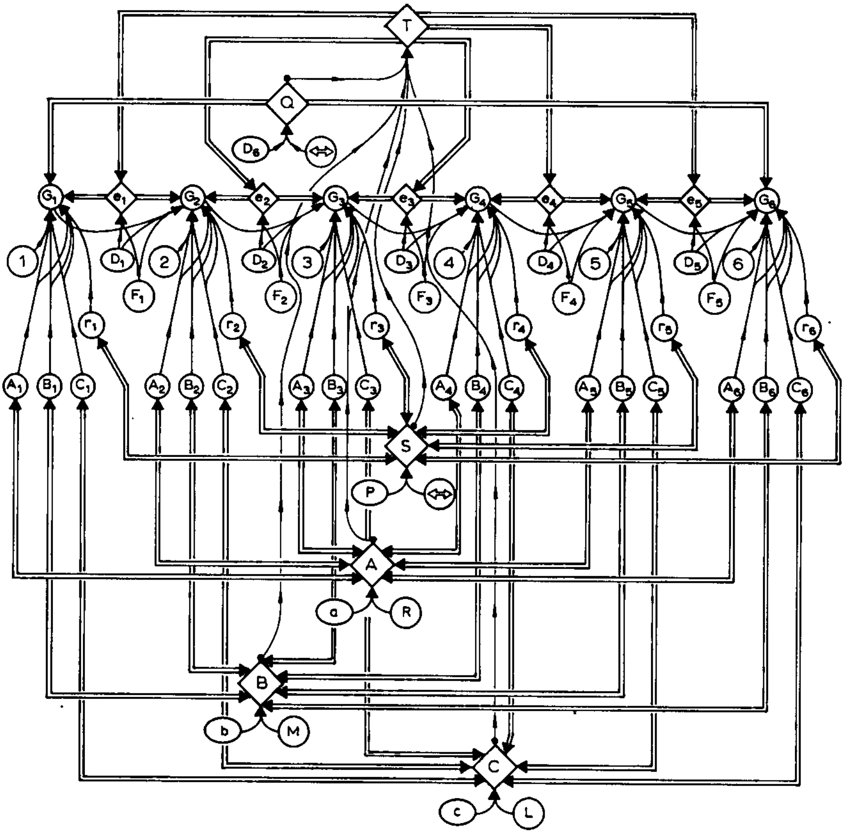


Fig. 11.1. Entailment structure for "Spy Ring History" test described in Chapter 3. The "Spy Ring Graphs" or connection networks are $G_1 \ldots G_6$ (only G_1 to G_5 are presented in test but G_6 may be inferred) for years 1880, 1885, 1890, 1895, 1900 (and, inferred only) 1905. A, B, and C are the countries' predicates; L (left), R (right), and M (middle of) being the systemic (i.e., geographical) component, and a, b, c an arbitrary (invented) series of semantic distinctions. $D_1 \ldots D_6$ are arbitrary (invented) distinctions between indexed eras. $F_1 \ldots F_5$ are the cartoon (graph product) functions establishing similarity component of between-era analogy relations $e_1 \ldots e_5$. Q, the cyclic part of the product, is determined by the isomorphism between G_1 and G_6 (the network in 1880 and in 1905). $A_1 \ldots A_6$, $B_1 \ldots B_6$, $C_1 \ldots C_6$ are countries predicates, arbitrarily distinguished in each era (1880 to 1900), provided they respect the geographical constraint, which is invariant. The graphs, G, may be generated by combining these predicates with the ordered-pair lists; 1, 2, 3, 4, 5 (recall 6 is not spelled out to a student), or by combining the predicates with role specifications $r_1 \ldots r_6$ sufficient to generate the behaviours of the "spies". S is a role isomorphism; the analogy relation that preserves roles but distinguishes different "spies" (by the arbitrary, or invented, distinction P). The entire system specification, T, can be learned in many ways; amongst others by a join of the analogy relations A, B, C, S, Q.

ceive the Spy Ring History as an *it*, and some of them do not do so. The latitude of the scenario allows any student to conceive the spies as characters, or even to characterise the social organisation of a spy ring or a country. Some students take advantage of this possibility and dramatise the system as a story involving P-Individuals, persons, or societies quite literally and non trivially pronominalised as "He" or "She". Notice, these students are acting as authors or dramatists. It is quite incidental that they act in this manner *in order to recall* some rather banal syntactic or systemic relations. It is far from incidental that whenever students act as dramatists they do *and must* to some degree participate in the enaction of their own drama.

Using the "compromise" techniques employed in the "learning to learn" experiments, it is certainly possible to exteriorise some facets of the student characterisation, and thus to gain some insight, albeit an inkling, of how characterisation proceeds. In other words, our data are not confined to verbal reports, though these are extremely valuable. For all that, a more general treatment of characterisation is required in order to support the contention that characters (as well as the context) can be represented adequately in a conversational domain.

3. A CLOSER LOOK AT THE CONVERSATIONAL LANGUAGE L

Even though I wrote them, I find the contents of this section quite strange and fully expect the reader to share this perplexity. So far as I can see, the argument holds water for all that, and an attempt is made to dissipate the feeling of oddity in the commentary that follows in Section 5.2 (some readers may prefer to look it over before continuing).

Any P-Individuals, A and B, have a language L in common, however primordial it may be. This is a conversational (or addressed programming) language, and it is an *interpreted* language; its universe of interpretation being a class of L-Processors.

By the same token, the representations π_A and π_B, of A and B in a conversational domain have something in common, and it is necessary to see what it is at this stage in the discussion. These entities (π_A, π_B) are static, not dynamic like A and B. But we wish to argue that what π_A and π_B have in common (regardless of any differences in their constitution or their interpretation) is in one sense the same as the communality between A and B; namely, the rudimentary elements of L.

Consider any non-trivial L metaphor. It appears in a conversational domain as one or more analogy relations, themselves *replicable* and coherent, between two or more sets of coherent propositions (Chapter 4). If the entailment structure of the conversational domain is augmented by a specification of the set of $\underline{\text{Proc}}^1$ (or, if preferred, of *DB* and *PB* operators as in Chapter 5) needed to execute structures that exist and to create further structures (i.e., the $\underline{\text{Prim}}^1$ of the previous monograph), and if the *BG* of the entailment structure of the conversational domain is augmented by the $\underline{\text{Proc}}^0$ needed for this same purpose (i.e., the $\underline{\text{Prim}}^0$ of the previous monograph), then the original structure, though still static, is of the form π_A or π_B. Let the conversational domain also contain the representation of a context Q made up of topic relations, T in Q, and to secure observability, let one L metaphor designate a personal analogy between π_A and π_B in Q. Certainly, the structure even at this point is static. However, if there is an L-Processor (or a set of them) in which the static encoding can be realised, it becomes an observable conversation between two or more P-Individuals, A and B. The question is, "What does it mean to realise π_A, π_B in Q, within an L-Processor." (And notice that the static encoding to be realised is augmented by a specification of $\underline{\text{Prim}}^1$ to execute derivations and $\underline{\text{Prim}}^0$ to execute explanations.)

L contains (at least) an operation sign, call it "$\Rightarrow$" to avoid specificity, which stands for implication or production or derivation. Although this sign is regarded as identical by any collection of P-Individuals A, B ..., this should not suggest that $\Rightarrow$, as judged impartially by an external observer, has the same meaning in A and B. On interpretation in an L-Processor, the operation sign $\Rightarrow$ stands for an *act*; something occurs. But, without further specification, this act may be a doing or an explication step or a derivation step.

L also contains at least an agreement sign, call it "$\Leftrightarrow$" to avoid specificity, which stands for correspondence. Although the sign is identical to any collection of P-Individuals A, B ..., this should not suggest that $\Leftrightarrow$, as judged impartially by an external observer, has the same meaning in A and in B (from his point of view, agreement is not identity). When $\Leftrightarrow$ is interpreted in an L-Processor, it indicates syntactic or systemic equivalence, but this may be an equivalence of doings or explanations or derivations.

The signs $\Rightarrow$ and $\Leftrightarrow$ appear in the conversational domain insofar as the derivation arcs in the entailment structure correspond to occurrences of $\Rightarrow$ when the static inscription is augmented by the Prim^1 (or the *DB*, *PB* operators), and they correspond to the delineation or execution of the *BG* when the static inscription is augmented by the Prim^0. Similarly, occurrences of $\Leftrightarrow$ mark systemic analogies; namely, groups of $\Rightarrow$ occurrences that are distinctly placed, but otherwise identical in form.

The compilation and interpretation of π_A, π_B, Q in an L-Processor is predication: a realisation of the semantic descriptors in the conversational domain. Some predication exists since L is an interpreted language. But, in *general*, it is ambiguous in respect of the interpretation of the imperative given to $\Rightarrow$ (as doings or derivations or thinkings, etc.) and the interpretation of $\Leftrightarrow$ (as various kinds of equivalence). With this interpretation ($\Rightarrow$ replacing derivation arcs by real derivations, or production arcs by real explanations), π_A and π_B in a purely *formal* sense become two or more P-Individuals, A and B, in the context of Q.

Under the *particular* circumstances specified, the realisation is, however, disambiguated and observable as a strict conversation (in the sense of this book and the previous monograph) between participants A and B. That is, L may be stratified by an external observer into levels L^1, L^0 and a free level (L^{-1} or L^2 as desired), and the A, B conversation is anchored upon the topics T in Q, which an external observer regards as fixed, and *the* conversational domain. That is, Q is the *support* of the previous monograph. Within that framework, L^1 occurrences of $\Rightarrow$ stand for cognitive acts or derivations; L^0 occurrences of $\Rightarrow$ for acts of modelling or explanation; and occurrences of $\Rightarrow$ in the free level (L^{-1} or L^2) stand for behaviours or the execution of models. Similarly, L^1 occurrences of $\Leftrightarrow$ signifies A, B cognitive agreement; L^0 occurrences of $\Leftrightarrow$ signify in A, B agreement over a model or an explanation; and occurrences of $\Leftrightarrow$ at the free level (L^{-1} or L^2) stand for A, B behavioural equivalence. But, further postulates are needed if the realisations A, B of π_A, π_B are to count *non-formally* as P-Individuals. These postulates are conditional.

(a) Even if $\pi_A = \pi_B$, their realisations are distinct ($A \neq B$). One obvious and common possibility is that π_A is realised in one L-

Processor α, and π_B in a different L-Processor β, and that α and β are distinguished independently (that is, α and β are distinctly M-Individuated in the sense of the previous monograph, for example, spatially demarcated brains).

(b) Even if $\pi_A = \pi_B$ and A, B are realised as P-Individuals in the same L-Processor (for example, an external observer does not see α and β as distinctly M-Individuated brains but as the same brain), it is still true that $A \neq B$. In other words, the predication of π_A and the predication of π_B carve out distinct universes of compilation and interpretation in the same processor.

4. CONDITIONS FOR INDIVIDUALITY

We sum up (a) and (b) as a principle of *privacy in the face of agreement*. Even if A and B are utterly agreed in respect of all topics T in Q, there are distinct individuals. Under $\Leftrightarrow$, occurrences of $\Rightarrow$ may be tagged $\Rightarrow_A$ or $\Rightarrow_B$. Equisignificantly, the predication (alias interpretation) of π_A is distinguished semantically from the predication or interpretation of π_B.

As soon as A and B operate upon Q, the conditions of a strict conversation are contravened, especially since further encodings (π^*_A, π^*_B) emerge when the conversational domain evolves (the "breeding" paradigm of Chapter 6). But "privacy in the face of agreement" is preserved.

5. WHY NOT CALL L-PROCESSORS BRAINS AND LEAVE IT AT THAT

Of (a) and (b), case (b) appears to be more general, and the evolution of P-Individuals beyond the confines of a strict and anchored conversation appears to be the rule. Otherwise, we might as well have said "brain" instead of "L-Processor" throughout.

One example will be sufficient to spell out the scope of these comments and some of their epistemological impact. The example stems from a series of carefully written papers by Lakatos (1968, 1973), which should be consulted for historical perspective, as well as a philosophically defensible statement. * My summary does

* As noted in the Introduction, another example is an educational system as it is conceived by Daniel. In that case, distinct πs would characterise the mores and career structures of educational systems which encouraged or discouraged analogical reasoning.

scant justice to the original, but so far as it goes, is accurate.

Lakatos argues that scientific development, though it does involve various well-accredited tactics such as Popperian falsification, has primarily to do with social organisations which he calls "research programs" and which roughly correspond to "schools of thought". These "organisations," whilst employing standard modes of inference and deduction in respect of particular hypotheses and data, are basically self-perpetuating; that is, they are coherent systems of belief which maintain their coherency very often by operations that do not have immediate recourse to factual validity. Lakatos cites and details numerous cases and pursues the development or evolution of several such organisations.

I propose that a "reserach program" in Lakatos' sense is a P-Individual with a representation of the form π (a role or character class). The realisation of π is an L-Processor (a societal one) but is neither a brain, nor even only a collection of brains, for the compilation and interpretation of π also involve current technologies and other inanimate components. Further, an adherent or advocate of π is a P-Individual with representation π_A, π_B ... in a context Q which includes at least some of π. Surely, π_A, π_B ... are realised L-Processors, but once again, these are not generally unique brains.

Perhaps A, B (the realisations of π_A, π_B) act as progenitors or theses about some or all of the beliefs in the realisation of π: by hypothesis, *all* theses are of this form for some A, B and some π. Such theses (and by hypothesis *only* such theses) are represented in conversational domains with A, B as subject matter experts.

Because of the caveats encompassing the modes of inference in π (and the interpretation of $\Rightarrow$ in its realisation), it is possible, likely, and perhaps necessary that π contains rival theses. Suppose these are S and T of the previous discussion and are espoused by A and B, respectively. We maintain that a representation of S and T of π in a conversational domain may only be understood if accompanied by a partial or complete representation of π_A and of π_B in a context Q which depicts the realisation of π (that is, Q is a story or scenario for the enactment of π on a par with the story or scenario in Fig. 11.1).

5.1. Monism and Pluralism

I can neither prove nor disprove the rectitude of these conjectures; they are advanced as plausible and useful means of throwing light upon certain epistemological issues and their claim to plausibility will be backed up by culling examples from other fields of educational concern (notably, the nature of educational media and developmental psychology). There is nothing which forces anyone to accept, or even consider, this view of things.

However, *if* the view is considered and deemed plausible enough to merit tentative acceptance, it *is* possible to avoid a species of pluralism (the P-Individual/M-Individual pluralism of the previous monograph; akin to, but not identical with, mind/body dualism) which is otherwise strongly suggested. Overall, I am proposing that *the* universe of compilation and interpretation *is* an L-Processor which may be locally carved up into portions α, β ... separated by regions in which only a more restricted interpretation of $\Rightarrow$ and of $\Leftrightarrow$ is possible i.e., processors of lesser capability. The carving up and local specialisation is due in the first place to the compilation and subsequent execution of encodements like "π_A in Q" or "π_B in Q" or (in a restricted but not essentially different case) like S or T. Since coded inscriptions (like π_A, π_B or S, T) are built in the last resort by progenitors A, B ... (the realisation of π_A, π_B in the L-Processor, albeit, with local compartments like α, β and their separating boundaries), we retrieve in evolution a systemic monism and with it the convenient permission to see each stage of the evolution as the creation, compilation, and execution of a program.

5.2. Commentary on the Previous Sections

As promised at the outset, I shall try to indicate why this argument, though basically sound, seems strange and curiously tortuous (to me, at any rate).

The trouble arises in working with distinctions between static entities, like entailment structures or other coded representations, and the dynamic entities which realise whatever is encoded in usually many, and always more than one, way (for example, representations are realised as programming or modelling operations, as program construction operations, as program executions). Simultaneously, we need to work with different kinds of particularity and generality, keeping them mentally distinct if the argument is to make sense. For example, it is necessary to distinguish π (the general organisation) from π_A (a particular organisation), whilst noting that π is (in a different sense) more general than a particular, spatially localised inscription of π; that π_A is more general than a spatially localised inscription of π_A; that particular inscriptions are realised more generally (in yet another different sense) by one or many processes, A, in any L-Processor; and that the realisation of a particular inscription of π may incorporate all of these processes.

Mental gymnastics of this kind are familiar enough in biology and genetics where global argument relies upon distinctions between general and particular organisations (genotype, phenotype); between organisations and static inscriptions in DNA or other hereditary material (the set of *possible* alleles, the alleles realised in the gene pool of a population, the genetic makeup of the chromosomes in a particular zygote); between static inscriptions and their realisation (organisms in a subspecific population, a particular organism including its growth and differentiation, as well as the manufacture of gametes that are fed back, both material wise and information wise, into the system). By custom, such gymnastics are not called for very often in psychology or epistemology since these subjects are reputed to exist in two forms: broadminded but deliciously soft, and hard but delightfully simple.

An equally barbed parody could have been aimed not too long ago at biology/genetics/evolutionary studies, as they were popularly conceived. But the content of such epigrams, for what it is worth, underlines a prevailing contentment with a limited field of enquiry, rather than making a substantive comment about our science.

My contention is that the problems germane to education tax the full apparatus of psychology and epistemology. If that apparatus is employed by assimilating systemic and information theoretic notions to harden the broad perspective, then global argument (which is mandatory for resolving the problems in question) does involve mentally elusive distinctions of the type encountered in biology or genetics.

It should be emphasised that the parallelism is intended to relate two ways of thinking and not to establish a similarity between the subject matters. Genetics and educational psychology have more differences than similarities. Some of them are very fundamental (for example, whereas the concept of an "organism" is fairly well defined in genetics until you consider its immunological as well as its spatial integrity, the concept of a "person" in psychology depends for most practical purposes upon the type of measurement and the enquiry in hand). Our main point is that interesting educational applications of psychology and epistemology demand a degree of sophistication which nowadays seems natural in genetics or biology. The strangeness of the argument in the last section is due to the fact that comparable ways of thinking are currently alien to education.

The question at issue is whether or not the trouble taken (over this theory or any other theory) is likely to pay dividends. We contend that this question can be answered in the affirmative and believe the discussion in the body of the book lends support to this view. However, as a concluding endeavour to press the point home, we shall turn to two educationally crucial matters (a useful theory of media and a useful interpretation of data from developmental studies) and show that the present approach leads to novel insights, hypotheses, etc., which could only be formulated within an inherently complex frame of reference, either this, or some equally difficult theory.

6. EDUCATIONAL MEDIA

With the exceptions referenced in the sequel, current attempts to classify media (as televisual, radio, written material, spoken utterance, mime, gesture, and so on) rely upon perceptual characteristics. The medium itself is regarded as a kind of signal channel linking spatially distinct transmitters or receivers (teachers and students, for example). Undeniably, this is a valid way of looking at media and the taxonomies derived from it are often valuable. But it is not the *only* way of looking at media and it is insufficiently general.

For example, studies based upon the signal channel scheme are seldom able to answer salient questions like, "Should this subject matter be purveyed by ETV or radio or by course modules?" or "What is lost or gained by transferring the *Goon Show/Sesame*

Street/Blue Peter from radio to television or vice versa?" It is relevant to remark, as people do, that "television provides a larger communication bandwidth than radio"; or that "books are at hand for reference, whereas radio transmissions are not"; but, however precise, these remarks are insufficient to furnish guidelines for the cost-beneficial deployment of media resources.

To deal with deeper questions, we need a broader theoretical base and a more subtle estimate of the degrees of freedom available to an educator/producer/director (or for that matter an advertiser) who employs the media to convey a message.

6.1. Prerequisites for a General Theory

McLuhan (1970) stated the prerequisites for a theory of media in two comments, "Media are extensions of the brain," and "The medium is the message". We have arrived at much the same conclusions by a different and possibly devious route. The advantages (if any) of our approach are that constructive recommendations, not unlike McLuhan's, can be issued from a theoretical and potentially quantitative platform and that the two superficially disjointed statements are seen as near complementary, at any rate as intimately related.

Let me translate "Media are extensions of the brain" as follows. (a) Media are precisely modelling facilities, qua processors in which programs are compiled, interpreted and executed as demonstrations or explanations or learning strategies. Modelling facilities act as extensions of the brain qua L-Processor and may, given a liberal design, approximate an L-Processor, or to go one step further: (b) Brains are distinct just because they are carved out of a pervasive L-Processor or general medium by more restricted and specialised regions (still modelling facilities but of more limited capability than an L-Processor).

6.2. Constraints Imposed Upon General Media

Of these propositions, (a) is relatively uncontentious and suggests a classification of media in terms of ability to accommodate demonstrations, etc.; that is, in terms of the interpretation which can be given to the L signs $\Rightarrow$ and $\Leftrightarrow$ together with the number of a-priori-independent subprocessors, each able to accommodate in parallel some different interpretation of $\Rightarrow$ and open to coupling or local synchronisation signified by the L sign $\Leftrightarrow$.

For example, the most restrictive facilities or media only permit the execution of compiled programs (the working of models) and thus accommodate no more than simple behaviours, in the limit the null or static "behaviour". The next category provides for the inscription and display of serial programs as well as permitting their execution. In order to represent analogy, several independent processors of this kind must be colligated in parallel. Each processor is able to accommodate a different (but L^0) interpretation of $\Rightarrow$, say $\Rightarrow_x$ and $\Rightarrow_y$, and the processors are coupled by a further facility giving an L^1 interpretation to $\Rightarrow$ and realising systemic equivalence $\Leftrightarrow$ between submodels realised by occurrences of $\Rightarrow_x$, and other submodels realised by occurrences of $\Rightarrow_y$. Scenarios, per se, are dynamic analogies (i.e., in the literal sense, parables) which can be accommodated within an indefinitely extensible medium of the type required to model analogies. Characterisation, on the other hand, involves a medium corresponding to an L-Processor, and story telling (though still a form of modelling) calls for the colligation of several L-Processors within the contextual frame of a scenario. It is not inordinately difficult to devise classifications of this sort, but further work is needed to determine a canonical and generally useful way of classifying the available degrees of freedom.

The degrees of freedom and the essential constraints upon each class of modelling facility can be realised in many kinds of fabric and using the attributes (visual, colour visual, auditory) of various modalities. Some embodiments are more convenient than others (it is no accident that we rely, in our own work, so much upon multiple image, visually oriented facilities, or that independence is conveniently represented by separation of sensory modalities). But, over a wide range of variation, the material factors and perceptual factors are not limiting. For example, it is often possible to tell a story, to depict it in a cartoon strip, or to mime it on television.

The crucial trick, which puts a bite into this way of thinking, is that modelling facilities (and, by hypothesis, media also) may either be represented and typified by spatial and physical construction (i.e., making an equipment like STATLAB, making an L-Processor), or with equal legitimacy and more general utility, by the constraints of the conversational domain which carries the "message". We have argued that the entailment mesh and the *BG* of a conversational domain represent any assertoric thesis and that if the thesis incorporates analogies (as it does, except for trivial cases), then the entailment mesh has distinct substructures determining a-priori-independent universes of compilation and interpretation, connected at a cognitive level by the analogy relations. * We have also argued, in the earlier part of this chapter, that characters and roles can be represented in a context Q (usually a plot or story) and that a context of this form can be represented in the entailment mesh given the augmentation of Section 3.

The present point is that the entailment mesh for any or all of those entities (theses, messages, or whatever) is sufficient to determine the modelling facility required to realise the entity(s) in question, and so in this sense to characterise the necessary medium. Moreover, if the most liberal kind of medium, an L-Processor, is available, then the entity(s) can be realised; either using its full capabilities or some restricted version.

6.3. Linguistic Status of Medium

This is probably a fair translation of (the intention behind) McLuhan's dictum, "The medium is the message". But it is possible to proceed further by invoking our own slightly cryptic proposition (b); that "the medium" is a pervasive L-Processor carved up into portions by boundaries that are more restricted processors. The carving or specialisation is determined by an (augmented) entailment mesh. Rephrasing the matter, a medium is the constrained universe of interpretation for a language of which the (augmented) entailment mesh is a semantic grammar (a point made in the previous monograph but emphasised in the present book). Conversely, the most general kind of conversational domain is an interpreted language L, of which particular versions correspond to demonstrations and learning strategies and P-

* In this respect it is instructive to build representations, as we have done, for popular non verbal entertainment. Disney's films (perhaps the best examples are his musical allegory sequence on the "Bobby Sox" movement but the "Samba" sequence is comparable) have rich interlacing and (non formally) rational structure.

Individuals (A, B) generated by particular behaviour graphs' *BG* entailment meshes and representations such as π_A or π_B. It is significant that L Metaphors designate analogical topics and that the class of analogies includes interpersonal analogies (the provocative transactions of the previous monograph, which play an attention directing as well as a communicative part). If all this were true (in the sense of useful and plausible), we have already advanced, though not as yet metricised, a general theory of media.

6.4. Relative Merits, Plausibility and Unification

On casual scrutiny, the suggestion of a pervasive L-Processor seems implausible if not outrageous. During maturation, adult human beings develop sensory and motor organs that effectively encapsulate their brains so that communication seems to involve an input/output bottleneck at the interface. Under these circumstances (or from this point of view), the notion of a medium as a substantially inert signal channel looks altogether more sensible. The difficulty is that perceptual studies employed to quantify the signal channel representation are bound to overemphasise the (real and undisputed) input/output bottleneck.

Such studies (rightly, in their own province) dissociate the linguistic and receptive functions. By virtue of the transmitter-channel-receiver paradigm, they deal only with the reception of signals which later on are internally symbolised and synthesised into percepts or concepts. Signal reception and signal processing have well-known limitations; for example, that words are read as strings of symbols. The appreciation of sights or sounds obeys similar sequential constraints, imposed by the sensory apparatus. The analysis of media along perceptual lines is based upon these findings; correctly, insofar as a medium is conventionally viewed as a signalling channel.

We regard this view as insufficient (not as inaccurate) by noting that an interpreted natural language is commonly used to relax the signal channel paradigm and create a situation in which distinct brains act as though they were a pervasive L-Processor. * The chief implements are attentional, provocative and metaphorical transactions; in this respect, the facts of everyday observation support

* Recall, a Fuzzily interpreted language (Goguen's hypothesis).

the general image developed in this Section, with L in the role of a natural language. Retrospectively, it looks as though human natural language has the calibre of an adaptation which compensates for the fact that adult brains are encapsulated by maturation, and allows them to function as though they were not.

6.5. Summary Discussion

On these grounds, our general theory of media stands out as quite a plausible candidate to complement, rather than vie with, the signal channel theory. It is necessary to show, of course, that L sufficiently approximates the richness of natural language. * If so, the general theory (pervasive L-Processor and all) has predictive power. Moreover, it opens up constructive possibilities for fabricating entirely novel types of media, some of which have been realised (for example, Chapter 8, Section 1, those due to De Fanti and Negroponte).

7. A CONVERSATIONAL VIEW OF CHILD PSYCHOLOGY

In Chapter 1, we emphasised the essential equivalence of conversations as we have described them, paired experiments, and Piagetian interviews. All of them are program sharing and/or programming operations, as well as contrivances for exteriorising cognition; they differ chiefly in the degree of constraint imposed as the price paid for external observations (and with it the extent to which concepts, etc., may be formally specified).

With these equivalences in mind, the following notion is by no means original, "The proper unit for study in developmental, as well as adult and/or educational, psychology is a conversation between P-Individuals" (the conversation also being a P-Individual in its own right).

Perhaps the most incisive statement of this principle appears in Luria (1961), the gist of Luria's lectures in 1958 at University College, London. On p. 20, Luria recalls Vygotsky's insistence upon paired experiments as the paradigmatic experimental situa-

* Or some liberalised and Fuzzily interpreted version of L; the present form does not meet this requirement.

tion. Luria revitalises and augments the dictum as follows: The entity which develops and is studied in psychology is a functional system (or a set of coherent functional systems) having their origins in socially encoded representations (p. 2). Paired experiments exteriorise functional systems and render them observable as they develop under physiological and environmental constraints, including the maturation of the human brain. Insofar as the child in a paired experiment has a brain which is only partially developed, whereas the other human participant, commonly an adult, has a fully developed brain, the influence of maturation can be factored out for special observation.

Given the proper equivalences, this point of view is not at odds with the Piaget school, or in fact the practice of most developmental psychologists who use conversational techniques (in contrast to the stimulus-response and constant-condition techniques which Luria calls "Static"). Perhaps because this approach is so widespread, the quite revolutionary consequences of Luria's basic statement appear to be overlooked. In order to highlight the issues involved, I shall "translate" Luria's statement and slightly extend it; using the equivalence between paired experiments and conversations (in our sense) to identify "Functional System" with either a "P-Individual or part of one," and to identify "stable or replicated functional system" with P-Individual.

(a) The classes of stable functional systems seen under development are P-Individuals A_1, A_2 ... which are exteriorised for observation either in paired experiments or conversations of the form A, B, Q (where B represents the participant experimenter and Q the context of a conversational domain), or (using the "cognitive reflector" construction in Fig. 6.1.) of the form A_1, A_2, Q.

(b) A_1, A_2 ... have their origin in socially encoded representations (characters, roles) π_{A1}, π_{A2} ...

(c) Since A_1, A_2 ... are integral symbolic systems, they may be expected to obey definite laws proper to such systems, notably, "fixity" as proposed in Chapter 2 and "breeding" (a form of symbolic evolution) as proposed in Chapter 6. This clause is an addition to the original statement but is in the same spirit (for example, Luria notes linguistic laws of much the same kind, and the Pavlovian laws governing the higher or linguistic signalling sytems).

(d) Human brains are integral, spatially localised concrete systems and are designated α_1, α_2 ... as proper units of observation (spatially localised; M-Individuated, using the nomenclature of the previous monograph). α_1, α_2 ... obey laws proper to concrete systems; for example, adaptation, Pavlovian first order conditioning, and habituation. In fact, we may go further than that, applying for example the general laws for concrete systems discussed by Miller (1973, 1974).

(e) α_1, α_2 ... have their origin in Genetic codes; call them $G_{\alpha 1}$, $G_{\alpha 2}$ ... and so on.

(f) As a result of maturation, α_1, α_2 ... commonly *acquire* the capabilities of L-Processors. For example, the embryonic nervous system is not an L-Processor, and the infant brain *becomes* such a thing quite gradually. It is a moot point whether all human brains *do* become L-Processors (see, for example, studies of extreme autism by Bettelheim (1967) and histories of isolation *). It may be true that $G_{\alpha 1}$, $G_{\alpha 2}$ do not necessarily generate L-Processors, and it is certainly true, as stressed repeatedly, that a human brain has many functions which do not involve L-Processing. If, and only if, α is an L-Processor can a P-Individual A or a conversation A_1, A_2, Q be executed in α. This is an extension of Luria's statement but seems to be fairly uncontentious.

(g) In general, A_1, A_2 ... are distributed under execution in several L-Processors; for example, in the paired experiment A, B, Q, if the respondent's brain is α and the participant experimenter's brain is β, then the execution of A is distributed over α, β, both of which are assumed to be L-Processors. If A or a conversation of the form A_1, A_2, Q is executed in one L-Processor or brain α, we say that A is spatially localised in α.

(h) Let α, β, γ be spatially localised concrete systems; α and β are brains; β is also an L-Processor. Let γ be an inanimate modelling facility such that α and γ jointly constitute an L-Processor. The conditions upon the spatial localisation of A are summarised in Table 11.1.

(i) Fuzzy Computation is the rule: non-Fuzzy Computation

* Edward Goldsmith was kind enough to lend me his remarkably comprehensive file of reports and tests of "wolf children" and other cases of human maturation in isolation from human contact. Scrutiny of these records (which vary from careful reporting to apocryphal anecdotes) indicates that linguistic exchange is needed to set up ingrained symbolic routines in the absence of which the brain is not an L-Processor.

TABLE 11.1

Spatial Localisation

Available Processors	Is L-Processor	Is Not L-Processor
α	A in α	A not in α
α, β	A in α or A in α, β	A in α, β
α, γ	A in α or A in α, γ	A in α, γ

(germane to formal schemes involving unique complementation and negation) is the exception. Formal schemes have value as the most efficient means of conducting other-than-analogical cognition. The generation of a character or role only *need* involve analogical processes. *Further Postulate* (but still in the spirit of Luria's statement): If a brain matures to become an L-Processor, it is able to accommodate (to compile and to execute) Fuzzy Procedures (in particular a character or a role) before it can accommodate non-Fuzzy Procedures.

(j) Let "child" mean a spatially distinct infant with brain α. From (f), a child cannot at birth accommodate a P-Individual A for which there is a social representation π_A: the mother-child or the family-child complex (α, β of (h)) *may* do so. The test for whether or not α is able to accommodate A, so that A may be spatially localised in α, is suggested by clause (i); namely, it is possible to show self-and-other recognition going on in α and evidenced by an internal conversation of the type A_1, A_2, Q (with A_1, A_2, factors of A). All studies of egocentricity and related phenomena appear to seek evidence of this kind. From (i), we predict that formal operations cannot be manifest as localised in α unless a character A_1, A_2, Q may also be localised in α.

(k) It follows, from the foregoing clauses, that a conversational approach to developmental studies (which is advocated by Dienes, Inhelder, Landa, Luria, Papert, Piaget and a host of other researchers) carries the following perspective as an at least implicit concomitant. Developmental Psychology is concerned with the incarnation of stable symbolic systems A_1, A_2 ... generated by social representations π_A, π_{A2} ... in a population of maturing concrete systems (brains) α_1, α_2, generated by genetic codes $G_{\alpha 1}$, $G_{\alpha 2}$ ... The execution of A may be spatially localised in α only insofar as α has matured as an L-Processor, and though special interest is attached to this case, the science also countenances distributed executions of A. On execution in α, the procedures of A modify the maturation of α, and vice versa, the constraints imposed by α at a certain stage of development modify A; say, "A becomes A*." This, in turn, leads to novel social representations $\pi_A{}^*$.

Systemic Monism (symbolic systems and concrete systems have basic laws of operation and development in common) has already been recommended. It is of material consequence insofar as the development of A (that is A becomes A*) may operate upon the coded representations of concrete systems ("G_α becomes $G_\alpha{}^*$," on a par with "π_A becomes $\pi_A{}^*$"). Until recently this transformation was inadmissable, at any rate, in practice.

It is worth noting that two mechanisms exist due to the development of our civilisation (in particular, due to research programs in Lakatos' sense). One mechanism is genetic engineering, applicable in case α_1, α_2 ... are brains. The other is the development of L-Processors, other than brains, able to accommodate P-Individuals.

It is hard to appreciate the gigantic impact of these two complementary developments, and it is important to recognise how radically they change the objects and perspectives of developmental studies in general and educational studies in particular. Notably, the universalist approach of Section 6.3. is seen, in this context at any rate, as more than a curiosity of possible academic interest. It is a viable and practicable way of dealing with reality.

8. COMPARATIVE STUDY OF DATA

It is instructive to compare data obtained by the conversational (paired experiment) technique and data from "static" studies, sometimes data obtained in the same laboratory. A gross comparison is given by Luria (1961) citing results from non-Fuzzy problem solving due to Minskaya (1954), the form of which is sketched in Fig. 11.2. Success is markedly higher at all ages if problem solution is preceded by paired experimentation, and the solution methods adopted by the conversational students are completely different, being integrated and purposeful, rather than fragmentary. In the conversational age/performance curve, we are looking at an overall summary of a P-Individual's ability to execute non-Fuzzy Programs, either in a pictorial/visual representation, or a formal/linguistic representation. In context, at least, it is fair to re-

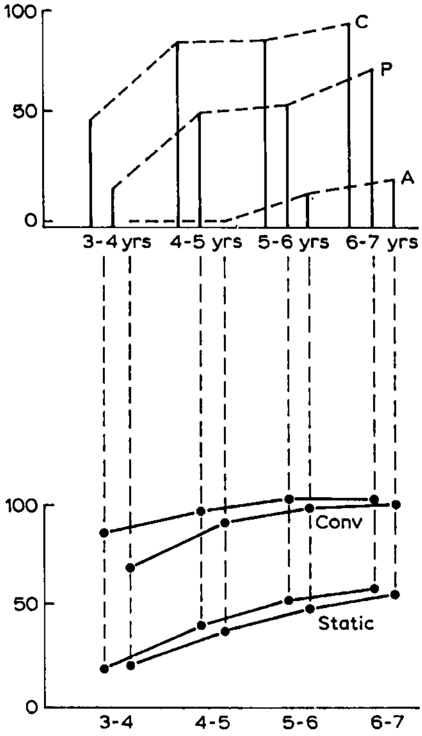


Fig. 11.2. Sketched from Luria (1961). Below: Age/performance curves for conversational (paired experiment) subjects and for static experiment subjects. Above: Relative performances for concrete, pictorial and algorithmic (linguistic) presentation. In each case vertical coordinate represents mean success, as a percentage, in problem solving task used for study by Minskaya. C = Concrete representation, A = Algorithmic representation, P = Pictorial representation.

gard pictorial/visual as one Fuzzy Transformation of a non-Fuzzy problem. In contrast, formal/linguistic is an algorithmic and non-Fuzzy Representation. The P-Individual is the child as augmented by the experimenter (A, B). Its locus is in the child's brain α, supplemented cooperatively by the experimenter's brain β. From time to time for test, execution is isolated in α.

8.1. Static Experiments

In contrast, consider the "static" performance/age curves. The experimental conditions now include a concrete/practical representation; meaning that there is a modelling facility (γ) in which problem solving programs may be compiled. Insofar as the programs are partially compiled in γ (that is, the relevant processor is the pair, α, γ), the results are fairly coherent; for the pictorial/visual and the formal/linguistic representation, they are increasingly fragmentary. In all cases, observation of the child as a functional system (Luria) or a P-Individual (present nomenclature) is imperfect since program execution is only incidentally exteriorised. With the possible exception of the concrete/practical data (where the behaviours in γ can be examined), the data primarily refer to the child's brain (α) in *its capacity as a non-Fuzzy Processor.* Moreover, by token of the attention lapses and distractions which occur repeatedly, information about α is adulterated by the co-existent compilation in α of a (Fuzzy) P-Individual A. This adulteration stays with the experimenter until A is able and willing to accept instructions that isolate some aspect of α (the problem of mental testing in preadolescents). Luria's own work upon the regulatory function of speech is a beautiful example of the latter kind of experiment. In order to illustrate the distinction, some of his results are overviewed in Table 11.2, as a profile of how α acquires the ability to act deductively and execute if-then-else statements.

By way of a summary, two quite distinct interpretations can (and should) be given to the experimental data from developmental studies. We maintain that the distinction is not a matter of fact (that human beings develop as two kinds of system) but depends upon the existence of two observational methods. It happens that the information obtainable by one method is maximised by expedients that adulterate the information obtainable by the other method.

TABLE 11.2

An Overview of Luria's Results and Interpretation in Terms of the Ability of Child's Brain to Deal with "If-Then-Else" or "Conditional Imperative" Statements in a Non-Fuzzy Program. The experiments are concerned with a situation in which a carefully recorded manual response (showing hesitation, etc.) is made to a visual stimulus and according to instructions. The situation is augmented by speech on the part of the experimenter or the child, and the overt or external utterances are regarded as parts of non-Fuzzy or algorithmic programs. Two modelling facilities (or two external-to-the-brain compilation media) are used: Overt feedback and the child's own speech.

Age	Findings	Proposed Interpretation of the Findings
6 Months to 18 Months	Speech initiates action but does not modify autonomous acts. "Press when light appears" results in intermittant pressing.	Brain acts as reactive device in respect of this task.
18 Months to 2.5 years	Specific reaction to speech or visual signal. Negation absent. ("Do not press if no light" often leads to more pressing.) If external feedback is provided (for example, bell rings after the pressing movement), reactions are discrete.	Brain can compile part of imperative implication but can process conditional imperative if, and only if, part of program is externally compiled and executed (the feedback loop).
2.5 Years to 4 Years	Role of feedback is taken over by child's speech. If he makes overt ejaculations after each act, these terminate act.	Child's own speech used as modelling facility. Compilation of simple conditional imperative, but if, and only if; overt vocal response is involved in execution.
4 Years to 5 Years	Overt speech internalised for simple task. For complex instruction like "Press n Times" or "press n times until". Overt speech is needed to regulate and negation is still unreliable.	Internal manipulation of simple conditional imperatives is possible; other instructions (nesting or sequencing) require augmentation or overt response.

TABLE 11.2 (continued)

Age	Findings	Proposed Interpretation of the Findings
5 Years to 6 Years	Negation handled adequately. Speech, if present, becomes overt. If the child repeats instructions, he can obey them for quite complex tasks.	Brain activity, with non-Fuzzy complementation (the proper acceptance of negation). Complete "If then or else" statements compiled and executed, but linking program uses speech as modelling facility for compilation and execution.
6 Years Onwards	Gradually, repetition of (program) instruction is covert rather than verbal.	Program is compiled and executed internally.

8.2. Discussion

Postulates (a) to (k) have predictive as well as descriptive potential insofar as they can be reapplied in complementary form to generate a series of interacting organisations. These organisations appear to recapitulate in system theoretic jargon the structures discovered and described by insightful developmental psychologists, many of them by Piaget and his collaborators. So, in particular, A will pass through many complex and context specific identities as A develops, and these identities can be classified; for example, A's body identity, A's world of Fuzzy (pictorial?) images, and A's world of formal procedures. The coexistence of such worlds (and the fact that the sequence is interlaced and context specific to begin with) leads to distinctions of the kind we have made between "descriptions of topics" and "topics".

Throughout (as may be inferred from (c) and (d)) an "external world," A's concept of what he has learned, is juxtaposed with an "internal world" of A's imaginatively generated procedures (Chapter 4). So it is that Luria's "functional systems" or our "P-Individuals" appear to evolve.

It is natural to ask whether, at this stage, there is a breakpoint marking a change in kind or quality of the basic entity A (not merely accretion, specialisation and generation). Our hunch is that just such a breakpoint occurs in "learning to learn" and that its resolution, in order to construct an essentially novel entity, is "innovation".

By "learning to learn" A imposes an internal structure on the environment, primarily upon the social environment. The crucial step (many aim operation is required) is "breeding" whereby $A \to A_1, A_2$ (Chapter 6). The compensating process, by which A as well as A_1, A_2 ... maintain integrity, is an agreement (common meaning resolution) together with "privacy in the face of agreement". Of these compensating steps, the former alone is sufficient to account for the act of innovation; the latter (so our notion goes) is responsible for the ownership of innovation. It is ownership in the peculiar sense that A has a world of ideas shared with others, albeit generated by their efforts, but from A's point of view as a participant in society, this world of ideas is his identity.

Appendices

APPENDIX A: INTUITION OPERATING RULES

The operating rules and transactions are discussed in the context of the "extended probability theory" thesis and the Lumped Modelling Facility STATLAB II. As a matter of convenience, the entailment structure representing this thesis is broken down into three modules that are often presented separately (the three miniature entailment structures). Though this is not a mandatory condition, the description is based on the assumption that one module is studied at once.

Ap. 1. The first rule to be accepted by the student is pragmatic. His *intention* is to learn the topics required in order to *understand* the uppermost topics in the entailment structure (the *head* topic of the previous monograph), and to do so in a manner permitted by the procedural rules indicated below.

Ap. 2. A student can explore any topic by pointing with an electrically connected stylus at the label representing this topic on the entailment structure. For this purpose, the entailment structure serves as a conceptual "map" of topics and their labels, disposed about a territory. The "map" is indexed by *descriptors*, which are displayed explicitly. The descriptors apparent in Fig. 1.3 are *depth* from the head topics taken to name the subject matter field (the "superordinate/subordinate" descriptor of the previous monograph): a descriptor with values *Re* = Real world of experiments; *Ab* = Abstract world of logical or mathematical constructs, and

An = Analogies involved in relations that underlie statistical inference. Finally, there is a descriptor, with values indicated as coloured columns, that discriminates the form of logical expression lying at the root of the topic.

In response to his explore enquiry, the student receives examples of the explored topic presented graphically by slides projected onto a screen (Fig. 1.1) using a random access projector. Each topic is associated with several examples determined by values assumed under different semantic *descriptors*, many of which are not displayed. For instance, the topic "simple random experiment" is exemplified in terms of "games of chance" and in terms of "behavioural experiments". The examples are also indexed by values of the descriptors that appear explicitly in the entailment structure, for example, the real world interpretation (Re) and the abstract world interpretation (Ab) which in the entailment structure correspond to the left and right hand half planes.

The descriptive examples (Fig. 1.5 is typical) are enriched both pictorially and by multiplicity of context (releaser function and humour). Descriptive examples do not delineate the underlying topic relation which is to be explained if the topic is addressed. But they do systematically discriminate the descriptor values (for example, "plant breeding," as distinct from "games of chance" or "behavioural experiments").

At this stage, the only caveat is that explorations do *not* peter out and that they *do* lead eventually to choice of some focus of attention which is dubbed an aim topic.

Ap. 3. To propose an aim, the student touches a different point on the topic label with his stylus. In response to a proposed aim, he receives a brief test administered by a confidence estimation device (BOSS, or Belief and Opinion Sampling System, Fig. 1.2). Questions cards, indexed by the topic number are inserted into the BOSS card reader, and the apparatus sequences responses and subsequent card insertions and computes a progressive estimate of correct degree of belief signifying that the student can genuinely describe (give veridical descriptor values to) the topic proposed as an aim. If so, the proposed aim is *validated* as a topic the student can appreciate (but not necessarily learn about), and it is instated as the current aim, of which by edict there may be only one. (From the previously considered rules, some one aim must be selected at any rate after an interval of exploration.) The only restriction upon a proposed aim selection is that no topic currently marked as understood is a legitimate candidate.

In return for selecting an aim, the student receives a display, through the illumination of green signal lamps attached to each topic, of the ways in which the aim topic may be derived from other topics; for example, Fig. A.1 shows the display presented if representative topic is cited as aim by the student and if the aim selection is validated by the system.

The display represents the "Entailment Set" (the union of the entailment kernels, as in the previous monograph) of the aim chosen, and consequently, all of the topics that might be learned in getting to know about this particular aim topic. The student is required (by a further rule) to select one or more goal topics, within the "Entailment Set" of his aim topic, as the topic(s) he intends to learn about and work upon. Notably, one possible goal is the aim topic itself.

Ap. 4. Before describing the goal selection procedure, it is necessary to look ahead at the placement of *understanding* markers (plugs with some circuitry inside them, shown in Fig. A.8) and to

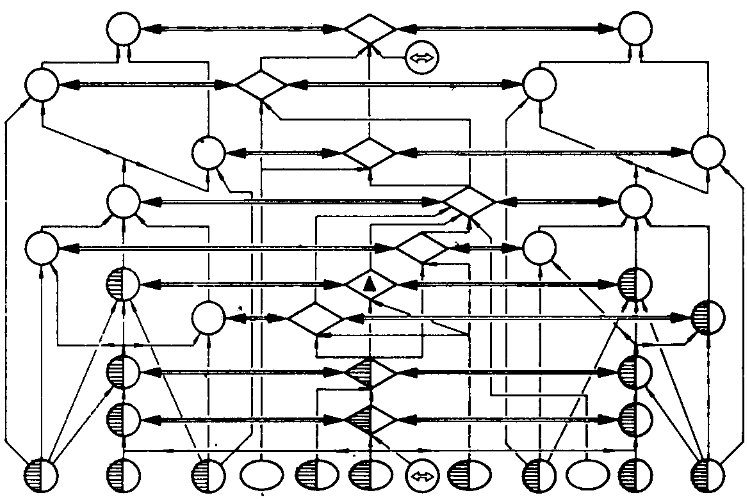


Fig. A.1. Aim = ▲, and left-shaded nodes with green signal lamp illuminated.

recall that any student has agreed to learn and understand all of the head topics. In INTUITION (though *not* in the more elaborate system of CASTE) we suppose that the student either (a) understands all of the lowermost (primitive) topics at the outset and is prepared to start learning from that point onwards, or (b) (an inherently more interesting possibility) that he declares his understanding of the other-than-primitive topics and engages in the "explain of explain" routine. In the latter case the topic is instated if and only if "explain of explain" is completed successfully.

Since case (a) is more easily described and pictured, we concentrate upon it at the moment and return to case (b) later. Now given case (a), all of the nodes of the primitive (lowermost) topics can be marked understood as an initial condition. Understanding is marked by inserting plugs (understanding markers). The result of doing so is to illuminate orange lamps (Fig. A.2) on the nodes of topics entailed by the collection of understood topics. Topics associated with illuminated orange signal lamps are known to the student as possible goals (distinct from legal goals, which will be introduced shortly).

Ap. 5. Return, after this brief digression, to the condition in which the student has selected and validated an aim topic so that topics

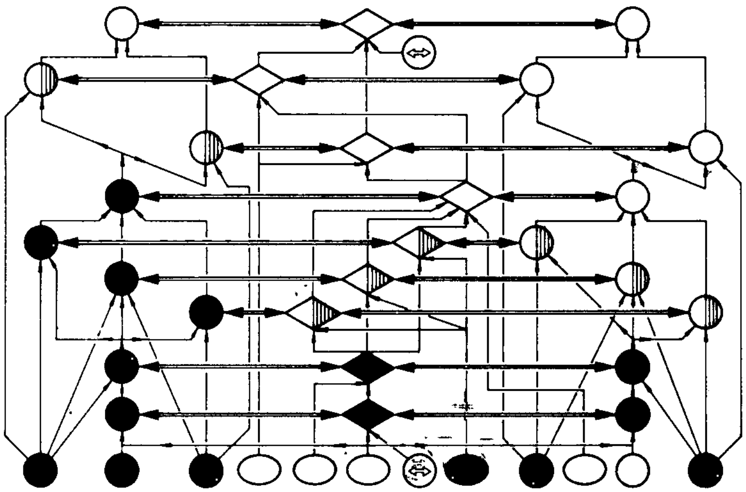


Fig. A.2. = Orange signal lamps (right-shaded nodes) showing possible goals for given distribution of understanding (■ nodes).

in the "Entailment Set" of the chosen aim are associated with an illuminated green signal lamp. If the prerequisites are also marked understood, some topics will thus be associated with both an orange *and* a green illuminated signal lamp. These are legal goals; that is, if the student satisfies the rule requiring him to select some (one or many) goals under his chosen aim topic, then selection of a legal goal or several legal goals will be accepted. The student will be able to access demonstrations and tutorial materials with respect to these topics and to learn about the underlying topic relations. A typical legal goal distribution, at the start of learning, is shown in Fig. A.3(A); a legal goal distribution later in learning (when more understood markers have been inserted) is shown in Fig. A.3(B); a still later legal goal distribution (and under a different choice of aim topic) is shown in Fig. A.3(C). Any attempt to select an illegal goal (any topic that is not marked by an orange and a green signal lamp) is automatically detected; the student receives an auditory signal and the equipment operation is locked until he dismantles the offending configuration either by changing his goal selection or occasionally by changing his aim selection.

In order to choose one or more goals the student must perform the following operations for each goal (Fig. A.4):

(a) Open the door bearing the topic name.

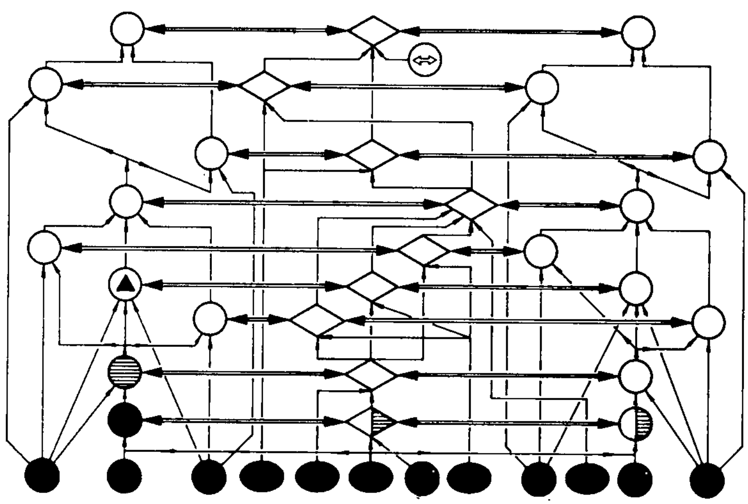


Fig. A.3(A)

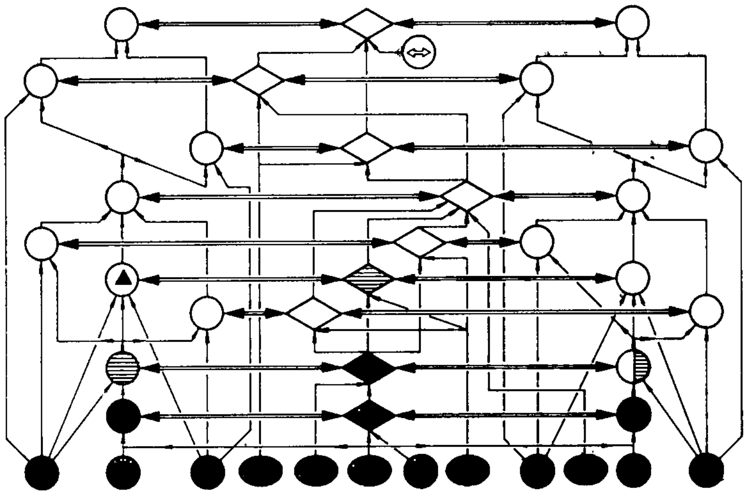


Fig. A.3(B)

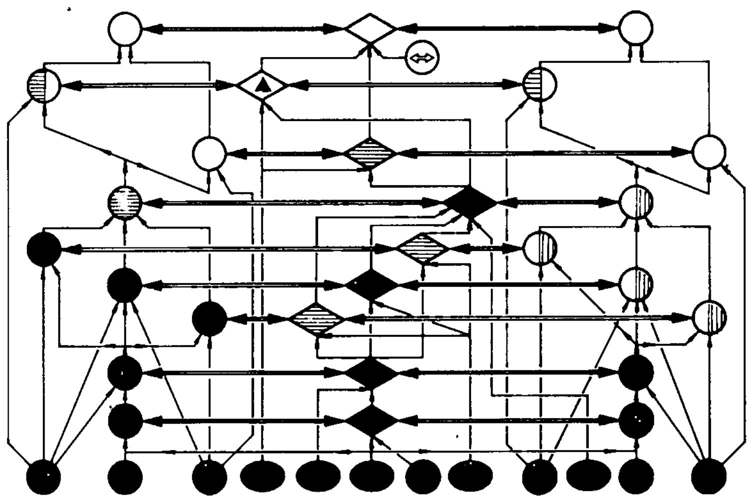


Fig. A.3(C) Aim ▲, Understood ■, fully-shaded nodes = legal goal nodes.

(b) Insert a goal probe (of which six are provided) into a socket thereby uncovered.

(c) Read the index numbers revealed on the reverse side of the topic door.

Regarding operation (c), there are two possibilities depending upon whether the goal topic is or is not representing an analogy relation. If not, the index number is unconditional and the topic position is associated with one orange signal lamp (the possibility so far described). If the topic does represent an analogical relation (for example, any topic in the central part of the display), there is a cluster of orange signal lamps above the topic label and although one of them is illuminated if the topic becomes a possible goal, the particular one depends (1) upon the configuration of understood markers achieved as a result of previous learning, and (2) upon the

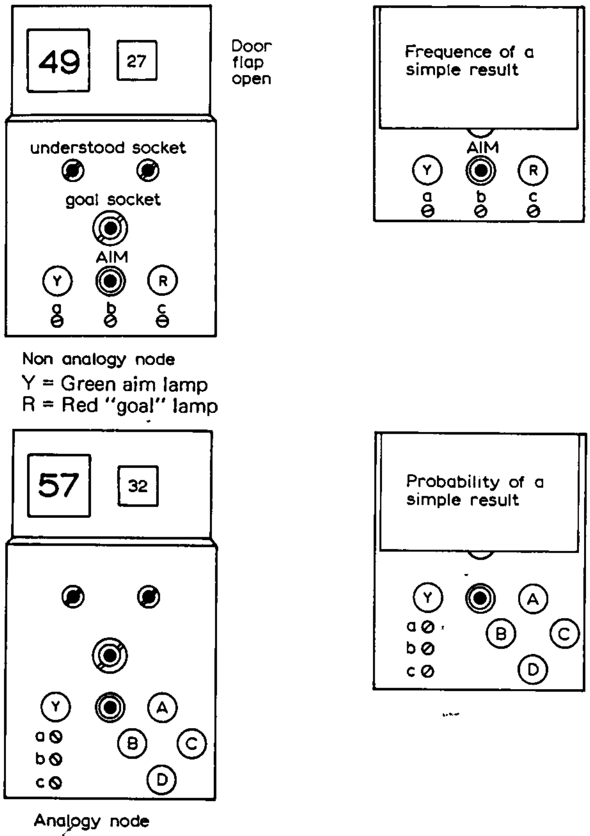


Fig. A.4. Concrete arrangements for nodes of an ordinary topic (above), or an analogical topic (below). a, b, c, are contacts for explore transaction probe. The goal signal lamp (R, above) is replaced by set of signal lamps (A, B, C, D, for 2 term analogical topic).

goal probe insertions currently made by the student.

In the former (unconditional) case, the one index number uniquely specifies demonstration and tutorial data files to which the student is given access. In the latter case, there are as many index numbers as orange signal lamps and the demonstration and tutorial data files are conditional upon the learning process. In Fig. A.4 there are four signal lamps and consequently four contingently accessible types of demonstration.

The conditionality arises because of the curiously complex structure of analogy relations discussed in Chapters 3, 4 and 6. In general, there may be various kinds and numbers of contingencies (for example, 4 to 12 in a thesis on "Heat Engines"), but for "Probability Theory" the contingencies are uniform in kind and readily stated. The configurations which illuminate the different signal lamps are summarised in Fig. A.5. All the analogy relations for this subject matter thesis have the same basic structure with two *terms* (the topics that are analogically related by the central topic), one term representing a "real world" topic and the other term an "abstract world" topic which is its mathematical image.

condition cited in text	left term topic	right term topic	goal lamps illuminated on analogical topic
I	+	+	A
II	+	—	B
III	—	+	C
IV	—	—	D

Fig. A.5. Conditions for learning 2 term analogy, marked as legal goal. + = understood, — = not understood.

Ap. 6. Presupposing a description of the next rule, the orange signal lamp illuminated on the analogical topic determines not only the type of demonstrations which the student can obtain but also the type of non-verbal explanation which the student will be required to produce in order eventually to mark the topic as being understood. Thus, consulting the conditionality table in Fig. A.5, lamp (A) is illuminated if and only if both terms of the analogy relation are understood; lamp (B) if the left hand term but not the right hand term; lamp (C) if the right term but not the left; finally, lamp (D) if neither the right nor the left hand term is already marked as understood but if either one or both of them is marked as a legal goal. If none of these conditions apply, then no lamp is illuminated as the topic is not a possible goal.

In Condition (I), the student (already *understanding* both terms) need only explain the analogy between them and he receives demonstrations only of this analogy relation. In condition (II), the demonstrations exhibit the right hand term by analogy with the left hand term, and an explanation of the analogical relation parallels this transformation. In Condition (III), the reverse applies; the left hand term is demonstrated by analogy with the right hand term and similarly explained. In each case there is a clear sense in which the student already knows one (Condition (II) or Condition (III)) or both of the terms (Condition (I)) before he tackles the analogy between these terms.

Condition (IV) is peculiar and interesting since it represents the case in which the student grasps the analogical relation to begin with and opts to explain the left hand or right hand terms by recourse to this analogy. In order to interpret his explanation, he is forced, before the analogy relation can be understood, to explain it by means of one or both of the terms (consequently placing himself in Condition (I), (II), or (III)). The practical consequence of this preference is that he is forced, before explaining the analogical topic, to mark one or both of the terms as a simultaneously entertained goal topic.

Ap. 7. The tutorial material consists in demonstrations of the topic(s) currently in focus as goals. For a topic T Behavioural Prescriptions (augmented by descriptive text) are derived from the Behaviour Graph BG(T) in the conversational domain, i.e., the Task Structure of *topic* T.

Given the proper index number, the student can access files (Fig. 1.1) containing layover cards (Fig. A.6) accompanied by written text. The layover card (or cards, if there is more than one goal) is placed in front of the fascia of the modelling facility STATLAB II, shown in Fig. 1.1, and outline labelled in Fig. A.6. The card itself, is a Behavioural Prescription, the written text (if any) serves as an accompanying description.

STATLAB II is a Lumped Modelling Facility containing six a priori independent processors (some electrically trivial, though their logical integrity is not).

STATLAB II (Fig. 1.1 and Fig. A.6) is divided into compart-

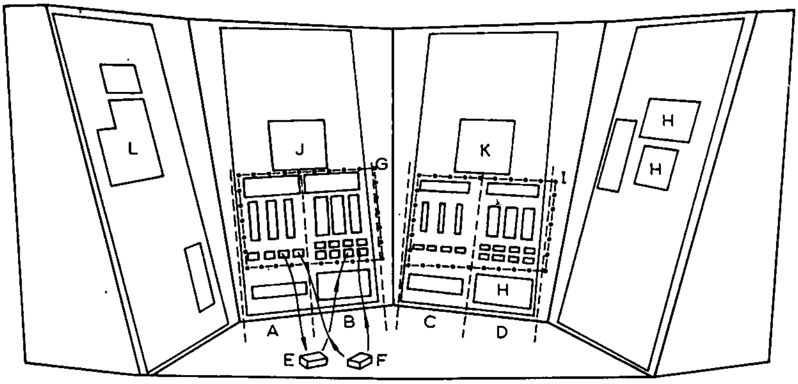


Fig. A.6. Outline of STATLAB II with layover cards. Parts of lumped modelling facility are: A, B, Distinct Universes of real results; C, D, Distinct universes of abstract events; E, F, conditional probability and delay units on Bench; G, Subsets of events; H, measures on event sets and arithmetic operators; J, Matrix of Joint Results; K, Bayesian Inference Summation and Matrix multiplication; L, unique and joint result counters, with marginal totals.

ments in register with the partitioning imposed upon the entailment structure by the descriptors. For example, (Fig. A.6) the lower left hand quadrant is concerned with topics bearing upon a temporally ordered "real world" (*Re*) of deterministic experiments; the lower right hand quadrant with "abstract world" (*Ab*) topics that bear upon set theoretic and atemporal images of deterministic experiments. The upper right quadrant contains topics concerned with frequencies of (temporal) results and their ratios, differences, contingent frequencies, etc. The upper left hand quadrant contains topics that are concerned with measures on abstract sets, conditional measures on product sets, etc. In fact, a finer grained partitioning is possible because the "real world" contains two a-priori-independent universes (in order to develop ideas of contingency, statistical independence, statistical dependence, and so on), and the "abstract world" contains two universes of a-priori-independent abstract images (for reprinting product experiments, conditional probability matrices, and Bayesian Inference).

The non-verbal explanation (or demonstration) of a non-analogical topic involves building a model in one compartment of STATLAB II and any analogical topic is explained by simultaneously executing two models that are analogically related, as required.

Ap. 8. Suppose that one or more layover cards have been removed from the file and placed in position. The student receives a demonstration by obeying the instructions on the card or the accompanying text material, and building a model on STATLAB according to this recipe. When he has done so, the model is executed to achieve some result (for example, to compute the frequencies and expected frequencies of results in an experiment).

For any topic there are several (often five or six) differently slanted demonstrations available in the original file and the student can access as many as he likes, in sequence. The INTUITION equipment keeps a list of the demonstrations of a topic that have been accessed by a student, until the topic in question has been successfully explained.

Ap. 9. When the *student* is satisfied that he comprehends the topic well enough to explain it, he enters the (non-verbal) explanation routine as follows:

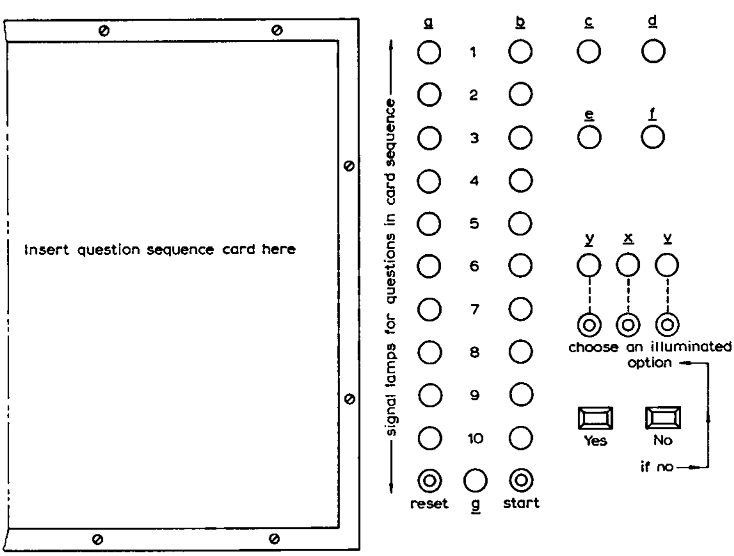


Fig. A.7. Check list questioning device with counter and choice of an option.

(a) All layover cards are returned to the file so that the panels of STATLAB are bare. The equipment is placed in a state that disallows any change of goal or aim, until the explanation is finished. Any attempt to change goal or aim locks the equipment and indicates contravention of this rule.

(b) Explanation is initiated by taking an instruction and check list sheet for the topic concerned and placing it in the check list reader (Fig. 1.1, shown schematically in Fig. A.7). After this point the demonstration file is inaccessible (any attempt to remove a layover card is detected, signalled as illegal, and locks the operation of the equipment).

(c) Card insertion resets a counter in the check list reader, provided that all the requisite conditions such as the existence of an aim and a goal are satisfied. As a result, an illuminated pointer is positioned against the first item in the check list.

(d) Each item in the check list consists in an instruction and a condition to be checked by the student. The instructions guide the student in building a model on STATLAB, which does the same thing as the demonstrations, but which is not identical with any demonstration he has received. This requirement is checked automatically by comparing the model with the set of demonstrations indicated by the demonstration list.

(e) If the student believes that a stage in model building is correct, then he presses the "Yes" button on the check list reader and the pointer moves to the next item. Before pressing the "Yes" button, the student may (and often does) execute the partial model he has built on STATLAB to convince himself that this part does whatever it ought to do.

(f) If he is in difficulties and anxious to start afresh, the student presses the "No" button, which returns the illuminated pointer to an invisible zero position and offers the following options (the lamps and the buttons at the base of the check list reader):

(A) Start a fresh explanation. In this case, the illuminated pointer moves to the first position (the most frequently chosen option).

(B) Obtain further demonstrations. In this case, the student is allowed access to the demonstration file provided the instruction and check list card is removed.

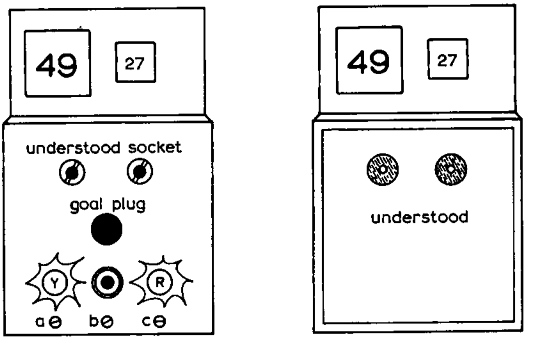
(C) Learn the topic in a different way. Choice of this option resets the entire collection of (internal) electrical register tags established since obtaining a demonstration layover card from the file and allows for change of aim or goal.

(g) Suppose the "No" button is never pressed and that, by pressing the "Yes" button for each item, the student eventually lends his approval to a sequence of partial models (one per item in the check list), and thus has built an entire model for the topic. The last item in the check list always guarantees that this *complete* model *is* executed or tested by the student (partial models may be but *need* not be executed). Pressing the "Yes" button at this point means that the student is satisfied with his model and submits it as a non-verbal explanation of the topic.

(h) Depending upon the experiment, we either accept the student's judgement of workability, invoke the judgement of a supervisor, or use the computer to check that execution of this model correctly satisfies the topic relation. In any case, provisions are made for sensing and tracing (electrically) all configurations of components, links, potentiometer settings, etc. A condensed form of this tracing data is used, in any case, to ensure that the submitted model is not identical with, hence possibly just a copy of, any demonstration on the stored demonstration list.

Ap. 10. Once a non-copied complete model is deemed *correct* (according to one or the other of these criteria) and has been submitted, the student is allowed access to an understood marker and is required to insert it into the entailment structure at the position of the topic which has been non-verbally explained. Simultaneously, the equipment rescinds the temporary (whilst explanation is in progress) edict that neither aim nor goal shall be changed. Insertion of the understood marker, which is based on a sizable plug, is only possible if the goal probe is removed (Fig. A.8). Further, insertion of an understood marker covers and obscures the green and orange signal lamps at the topic position. The student's activities are monitored and mistakes (such as placing the understood marker in a different topic position) lock the operation of the equipment and give rise to a signal.

Ap. 11. Repetition of these operations until the uppermost topics are understood (as agreed by the student in Clause 1) gives rise to



Red and yellow lamps illuminated and goal plug inserted for legal goal

Understood plug inserted covers up all lamps and sockets

Fig. A.8. Topic node arrangements with, and without, understanding plug inserted.

a series of marker distributions on the entailment structure which is visible to the student as he learns and makes him aware of *how* he *did learn*, as a visual pattern. The distribution of marker patterns and transactions generated in the course of learning is a *learning strategy* and is characteristic of the student.

APPENDIX B: A SIMPLE MODEL FOR AN L-PROCESSOR

The L-Processor is a modular computing machine, the components of which, and their integrity and persistence, depend upon an evolutionary process like Fogel, Owens, and Walsh's simulation (Section 7). The finite state machines are the modular automata which, in such a system, replace indexed storage. They, and their weak interactions, constitute the *PC* operations. But, because the interaction terms are involved in the *PC* specification, a collection of modular automata has a definite L-Processor organisation.

One tangible realisation of a computing medium made up from modular automata is a so called tesselation surface (Fig. B.1): a collection of cells, each containing an automaton, interacting by a neighbour function. (For example, the input to cell ϕ is a function of the previous states assumed by the automata right, left, up, and

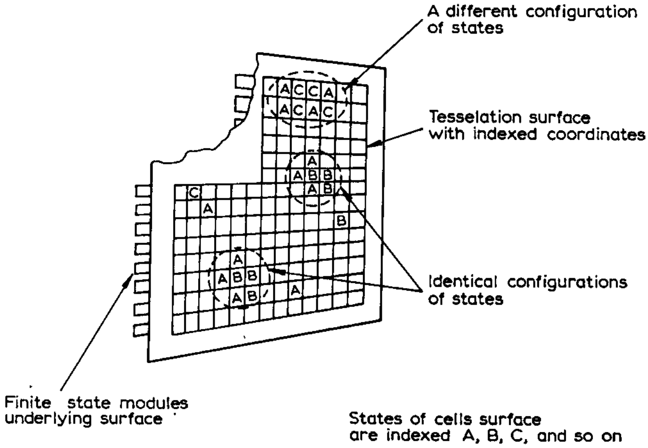


Fig. B.1. Tesselation surface.

down, adjacent to ϕ.)* Arrangements of this kind are used to represent reproducing automata. The entity being "reproduced" is a *configuration* of states of modular automata, irrespective of where upon the surface it is located. The surface, in other words, is a *computing medium* (a taciturn system) inhabited by procedures (here configurations) which survive or decay. Under the immediate interpretation, *understanding* and memories figure as *configurations*, the *DB/PB* operations as the dynamics of strong interaction between *configurations*, and these strong interactions, in turn, as the L transactions of a language oriented system.

Finally, the compilation Inter of a Program Prog (to realise Proc = ⟨Prog, Inter⟩) is that activity in the states of certain modular automata which induces a state of a configuration (Prog) and does so for each state of Prog. Hence Inter belongs to the class of *PC* operations, any Proc has a *PC* component in it as required by the overall theory.

* Probably the simplest tesselation system is Conway's (1971) "Life" Simulation, but it is marginially adequate for the present purpose. Other more elaborate tesselation systems are described in Burkes (1970). To exemplify the notion, several systems have been simulated with one additional, mathematically irritating but *essential* property; namely, that a modular automaton is never sessile, i.e., the automata act as oscillators damped by weak *interaction* with their neighbours.

In such an arrangement, it is possible to vary the composition of the automata (they may be uniform or varied), the neighbour function (it may be homogeneous at all points on the surface, or not), and less plausibly, when it comes to physical interpretation, the dimensionality of the tesselation surface. Any or all of these parameters constitute "patterns of L-Processor organisation", as the phrase was used in the paragraph before last. Moreover, at least one of these parameters is varied if the modular automata are produced and refurbished by an evolutionary style process (Fig. B.2).

Provided certain limiting conditions are respected, these variations do not influence what may be computed by *configurations* on the tesselation surface (for example, reproductive Turing automata can be represented in any such system). But the parametric variations *do* profoundly influence how the computation takes place, and it is surely possible to set the parameters (in many ways, in fact) to capture each competence profile. Moreover, the parameter setting may be (and in Fig. B.2 it will be) determined adaptively, as required if this picture of things is to match the observations of other researchers or of our own group.

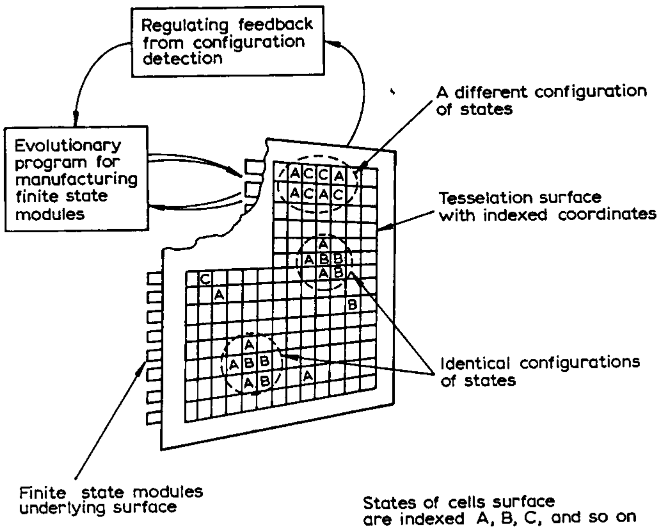


Fig. B.2. Tesselation surface with finite state machine components (the modules) constructed and maintained by an "Evolutionary" program.

That is, depending upon the characteristics of the *computing medium* (or candidate as an L-Processor), a *DB* operator (if it exists) will be a *GDB* or an *LDB*; similarly, a *PB* operator (if it exists) will be a *GPB* or an *LPB*. This global or local propensity is the least readily modified; the effectiveness of *DB* and the effectiveness of *PB* operations depend by hypothesis upon the steady state densities of *DB* and *PB* amongst the population of programs under execution in this medium.

We do not hold that L-Processors (in particular brains) actually *are* organisations on a tesselation surface; the tesselation surface was introduced as a familiar example. But any set of interacting modular automata have communication and control connections equivalent to neighbour functions and the like, and we do maintain that L-Processors are just such systems. †

† It is worth noting that a number of telling parallels exist between processors of this kind and biological or physiological systems. For example, Goodwin's (1963) discussion of cellular metabolism makes a clear distinction between weak interactions through pools of metabolites (reaction products and precursors) and the strong interactions implicating DNA, RNA, the Ribosomes, and Enzyme synthetic processes, which may be regarded as *DB/PB* replicable procedures executed in the milieu of the cell. Pringle (1951) and Beurle (1954, 1959) entertained similar notions with specific interpretations in Brain Dynamics; so, with some variations, did Hebb (1949). Since that era, a host of comparable formulations has been devised in diverse fields; for example, neurophysiology, molecular biology, biochemistry, genetics, ethology and ecology. One fascinating example which has recently aroused lively interest (see, for instance, the proceedings of the December 1974, Faraday Society Symposium, No. 9) is a system of spontaneous chemical oscillations in a dish of Belousov reagent. (Bromate ion in sulphuric acid solution with malonic acid and reducible manganese or cerium ions).

It is quite important to recognise that these conditions are the norm (for otherwise the argument seems curiously outlandish) and that the more familiar cases of serial execution are specially contrived and seldom encountered in nature. That is, most natural systems are *not* subject to the limitation, "stop execution whilst rewriting a program, and stop rewriting whilst execution is in progress," which we used in the first monograph to *delineate* the class of serial modelling facilities to which this very special caveat properly applies (the "t clock" and the "τ clock" convention). See also Pask (1961, reprinted 1968, 1972) and Pask (1975a) as well as Ben Eli (Brunel University thesis, 1976).

APPENDIX C: DETAILS OF COMPROMISE PROCEDURES FOR "LEARNING TO LEARN" EXPERIMENTS

The following compromise procedures stem from an application of entailment structure techniques devised and successfully pilot studied by Dr. R. Glanville and his colleagues in the context of an architecture school.

Session A

After students have studied the texts, lists of topic names, to which others may be added by individuals, are handed out, and each student is asked to rate the topics as follows: + if he thinks he can explain the topic, ? if he is doubtful, — if he cannot, and * if the topic is irrelevant (some seemingly irrelevant topic names are given in the list, together with some "spare" locations to be filled by additional topic names). The students are asked to show by directed arcs how they conceive the topic to be "connected". Typical results are shown in Fig. C.1 and Fig. C.2. Students are next given a sheet on which the experimenter has encircled the topic names which each *individual* student thinks he is able to explain (hence, each student has a "personalised" sheet), and the class is asked to construct a similar connection graph for these topics only (Fig. C.3 and Fig. C.4). Essay questions, as well as interviews, are used to check that students who *say* they can explain a topic *can*, in fact, do so.

Interim A, B

Between Session A and Session B, the connection graphs are computer processed to give a pruned version in which the analogical convention is inserted.

Session B

At the start of the training session, the computer processed, individual connection graphs are returned as "feedback" and are used in the "learning to learn" exercises. Further and more sophisticated structures are built up as the various principles are introduced. Amongst other things, the distinction between formal and analogical derivations is established.

A = Law of 3
B = Law of 7
C = Collig + 1
D = Collig + 2
E = Activation
F = Active force
G = Passive force
H = Neutral force
I = Animus
J = Anima
K = Tao
L = Yan
M = Ying
N = Hsing
O = Ming
P = Interpen of opps.
Q = Neg of neg
R = Trans quant to qual
S = Satori + 48
T = Satori + 24
U = Satori + 12
V = Satori + 6
W = Satori + 3
Y = Predestination

a = recepts
b = percepts
c = concepts
d = self consciousness
e = cosmic consciousness
f = simple consciousness
g = self
h = God
i = taboo of taboos
j = unit of evolution
k = creatura
l = intelligent universe
m = intuition
n = cybernetic system
o = unit of mind
p = unit of information
q = diff that makes a diff
r = difference
s = sap
t = programming
u = territory
v = any concepts

		Student 1	Student 2	
Session A		1	2	All topics
		3	4	+ only topics
Session C		5	6	All topics
		7	8	+ only topics

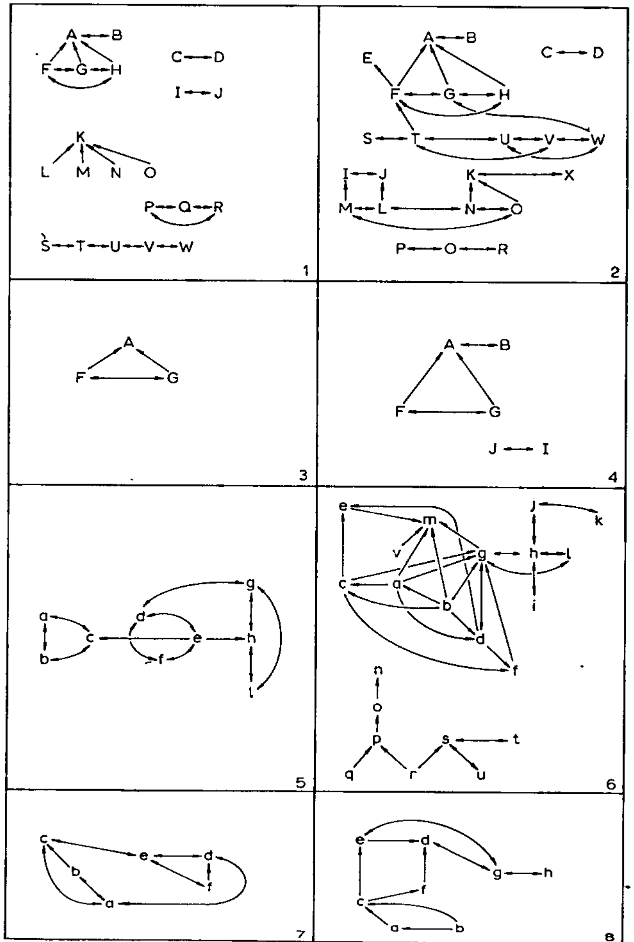


Fig. C1—8.

Interim B, C

Graphs obtained from the students during the training session B are individually processed and returned as "feedback" before the students start to learn the Session C texts.

Session C

After studying the texts, students are subjected to a repetition of the procedures described for Session A. At this stage, they have a more sophisticated repertoire of representational techniques and have been exposed to the "learning to learn" training of Session B. Clearly, the representation skill and "learning to learn" are interrelated, but not identical. Typical connection graphs are shown in Fig. C.5 and Fig. C.6 ("all" topics) and Fig. C.7 and Fig. C.8 (+ marked topics).

Revisiting

These graphs are personally processed and returned as "feedback" somewhat later. If possible, we use "feedback" delivery to ask for repertory grid descriptors elicited over terms of the analogies, and students who cooperate in this matter assign values to their own descriptors.

Discussion

Apart from comparing factual and explanatory responses, it is possible to obtain indices of complexity and coherence over the individual connection graphs. Hence, we are in a position to observe "learning to learn" (students for whom increased understanding after the training session is accompanied by an ability to represent the subject matter), different types of representation, and when they occur, the defects in learning such as "Globetrotting". The latter condition, for example, is detected by noting no difference between the "can explain" connectivity and the "all topic connectivity"; at a more detailed level, by specific "false analogy" patterns. Such deficiencies are generally reduced by training in the "learning to learn" session, and for some of the students virtually eliminated.

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